ALGEBRAS GROUPS AND GEOMETRIES 35 283 -318 (2018)
DOI 10.29083/AGG.35.03.2018

FOUR INDEPENDENT EIGENVECTORS AND THEIR PROPERTIES FOR THE PROBLEM OF ACOUSTIC 4P-SH-WAVE PROPAGATION INCORPORATING GRAVITATIONAL PHENOMENA

Aleksey Anatolievich Zakharenko
International Institute of Zakharenko Waves (IIZWs)
660014, ul. Chaikovskogo, 20-304
Krasnoyarsk, Russia
aazaaz@inbox.ru

Received October 21, 2019

Abstract

This theoretical investigation has obtained four independent pairs of main eigenvectors called the *E*-, *H*-, *G*-, and *F*-eigenvectors. Their properties are also demonstrated. The obtained eigenvectors together with the eigenvalues represent the solutions of the coupled equations of motion when the shear-horizontal (SH) acoustic wave propagation in the 6 *mm* solids is coupled with the electrical, magnetic, gravitational, and cogravitational potentials, i.e. 4P-SH-waves. To obtain suitable forms of the eigenvectors is very important because they form the final solutions of the propagation velocities, with which the acoustic waves propagate in the solids. It is expected that their proper forms can demonstrate dominating factors causing the acoustic wave propagation, i.e. the physical properties of the wave processes. To know the acoustic wave propagation velocities in the solids is one of very important engineering problems in the signal processing technology. This allows the constitution of various technical devices such as filters, sensors, delay lines, energy harvesting and wireless devices, etc. The successful development of the theoretical, mathematical, experimental, engineering, and technological directions of investigations of the four-potential waves and their applications can result in the instant interplanetary (interstellar and even intergalactic) communications, for instance, the instant interplanetary Internet. This is possible because the evaluated speeds of the recently discovered fast four-potential waves propagating in a vacuum can be thirteen orders faster than the speed of light, i.e. the speed of the electromagnetic waves representing two-potential waves.

Keywords: Transversely isotropic continuous media; Acoustic wave propagation; Gravitational effects; Four potential coupling problem.

1 Introduction

In the jubilee year 2016, it was developed the theory [1] that incorporates the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems in the wave motion process. As a result, the new four-potential shear-horizontal surface acoustic wave (4P-SH-SAW) [1] was discovered when the acoustic wave propagates in the transversely isotropic material of symmetry class 6 *mm*. It was a centenary jubilee of the prediction of the existence of the gravitational waves by Albert Einstein in 1916 [2]. To this jubilee, a very large international research team has experimentally proved the existence of the gravitational waves. Their experimental results [3] published in 2016 have soundly confirmed the existence of the theoretically predicted gravitational waves.

Developed theory [1] uses this experimental fact of the propagation of the gravitational waves in the free space (vacuum) at the speed of light. It is well-known that the electromagnetic waves also propagate in a vacuum at the speed of light. The speed of light was precisely measured only in the 1970s and now can be found in any modern reference book on physics. For instance, the measured value of the speed of light can be found in popular reference book [4]. So, it is possible to write down the following value of the speed of light: $C_L = (\varepsilon_0\mu_0)^{-1/2} \sim (\gamma_0\eta_0)^{-1/2} = 2.99792458 \times 10^8$ [m/s], where ε_0, μ_0, γ_0, and η_0 are the electric constant, magnetic constant, gravitic constant, and cogravitic constant for a vacuum, respectively. A gravitational and electromagnetic analogy was first stated by Heaviside [5] and then developed by many researchers. The famous book by Jefimenko [6] operates with gravitation and cogravitation to develop Newton's theory of gravitation to its physical and mathematical conclusion. So, the purely gravitational analogy to the theory of electromagnetism is now called the theory of the gravitoelectromagnetism. It is expected that the gravitic constant γ_0 and the cogravitic constant η_0 for a vacuum can be precisely measured in this century and then recorded in physics reference books.

Created theory [1] relating to numerical experiment takes into account the fact that the speed of the electromagnetic waves in both a solid and a vacuum is five orders faster than the speed of any acoustic wave in a solid. As a result, a quasi-static approximation is used in the theory of acoustic wave propagation in continuous solid media. In the solids, the slow speed of the new 4P-SH-SAW [1] can naturally depend on the speed of the electromagnetic wave. However, it also depends on the speed of new fast waves that can be thirteen orders faster than the speed of light. These new fast waves can also propagate in a vacuum with the following speeds: $\Lambda_{01} = (\zeta_0 \lambda_0)^{-1/2} \to 10^{13} C_L$ and $\Lambda_{02} = (\xi_0 \beta_0)^{-1/2} \to 10^{13} C_L$, where the vacuum parameters ζ_0, λ_0, ξ_0, and β_0 are called the gravitoelectric, cogravitomagnetic, cogravitoelectric, and gravitomagnetic constants, respectively. These vacuum parameters relating to the gravitational phenomena must be also measured and recorded in any modern reference book on physics.

The existence of the new fast waves propagating in a vacuum with the speeds Λ_{01} and Λ_{02} can be true because the famous Russian astronomer and astrophysicist Kozyrev [7] has created his own tools, with which he has observed true positions of stars from his observatory. It is obvious that astronomers use optic tools to observe stars. As a result, they can for instance observe a star distant from the Earth on several light years at its position it was several light years ago. There are a lot of stars distant from the Earth on 10, 100, 1000 light years. Therefore, any optic tools for observation of stars cannot record true positions of the stars. To observe the true position of any star, Kozyrev [7] has first used some optic tools to observe a star. After that, he has calculated the true position of the star and used his own tools to observe this star at the true position. This means that there is something that can propagate in a vacuum significantly faster than the speed of light. To explain this phenomenon, Kozyrev has developed his own theory called the theory of time.

Kozyrev is not the single researcher who has found that there is something significantly faster than the speed of light. The French mathematician Pierre-Simon Laplace [8], also known as the French Newton, has evaluated the stability of the Solar system. He has investigated the motion of space bodies in the Solar system and concluded that gravity must propagate millions of times faster than light, since otherwise violation of Newton's law of universal gravitation would be observed. It is worth noting that Newton's law of universal gravitation assumes an infinitely large speed of gravity, i.e. the gravity can instantly propagate. However, the first attempt to explain Newton's law of gravitation was done by Nicolas Fatio de Duillier in 1690 and by Georges-Louis Le Sage in 1748. This is a kinetic theory of gravity originally proposed by Fatio and later developed by Le Sage. The theory proposed a mechanical explanation for Newton's gravitational force in terms of streams of tiny unseen particles impacting all material objects from all directions [9]. Concerning the movements of planets in the Solar system in the fluid consisting of the particles (i.e. in the free space), Le Sage [9] has used an argument to demonstrate that the speed of the fluid must be thirteen orders faster than the speed of light. To review all existing evaluations of the speed of gravity and possible gravitational phenomena does not represent the main purpose of this study. Therefore, it is necessary to return to modern theory [1] because the thermodynamic functions and the thermodynamic variables were used to develop this theory.

The hottest application of the new fast waves propagating in a vacuum at the speeds Λ_{01} and Λ_{02} is the instant interplanetary communication [10, 11] inside the Solar system, for instance, the space Internet. The possible applications proposed in paper [10] for the new fast waves propagating in the continuous medium such as a vacuum is the natural consequence from theory [1] developed for the continuous medium such as a solid. Theory [1] considers the acoustic wave propagation in the smart solid materials such as the piezoelectromagnetics (magnetoelecroelastics) with taking into account the gravitational phenomena. In this case, the gravitational phenomena significantly complicate the theoretical developments compared with the case of pure piezoelectromagnetics without the phenomena [12, 13, 14]. However, it is necessary to treat the gravitational phenomena because the propagation of acoustic waves means energy transmission between the wave generator and the wave detector. One century ago Einstein has postulated that any kind of energy (and any change in energy) is coupled with gravitation. Therefore, theory [1] treats the gravitational and cogravitational subsystems in addition to the mechanical, electrical, and magnetic subsystems. As a result, the wave motions are caused by exchange interactions (i.e. some energetic exchanges) among the subsystems. It is expected that such energetic system consisting of five subsystems can be more reach than the purely piezoelectromagnetic system consisting of only three subsystems such as the mechanical, electrical, and magnetic subsystems. It is natural because piezoelectromagnetics with (without) taking into account the gravitational phenomena can be used instead of pure piezoelectrics.

Today, piezoelectrics are utilized in various technical devices such as filters, (chemical and biological) sensors, delay lines, energy harvesting and wireless devices, etc. They are also used in the cutting-edge technologies of mobile communication and wireless sensors [15, 16]. The piezoelectric materials are also studied for applications in energy harvesting devices [16, 17, 18, 19]. However, smart piezoelectromagnetic materials are more preferable for constitution of the wireless devices [20, 21, 22] in comparison with the conventional piezoelectrics. It is also expected that the piezogravitocogravitoelectromagnetic materials [1] with the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems can be more reach regarding the energy harvesting and energy conversion. Recently, Füzfa [23] has found a method to study interactions of the electrical or magnetic subsystem with the gravitational or cogravitational subsystem at the laboratory conditions on the Earth instead of very expensive space experiments. The theoretical, mathematical, experimental, engineering, and many relative investigations can be useful for the development of infrastructure including the technical devices for generating, receiving, and signal processing of the new fast waves for the new era of the instant interplanetary communication.

The theoretical investigations were initiated in pioneer paper [1]. However, this created theory has released some mathematical problems that must be also resolved. One of the most important mathematical problems is the finding of proper eigenvectors because their forms can result in final solutions for the propagating velocities of the acoustic waves. Theoretical work [24] has demonstrated only half of the twenty-four possible eigenvectors that can be obtained in the framework of theory [1]. However, all of them can be reduced to four main independent eigenvectors. To obtain these four eigenvectors called the *E*-, *H*-, *G*-, and *F*-eigenvectors and their properties is the main purpose of this study. The theoretical part and mathematical methods are very important even in the case of the acoustic wave propagation in the pure piezoelectrics. To find propagation velocities in different structures represents an engineering problem. There is famous book by Dieulesaint and Royer [25] concerning applications of the elastic waves in the piezoelectric solids to signal processing. Therefore, original theory [1] represents a rudiment for further development of the theory of the acoustic wave propagation in the piezogravitocogravitoelectromagnetic solids guided by the surface [1], interface [26], and plate (thin film) [27]. In all these cases, it is very important to know eigenvalues and eigenvectors representing the solutions of the corresponding coupled equations of motion.

2 The equations of motion and solutions

To construct the coupled equations of motion, it is necessary first of all to treat the suitable thermodynamic functions and thermodynamic variables for an adiabatic process in solid continuous media. For the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems, the thermodynamic functions are the mechanical stress, electrical induction, magnetic induction, gravitational induction, and cogravitational induction. The thermodynamic variables are the mechanical strain, electrical field, magnetic field, gravitational field, and cogravitational field. So, this is a multidisciplinary research arena. The mechanical parameters came from the theory of elasticity, the electrical and magnetic parameters from the theory of electromagnetism, and the gravitational and cogravitational parameters from the theory of gravitoelectromagnetism. There is some analogy between the theories of gravitation and the electromagnetism [5]. As a result, some analogy can be found between the theories of electricity and gravitation (i.e. gravitoelectricity) and between the theories of magnetism and cogravitation [6] (i.e. gravitomagnetism [28]). This part of theory is quite large and the reader can find it in paper [1]. However, it is also necessary to mention that the quasi-static approximation must be applied to obtain the coupled equations of motion in the form of a set of second-order partial differential equations. This approximation is used because the speed of the electromagnetic waves in a solid is close to the speed of light in a vacuum and five orders faster than the speed of any acoustic wave.

To resolve the second-order partial differential equations, it is natural to exploit the plane wave solutions. These solutions naturally transform the differential form of the coupled equations of motion into the tensor form of the equations. In the common case of the acoustic wave propagation in an anisotropic solid, the tensor form can be written down in the following compact expression called the modified Green-Christoffel equation [1]: $\left(GL_{IJ} - \delta_{IJ}\rho V_{ph}\right)U_I^0 = 0$, where the indices I and J run from 1 to 7 and ρ is the mass density listed in table 1. The phase velocity V_{ph} will be defined below by expression (7). GL_{IJ} stands for the components of the modified symmetric tensor [1] and δ_{IJ} represents the Kronecker delta-function with the following conditions: $\delta_{IJ} = 1$ for $I = J < 4$, $\delta_{IJ} = 0$ for $I \neq J$, and $\delta_{44} = \delta_{55} = \delta_{66} = \delta_{77} = 0$. Also, parameters U_I^0 represent the components of the eigenvector $(U_1^0, U_2^0, U_3^0, U_4^0, U_5^0, U_6^0, U_7^0)$. This tensor form of the coupled equations of motion represents the common problem for determination of the eigenvalues and eigenvectors. This common problem is very complicated and can be numerically resolved. However there are the high-symmetry propagation directions in solid materials with certain crystal symmetries [25, 29] that allow significant simplifications leading to some analytical solutions.

Table 1: The material parameters of the transversely isotropic material (crystal) of symmetry class 6 *mm* and their physical dimensions.

Material parameter	**Symbol [dimension]**
Mass density	ρ [kg/m^3]
Elastic stiffness constant	$C = C_{44} = C_{66}$ [kg/(m×s^2)], [N/m^2]
Piezoelectric constant	$e = e_{16} = e_{34}$ [kg$^{1/2}$/m$^{3/2}$], [C/m^2]
Piezomagnetic coefficient	$h = h_{16} = h_{34}$ [kg$^{1/2}$/(m$^{1/2}$×s)], [N/(A×m)], [T]
Piezogravitic constant	$g = g_{16} = g_{34}$ [kg/m^2]
Piezocogravitic coefficient	$f = f_{16} = f_{34}$ [s^{-1}]
Electric constant	$\varepsilon = \varepsilon_{11} = \varepsilon_{33}$ [s^2/m^2], [F/m]
Magnetic constant	$\mu = \mu_{11} = \mu_{33}$ [-], [N×s^2/C^2], [H/m], [N/A^2], [T×m/A], [Wb/(A×m)], [V×s/(A×m)]
Electromagnetic constant	$\alpha = \alpha_{11} = \alpha_{33}$ [s/m], [N×s/(V×C)]
Gravitic constant	$\gamma = \gamma_{11} = \gamma_{33}$ [kg×s^2/m^3], [kg^2/(N×m^2)]
Cogravitic constant	$\eta = \eta_{11} = \eta_{33}$ [m/kg]
Gravitocogravitic constant	$\vartheta = \vartheta_{11} = \vartheta_{33}$ [s/m]
Gravitoelectric constant	$\zeta = \zeta_{11} = \zeta_{33}$ [kg$^{1/2}$×s^2/m$^{5/2}$], [C×kg/(J×m)]
Cogravitoelectric constant	$\xi = \xi_{11} = \xi_{33}$ [s/(kg$^{1/2}$×m$^{1/2}$)], [m/Wb]
Gravitomagnetic constant	$\beta = \beta_{11} = \beta_{33}$ [kg$^{1/2}$×s/m$^{3/2}$], [T×kg×m/J]
Cogravitomagnetic constant	$\lambda = \lambda_{11} = \lambda_{33}$ [m$^{1/2}$/kg$^{1/2}$], [T×m^3/(C×Wb)]

For a solid of symmetry class 6 *mm*, the high-symmetry propagation directions on the certain cuts of the solid are well-known. According to review paper [30], the sixfold symmetry axis must be directed along the x_2-axis, perpendicular to both the wave propagation direction towards the x_1-axis and the surface normal along the x_3-axis. The rectangular coordinate system $\{x_1, x_2, x_3\}$ and the corresponding wavevector components $\{k_1, k_2, k_3\}$ are used. For this case, the modified Green-Christoffel equation representing a set of seven homogeneous equations splits into two independent sets. The first set of two homogeneous equations with the eigenvector (U_1^0, U_3^0) corresponds to the purely mechanical wave with the in-plane polarization. The second set of five homogeneous equations with the eigenvector $(U_2^0, U_4^0, U_5^0, U_6^0, U_7^0)$ represents the main interest in this study because here the acoustic wave propagation is coupled with the following four potentials: the electrical φ, magnetic ψ, gravitational Φ, and cogravitational Ψ potentials. This is the case of propagation of the anti-plane polarized four-potential shear-horizontal (4P-SH) acoustic waves.

For this case, it is possible to deal only with the suitable *GL*-tensor components of the modified Green-Christoffel equation [1]. Therefore, the coupled equations of motion describing the 4P-SH-wave propagation coupled with the four potentials can be then expressed by the following five homogeneous equations:

$$\begin{pmatrix} GL_{22}-\rho V_{ph}^2 & GL_{24} & GL_{25} & GL_{26} & GL_{27} \\ GL_{42} & GL_{44} & GL_{45} & GL_{46} & GL_{47} \\ GL_{52} & GL_{54} & GL_{55} & GL_{56} & GL_{57} \\ GL_{62} & GL_{64} & GL_{65} & GL_{66} & GL_{67} \\ GL_{72} & GL_{74} & GL_{75} & GL_{76} & GL_{77} \end{pmatrix} \begin{pmatrix} U_2^0 \\ U_4^0 \\ U_5^0 \\ U_6^0 \\ U_7^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (1)$$

With the material parameters listed in table 1, equations (1) must be rewritten as follows:

$$\begin{pmatrix} C\left[m-(V_{ph}/V_{t4})^2\right] & em & hm & gm & fm \\ em & -\varepsilon m & -\alpha m & -\zeta m & -\xi m \\ hm & -\alpha m & -\mu m & -\beta m & -\lambda m \\ gm & -\zeta m & -\beta m & -\gamma m & -\vartheta m \\ fm & -\xi m & -\lambda m & -\vartheta m & -\eta m \end{pmatrix} \begin{pmatrix} U^0 \\ \phi^0 \\ \psi^0 \\ \Phi^0 \\ \Psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (2)$$

where

$$m = 1 + n_3^2 \qquad (3)$$

$$n_3 = k_3/k \qquad (4)$$

$$(U^0, \phi^0, \psi^0, \Phi^0, \Psi^0) = (U_2^0, U_4^0, U_5^0, U_6^0, U_7^0) \qquad (5)$$

$$V_{t4} = \sqrt{C/\rho} \qquad (6)$$

$$V_{ph} = \omega/k \tag{7}$$

All the suitable eigenvalues n_3 and the corresponding eigenvectors respectively defined by expressions (4) and (5) must be obtained by resolving equation (2). The velocity V_{t4} defined by expression (6) represents the speed of the purely mechanical bulk SH-wave. The phase velocity V_{ph} defined by expression (7) is proportional to the angular frequency ω and inversely proportional to the wavenumber k in the propagation direction.

Expanding the determinant of the coefficient matrix in equations (2), it is possible to obtain a secular equation representing a polynomial that must be equal to zero. To find all the possible polynomial roots n_3, it is convenient to rewrite the polynomial in the form of several suitable cofactors in order that each cofactor contains two polynomial roots n_3. Therefore, it is possible to write down the following convenient form consisting of five cofactors:

$$m \times m \times m \times m \times \det(V_{ph}) = 0 \tag{8}$$

where

$$\det(V_{ph}) = \begin{vmatrix} C\left[m - (V_{ph}/V_{t4})^2\right] & e & h & g & f \\ em & -\varepsilon & -\alpha & -\zeta & -\xi \\ hm & -\alpha & -\mu & -\beta & -\lambda \\ gm & -\zeta & -\beta & -\gamma & -\vartheta \\ fm & -\xi & -\lambda & -\vartheta & -\eta \end{vmatrix} \tag{9}$$

It is clearly seen that the first four cofactors in equation (8) are identical. Therefore, they actually release the following four pairs of identical eigenvalues:

$$n_3^{(1,2)} = n_3^{(3,4)} = n_3^{(5,6)} = n_3^{(7,8)} = \pm j \tag{10}$$

where $j = \sqrt{-1}$ is the imaginary unity.

Expanding determinant (9) leads to the following fifth pair of the eigenvalues:

$$n_3^{(9,10)} = \pm j\sqrt{1 - (V_{ph}/V_{temgc})^2} \tag{11}$$

where

$$V_{temgc} = \sqrt{C/\rho}\left(1 + K_{emgc}^2\right)^{1/2} \quad (12)$$

$$K_{emgc}^2 = \frac{A_1}{A_2} \quad (13)$$

$$A_1 = e^2(\mu\gamma\eta + 2\beta\lambda\vartheta - \lambda^2\gamma - \beta^2\eta - \vartheta^2\mu) + h^2(\varepsilon\gamma\eta + 2\zeta\xi\vartheta - \vartheta^2\varepsilon - \zeta^2\eta - \xi^2\gamma) + g^2(\varepsilon\mu\eta + 2\alpha\xi\lambda - \lambda^2\varepsilon - \alpha^2\eta - \xi^2\mu) + f^2(\varepsilon\mu\gamma + 2\alpha\beta\zeta - \beta^2\varepsilon - \alpha^2\gamma - \zeta^2\mu) + 2eh(\vartheta^2\alpha + \zeta\beta\eta + \xi\gamma\lambda - \alpha\gamma\eta - \zeta\lambda\vartheta - \xi\beta\vartheta) + 2eg(\alpha\beta\eta + \lambda^2\zeta + \xi\vartheta\mu - \alpha\lambda\vartheta - \zeta\mu\eta - \xi\beta\lambda) + 2ef(\alpha\gamma\lambda + \zeta\vartheta\mu + \beta^2\xi - \alpha\beta\vartheta - \zeta\beta\lambda - \xi\mu\gamma) + 2hg(\varepsilon\lambda\vartheta + \zeta\alpha\eta + \xi^2\beta - \varepsilon\eta\beta - \zeta\lambda\xi - \xi\vartheta\alpha) + 2hf(\varepsilon\beta\vartheta + \zeta^2\lambda + \xi\alpha\gamma - \varepsilon\lambda\gamma - \zeta\vartheta\alpha - \xi\zeta\beta) + 2gf(\varepsilon\beta\lambda + \alpha^2\vartheta + \xi\mu\zeta - \varepsilon\mu\vartheta - \alpha\zeta\lambda - \alpha\beta\xi) \quad (14)$$

$$A_2 = C(\varepsilon\mu - \alpha^2)(\gamma\eta - \vartheta^2) + C(\beta^2\xi^2 - \xi^2\mu\gamma - \beta^2\varepsilon\eta) + C(\lambda^2\zeta^2 - \lambda^2\varepsilon\gamma - \zeta^2\mu\eta) + 2C(\gamma\alpha\xi\lambda + \eta\alpha\beta\zeta + \varepsilon\beta\lambda\vartheta + \mu\zeta\xi\vartheta - \zeta\xi\beta\lambda - \alpha\zeta\lambda\vartheta - \alpha\beta\xi\vartheta) \quad (15)$$

Table 2: The physical dimensions of factors M_{ee}, M_{hh}, M_{gg}, M_{ff}, M_{eh}, M_{eg}, M_{ef}, M_{hg}, M_{hf}, M_{gf} and CEMGCMC coupling mechanisms M_1, M_2, M_3, M_4, M_5.

Factor	Dimension
M_{ee}	s^2/m^2
M_{hh}	s^4/m^4
M_{gg}	$s^2/(kg\times m)$
M_{ff}	$kg\times s^4/m^5$
M_{eh}	s^3/m^3
M_{eg}	$s^2/(kg^{1/2}\times m^{3/2})$
M_{ef}	$kg^{1/2}\times s^3/m^{7/2}$
M_{hg}	$s^3/(kg^{1/2}\times m^{5/2})$
M_{hf}	$kg^{1/2}\times s^4/m^{9/2}$
M_{gf}	s^3/m^3
M_1	$kg^{1/2}\times s^2/m^{7/2}$
M_2	$kg^{1/2}\times s^3/m^{9/2}$
M_3	s^2/m^3
M_4	$kg\times s^3/m^5$
M_5	s^4/m^4
eM_1, hM_2, gM_3, fM_4, CM_5	$kg\times s^2/m^5$

In definition (12), the velocity V_{temgc} of the shear-horizontal bulk acoustic wave (SH-BAW) is coupled with the electrical φ, magnetic ψ, gravitational Φ, and cogravitational Ψ potentials. This velocity depends on the nondimensional parameter K^2_{emgc} (13) called the coefficient of the electromagnetogravitocogravitomechanical coupling (CEMGCMC). It is useful to rewrite the parameter K^2_{emgc} (13) as a function of the CEMGCMC coupling mechanisms M_1, M_2, M_3, M_4, and M_5. Their physical dimensions are listed in table 2. So, the parameter K^2_{emgc} reads:

$$K^2_{emgc} = \frac{eM_1+hM_2+gM_3+fM_4}{CM_5} \tag{16}$$

where

$$M_1 = eM_{ee} + hM_{eh} + gM_{eg} + fM_{ef} \tag{17}$$

$$M_2 = eM_{eh} + hM_{hh} + gM_{hg} + fM_{hf} \tag{18}$$

$$M_3 = eM_{eg} + hM_{hg} + gM_{gg} + fM_{gf} \tag{19}$$
$$M_4 = eM_{ef} + hM_{hf} + gM_{gf} + fM_{ff} \tag{20}$$

$$\begin{aligned} M_5 &= \varepsilon M_{ee} + \alpha M_{eh} + \zeta M_{eg} + \xi M_{ef} = \alpha M_{eh} + \mu M_{hh} + \beta M_{hg} + \lambda M_{hf} = \\ &\zeta M_{eg} + \beta M_{hg} + \gamma M_{gg} + \vartheta M_{gf} = \xi M_{ef} + \lambda M_{hf} + \vartheta M_{gf} + \eta M_{ff} = \\ &(\varepsilon\mu - \alpha^2)(\gamma\eta - \vartheta^2) + (\beta\xi - \lambda\zeta)^2 - (\xi^2\mu\gamma + \beta^2\varepsilon\eta + \lambda^2\varepsilon\gamma + \zeta^2\mu\eta) + \\ &2(\gamma\alpha\xi\lambda + \eta\alpha\beta\zeta + \varepsilon\beta\lambda\vartheta + \mu\zeta\xi\vartheta - \alpha\zeta\lambda\vartheta - \alpha\beta\xi\vartheta) \end{aligned} \tag{21}$$

because

$$M_{ee} = \mu\gamma\eta + 2\beta\lambda\vartheta - \lambda^2\gamma - \beta^2\eta - \vartheta^2\mu \tag{22}$$

$$M_{hh} = \varepsilon\gamma\eta + 2\zeta\xi\vartheta - \vartheta^2\varepsilon - \zeta^2\eta - \xi^2\gamma \tag{23}$$

$$M_{gg} = \varepsilon\mu\eta + 2\alpha\xi\lambda - \lambda^2\varepsilon - \alpha^2\eta - \xi^2\mu \tag{24}$$

$$M_{ff} = \varepsilon\mu\gamma + 2\alpha\beta\zeta - \beta^2\varepsilon - \alpha^2\gamma - \zeta^2\mu \tag{25}$$

$$M_{eh} = \zeta\beta\eta + \xi\gamma\lambda + \vartheta^2\alpha - \alpha\gamma\eta - \zeta\lambda\vartheta - \xi\beta\vartheta \qquad (26)$$

$$M_{eg} = \alpha\beta\eta + \xi\vartheta\mu + \lambda^2\zeta - \alpha\lambda\vartheta - \zeta\mu\eta - \xi\beta\lambda \qquad (27)$$

$$M_{ef} = \alpha\gamma\lambda + \zeta\vartheta\mu + \beta^2\xi - \alpha\beta\vartheta - \zeta\beta\lambda - \xi\mu\gamma \qquad (28)$$

$$M_{hg} = \varepsilon\lambda\vartheta + \zeta\alpha\eta + \xi^2\beta - \alpha\xi\vartheta - \zeta\lambda\xi - \varepsilon\eta\beta \qquad (29)$$

$$M_{hf} = \varepsilon\beta\vartheta + \xi\alpha\gamma + \zeta^2\lambda - \alpha\zeta\vartheta - \zeta\xi\beta - \varepsilon\lambda\gamma \qquad (30)$$

$$M_{gf} = \varepsilon\beta\lambda + \xi\mu\zeta + \alpha^2\vartheta - \alpha\zeta\lambda - \alpha\beta\xi - \varepsilon\mu\vartheta \qquad (31)$$

The physical dimensions of parameters (22)-(31) are listed in table 2. To obtain the eigenvector components, it is necessary to resolve equation (2). This was done in paper [24]. However, paper [24] has demonstrated only half of twenty-four possible eigenvectors. Moreover, each of the twenty-four eigenvectors can be reduced to one of four main independent eigenvectors that will be obtained together with their properties in the next four sections. These independent eigenvectors are called the *F*-eigenvectors, *G*-eigenvectors, *H*-eigenvectors, and *E*-eigenvectors. In conclusion to this section, it is useful to write down some useful expressions for the nondimensional material parameters defined below by equalities from (32) to (41). These material parameters are used in the following four sections. For them, it is necessary to mention that the coefficient K^2_{emgc} (CEMGCMC) is defined by expression (16) and all the material constants are listed in table 1. These nondimensional material parameters read:

$$K_E = 1 - \frac{K_e^2}{K_{emgc}^2}, K_e^2 = \frac{e^2}{C\varepsilon} \qquad (32)$$

$$K_M = 1 - \frac{K_m^2}{K_{emgc}^2}, K_m^2 = \frac{h^2}{C\mu} \qquad (33)$$

$$K_G = 1 - \frac{K_g^2}{K_{emgc}^2}, K_g^2 = \frac{g^2}{C\gamma} \qquad (34)$$

$$K_F = 1 - \frac{K_f^2}{K_{emgc}^2}, K_f^2 = \frac{f^2}{C\eta} \quad (35)$$

$$K_A = 1 - \frac{K_\alpha^2}{K_{emgc}^2}, K_\alpha^2 = \frac{eh}{C\alpha} \quad (36)$$

$$K_T = 1 - \frac{K_\vartheta^2}{K_{emgc}^2}, K_\vartheta^2 = \frac{gf}{C\vartheta} \quad (37)$$

$$K_B = 1 - \frac{K_\beta^2}{K_{emgc}^2}, K_\beta^2 = \frac{hg}{C\beta} \quad (38)$$

$$K_Z = 1 - \frac{K_\zeta^2}{K_{emgc}^2}, K_\zeta^2 = \frac{eg}{C\zeta} \quad (39)$$

$$K_S = 1 - \frac{K_\xi^2}{K_{emgc}^2}, K_\xi^2 = \frac{ef}{C\xi} \quad (40)$$

$$K_L = 1 - \frac{K_\lambda^2}{K_{emgc}^2}, K_\lambda^2 = \frac{hf}{C\lambda} \quad (41)$$

3 The *F*-eigenvectors and their properties

The first pair of the eigenvectors called the first and second *F*-eigenvectors is defined below by expressions (42) and (43), respectively. Eigenvector (42) relates to eigenvalues (10) and eigenvector (43) relates to eigenvalues (11). For eigenvector (42), the eigenvector components correspondingly depend on the parameters M_{ef}, M_{hf}, M_{gf}, and M_{ff} that are defined by expressions (28), (30), (31), and (25), respectively, and contain the subscript "*f*" symbolizing the *F*-eigenvectors. So, the *F*-eigenvectors can be written in the following convenient forms:

$$\begin{pmatrix} U^{0(1)} \\ \phi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(2)} \\ \phi^{0(2)} \\ \psi^{0(2)} \\ \Phi^{0(2)} \\ \Psi^{0(2)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \phi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(4)} \\ \phi^{0(4)} \\ \psi^{0(4)} \\ \Phi^{0(4)} \\ \Psi^{0(4)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \phi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(6)} \\ \phi^{0(6)} \\ \psi^{0(6)} \\ \Phi^{0(6)} \\ \Psi^{0(6)} \end{pmatrix} =$$

$$\begin{pmatrix} U^{0(7)} \\ \phi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix} = \begin{pmatrix} U^{0(8)} \\ \phi^{0(8)} \\ \psi^{0(8)} \\ \Phi^{0(8)} \\ \Psi^{0(8)} \end{pmatrix} = \begin{pmatrix} U_f^{0(7)} = 0 \\ \phi_f^{0(7)} = M_{ef} \\ \psi_f^{0(7)} = M_{hf} \\ \Phi_f^{0(7)} = M_{gf} \\ \Psi_f^{0(7)} = M_{ff} \end{pmatrix} \qquad (42)$$

$$\begin{pmatrix} U^{0(9)} \\ \phi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^{0(10)} \\ \phi^{0(10)} \\ \psi^{0(10)} \\ \Phi^{0(10)} \\ \Psi^{0(10)} \end{pmatrix} = \begin{pmatrix} U_f^{0(9)} \\ \phi_f^{0(9)} \\ \psi_f^{0(9)} \\ \Phi_f^{0(9)} \\ \Psi_f^{0(9)} \end{pmatrix} \qquad (43)$$

where

$$U_f^{0(9)} = \frac{e\phi_f^{0(9)} + h\psi_f^{0(9)} + g\Phi_f^{0(9)} + f\Psi_f^{0(9)}}{CK_{emgc}^2} = \frac{M_4}{CK_{emgc}^2} \qquad (44)$$

$$\phi_f^{0(9)} = \gamma\alpha\lambda K_G K_A K_L + \mu\vartheta\zeta K_M K_T K_Z + \beta^2\xi K_B{}^2 K_S - \alpha\beta\vartheta K_A K_B K_T - \beta\lambda\zeta K_B K_L K_Z - \mu\gamma\xi K_M K_G K_S = \phi_f^{0(7)} - \gamma\alpha\lambda\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) - \mu\vartheta\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \beta^2\xi\left(2\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \alpha\beta\vartheta\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) + \beta\lambda\zeta\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \mu\gamma\xi\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) \qquad (45)$$

$$\psi_f^{0(9)} = \gamma\alpha\xi K_G K_A K_S + \lambda\zeta^2 K_L {K_Z}^2 + \varepsilon\beta\vartheta K_E K_B K_T - \alpha\vartheta\zeta K_A K_T K_Z - \beta\xi\zeta K_B K_S K_Z - \varepsilon\gamma\lambda K_E K_G K_L = \psi_f^{0(7)} - \gamma\alpha\xi\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_a^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) - \lambda\zeta^2\left(\frac{K_\lambda^2}{K_{emgc}^2} + 2\frac{K_\zeta^2}{K_{emgc}^2}\right) - \varepsilon\beta\vartheta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) + \alpha\vartheta\zeta\left(\frac{K_a^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \beta\xi\zeta\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \varepsilon\gamma\lambda\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) \quad (46)$$

$$\Phi_f^{0(9)} = \alpha^2\vartheta {K_A}^2 K_T + \mu\xi\zeta K_M K_S K_Z + \varepsilon\beta\lambda K_E K_B K_L - \alpha\lambda\zeta K_A K_L K_Z - \alpha\beta\xi K_A K_B K_S - \varepsilon\mu\vartheta K_E K_M K_T = \Phi_f^{0(7)} - \alpha^2\vartheta\left(2\frac{K_a^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) - \mu\xi\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \varepsilon\beta\lambda\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) + \alpha\lambda\zeta\left(\frac{K_a^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \alpha\beta\xi\left(\frac{K_a^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \varepsilon\mu\vartheta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) \quad (47)$$

$$\Psi_f^{0(9)} = 2\alpha\beta\zeta K_A K_B K_Z + \varepsilon\mu\gamma K_E K_M K_G - \gamma\alpha^2 K_G {K_A}^2 - \mu\zeta^2 K_M {K_Z}^2 - \varepsilon\beta^2 K_E {K_B}^2 = \Psi_f^{0(7)} - 2\alpha\beta\zeta\left(\frac{K_a^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \varepsilon\mu\gamma\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_m^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2}\right) + \gamma\alpha^2\left(\frac{K_g^2}{K_{emgc}^2} + 2\frac{K_a^2}{K_{emgc}^2}\right) + \mu\zeta^2\left(\frac{K_m^2}{K_{emgc}^2} + 2\frac{K_\zeta^2}{K_{emgc}^2}\right) + \varepsilon\beta^2\left(\frac{K_e^2}{K_{emgc}^2} + 2\frac{K_\beta^2}{K_{emgc}^2}\right) \quad (48)$$

It is clearly seen in expressions (45), (46), (47), and (48) that the eigenvector components $\phi_f^{0(9)}$, $\psi_f^{0(9)}$, $\Phi_f^{0(9)}$, and $\Psi_f^{0(9)}$ depend on the corresponding eigenvector components $\phi_f^{0(7)}$, $\psi_f^{0(7)}$, $\Phi_f^{0(7)}$, and $\Psi_f^{0(7)}$. This manifests that both eigenvectors (42) and (43) are naturally called the *F*-eigenvectors. The analogical reason was used for distinguishing of the rest three independent eigenvectors called the *G*-, *H*-, and *E*-eigenvectors in the following three sections.

With the CEMGCMC coupling mechanisms M_4 and M_5 defined by expressions (20) and (21), respectively, the properties of the corresponding components of the *F*-eigenvectors can be demonstrated by all the expressions written below in this section. For the linear combinations of the components of the first *F*-eigenvector, the following five expressions disclose some useful properties:

$$e\phi_f^{0(7)} + h\psi_f^{0(7)} + g\Phi_f^{0(7)} + f\Psi_f^{0(7)} = e\phi_f^{0(9)} + h\psi_f^{0(9)} + g\Phi_f^{0(9)} + f\Psi_f^{0(9)} = eM_{ef} + hM_{hf} + gM_{gf} + fM_{ff} = M_4 \quad (49)$$

$$\varepsilon\phi_f^{0(7)} + \alpha\psi_f^{0(7)} + \zeta\Phi_f^{0(7)} + \xi\Psi_f^{0(7)} = \varepsilon K_E\phi_f^{0(9)} + \alpha K_A\psi_f^{0(9)} + \zeta K_Z\Phi_f^{0(9)} + \xi K_S\Psi_f^{0(9)} = 0 \quad (50)$$

$$\alpha\phi_f^{0(7)} + \mu\psi_f^{0(7)} + \beta\Phi_f^{0(7)} + \lambda\Psi_f^{0(7)} = \alpha K_A\phi_f^{0(9)} + \mu K_M\psi_f^{0(9)} + \beta K_B\Phi_f^{0(9)} + \lambda K_L\Psi_f^{0(9)} = 0 \quad (51)$$

$$\zeta\phi_f^{0(7)} + \beta\psi_f^{0(7)} + \gamma\Phi_f^{0(7)} + \vartheta\Psi_f^{0(7)} = \zeta K_Z\phi_f^{0(9)} + \beta K_B\psi_f^{0(9)} + \gamma K_G\Phi_f^{0(9)} + \vartheta K_T\Psi_f^{0(9)} = 0 \quad (52)$$

$$\xi\phi_f^{0(7)} + \lambda\psi_f^{0(7)} + \vartheta\Phi_f^{0(7)} + \eta\Psi_f^{0(7)} = \xi M_{ef} + \lambda M_{hf} + \vartheta M_{gf} + \eta M_{ff} = M_5 \quad (53)$$

For the linear combinations of the components of the second *F*-eigenvector, the following equalities can be obtained:

$$\varepsilon\phi_f^{0(9)} + \alpha\psi_f^{0(9)} + \zeta\Phi_f^{0(9)} + \xi\Psi_f^{0(9)} = -\varepsilon K_E\phi_f^{0(7)} - \alpha K_A\psi_f^{0(7)} - \zeta K_Z\Phi_f^{0(7)} - \xi K_S\Psi_f^{0(7)} = \frac{e}{CK_{emgc}^2}\left(e\phi_f^{0(7)} + h\psi_f^{0(7)} + g\Phi_f^{0(7)} + f\Psi_f^{0(7)}\right) = \frac{eM_4}{CK_{emgc}^2} \quad (54)$$

$$\alpha\phi_f^{0(9)} + \mu\psi_f^{0(9)} + \beta\Phi_f^{0(9)} + \lambda\Psi_f^{0(9)} = -\alpha K_A\phi_f^{0(7)} - \mu K_M\psi_f^{0(7)} - \beta K_B\Phi_f^{0(7)} - \lambda K_L\Psi_f^{0(7)} = \frac{h}{CK_{emgc}^2}\left(e\phi_f^{0(7)} + h\psi_f^{0(7)} + g\Phi_f^{0(7)} + f\Psi_f^{0(7)}\right) = \frac{hM_4}{CK_{emgc}^2} \quad (55)$$

$$\zeta\phi_f^{0(9)} + \beta\psi_f^{0(9)} + \gamma\Phi_f^{0(9)} + \vartheta\Psi_f^{0(9)} = -\zeta K_Z\phi_f^{0(7)} - \beta K_B\psi_f^{0(7)} - \gamma K_G\Phi_f^{0(7)} - \vartheta K_T\Psi_f^{0(7)} = \frac{g}{CK_{emgc}^2}\left(e\phi_f^{0(7)} + h\psi_f^{0(7)} + g\Phi_f^{0(7)} + f\Psi_f^{0(7)}\right) = \frac{gM_4}{CK_{emgc}^2} \quad (56)$$

$$\xi K_S\phi_f^{0(9)} + \lambda K_L\psi_f^{0(9)} + \vartheta K_T\Phi_f^{0(9)} + \eta K_F\Psi_f^{0(9)} = 0 \quad (57)$$

Using property (57), it is possible to write down the following equalities:

$$\xi\phi_f^{0(9)} + \lambda\psi_f^{0(9)} + \vartheta\Phi_f^{0(9)} + \eta\Psi_f^{0(9)} = \xi\frac{K_\xi^2}{K_{emgc}^2}\phi_f^{0(9)} + \lambda\frac{K_\lambda^2}{K_{emgc}^2}\psi_f^{0(9)} + \vartheta\frac{K_\vartheta^2}{K_{emgc}^2}\Phi_f^{0(9)} + \eta\frac{K_f^2}{K_{emgc}^2}\Psi_f^{0(9)} = \frac{f}{CK_{emgc}^2}\left(e\phi_f^{0(9)} + h\psi_f^{0(9)} + g\Phi_f^{0(9)} + f\Psi_f^{0(9)}\right) = \frac{f}{CK_{emgc}^2}\left(e\phi_f^{0(7)} + h\psi_f^{0(7)} + g\Phi_f^{0(7)} + f\Psi_f^{0(7)}\right) = \xi\frac{K_\xi^2}{K_{emgc}^2}\phi_f^{0(7)} + \lambda\frac{K_\lambda^2}{K_{emgc}^2}\psi_f^{0(7)} + \vartheta\frac{K_\vartheta^2}{K_{emgc}^2}\Phi_f^{0(7)} + \eta\frac{K_f^2}{K_{emgc}^2}\Psi_f^{0(7)} = \frac{fM_4}{CK_{emgc}^2} \quad (58)$$

4 The *G*-eigenvectors and their properties

The explicit forms of the second pair of the independent eigenvectors called the *G*-eigenvectors are obtained below in expressions (59) and (60). For first *G*-eigenvector (59), the eigenvector components correspondingly depend on the parameters M_{eg}, M_{hg}, M_{gg}, and M_{gf} that are defined by expressions (27), (29), (24), and (31), respectively, and contain the subscript "*g*" symbolizing the *G*-eigenvectors. So, the first and second *G*-eigenvectors respectively read:

$$\begin{pmatrix} U^{0(1)} \\ \phi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(2)} \\ \phi^{0(2)} \\ \psi^{0(2)} \\ \Phi^{0(2)} \\ \Psi^{0(2)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \phi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(4)} \\ \phi^{0(4)} \\ \psi^{0(4)} \\ \Phi^{0(4)} \\ \Psi^{0(4)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \phi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(6)} \\ \phi^{0(6)} \\ \psi^{0(6)} \\ \Phi^{0(6)} \\ \Psi^{0(6)} \end{pmatrix} =$$

$$\begin{pmatrix} U^{0(7)} \\ \phi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix} = \begin{pmatrix} U^{0(8)} \\ \phi^{0(8)} \\ \psi^{0(8)} \\ \Phi^{0(8)} \\ \Psi^{0(8)} \end{pmatrix} = \begin{pmatrix} U_g^{0(7)} = 0 \\ \phi_g^{0(7)} = M_{eg} \\ \psi_g^{0(7)} = M_{hg} \\ \Phi_g^{0(7)} = M_{gg} \\ \Psi_g^{0(7)} = M_{gf} \end{pmatrix} \tag{59}$$

$$\begin{pmatrix} U^{0(9)} \\ \phi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^{0(10)} \\ \phi^{0(10)} \\ \psi^{0(10)} \\ \Phi^{0(10)} \\ \Psi^{0(10)} \end{pmatrix} = \begin{pmatrix} U_g^{0(9)} \\ \phi_g^{0(9)} \\ \psi_g^{0(9)} \\ \Phi_g^{0(9)} \\ \Psi_g^{0(9)} \end{pmatrix} \tag{60}$$

where

$$U_g^{0(9)} = \frac{e\phi_g^{0(9)} + h\psi_g^{0(9)} + g\Phi_g^{0(9)} + f\Psi_g^{0(9)}}{CK_{emgc}^2} = \frac{M_3}{CK_{emgc}^2} \tag{61}$$

$$\phi_g^{0(9)} = \eta\alpha\beta K_F K_A K_B + \mu\vartheta\xi K_M K_T K_S + \lambda^2\zeta {K_L}^2 K_Z - \alpha\vartheta\lambda K_A K_T K_L - \mu\eta\zeta K_M K_F K_Z - \beta\lambda\xi K_B K_L K_S = \phi_g^{0(7)} - \eta\alpha\beta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2}\right) - \mu\vartheta\xi\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) - \lambda^2\zeta\left(2\frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \alpha\vartheta\lambda\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) + \mu\eta\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \beta\lambda\xi\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) \tag{62}$$

$$\psi_g^{0(9)} = \eta\alpha\zeta K_F K_A K_Z + \beta\xi^2 K_B K_S{}^2 + \varepsilon\vartheta\lambda K_E K_T K_L - \alpha\vartheta\xi K_A K_T K_S - \lambda\xi\zeta K_L K_S K_Z - \varepsilon\eta\beta K_E K_F K_B = \psi_g^{0(7)} - \eta\alpha\zeta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \beta\xi^2\left(\frac{K_\beta^2}{K_{emgc}^2} + 2\frac{K_\xi^2}{K_{emgc}^2}\right) - \varepsilon\vartheta\lambda\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) + \alpha\vartheta\xi\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \lambda\xi\zeta\left(\frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \varepsilon\eta\beta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2}\right) \quad (63)$$

$$\Phi_g^{0(9)} = 2\alpha\lambda\xi K_A K_L K_S + \varepsilon\mu\eta K_E K_M K_F - \eta\alpha^2 K_F K_A{}^2 - \mu\xi^2 K_M K_S{}^2 - \varepsilon\lambda^2 K_E K_L{}^2 = \Phi_g^{0(7)} - 2\alpha\lambda\xi\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) - \varepsilon\mu\eta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_m^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2}\right) + \eta\alpha^2\left(\frac{K_f^2}{K_{emgc}^2} + 2\frac{K_\alpha^2}{K_{emgc}^2}\right) + \mu\xi^2\left(\frac{K_m^2}{K_{emgc}^2} + 2\frac{K_\xi^2}{K_{emgc}^2}\right) + \varepsilon\lambda^2\left(\frac{K_e^2}{K_{emgc}^2} + 2\frac{K_\lambda^2}{K_{emgc}^2}\right) \quad (64)$$

$$\Psi_g^{0(9)} = \alpha^2\vartheta K_A{}^2 K_T + \mu\xi\zeta K_M K_S K_Z + \varepsilon\beta\lambda K_E K_B K_L - \alpha\lambda\zeta K_A K_L K_Z - \alpha\beta\xi K_A K_B K_S - \varepsilon\mu\vartheta K_E K_M K_T = \Psi_g^{0(7)} - \alpha^2\vartheta\left(2\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) - \mu\xi\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \varepsilon\beta\lambda\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) + \alpha\lambda\zeta\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \alpha\beta\xi\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \varepsilon\mu\vartheta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) \quad (65)$$

The properties of the corresponding *G*-eigenvectors' components can be illuminated by all the equalities written below in this section, where the CEMGCMC coupling mechanism M_3 is defined by expression (19). For the natural linear combinations of the components of the first *G*-eigenvector, the reader can find the following five expressions:

$$e\phi_g^{0(7)} + h\psi_g^{0(7)} + g\Phi_g^{0(7)} + f\Psi_g^{0(7)} = e\phi_g^{0(9)} + h\psi_g^{0(9)} + g\Phi_g^{0(9)} + f\Psi_g^{0(9)} = eM_{eg} + hM_{hg} + gM_{gg} + fM_{gf} = M_3 \quad (66)$$

$$\varepsilon\phi_g^{0(7)} + \alpha\psi_g^{0(7)} + \zeta\Phi_g^{0(7)} + \xi\Psi_g^{0(7)} = \varepsilon K_E\phi_g^{0(9)} + \alpha K_A\psi_g^{0(9)} + \zeta K_Z\Phi_g^{0(9)} + \xi K_S\Psi_g^{0(9)} = 0 \quad (67)$$

$$\alpha\phi_g^{0(7)} + \mu\psi_g^{0(7)} + \beta\Phi_g^{0(7)} + \lambda\Psi_g^{0(7)} = \alpha K_A\phi_g^{0(9)} + \mu K_M\psi_g^{0(9)} + \beta K_B\Phi_g^{0(9)} + \lambda K_L\Psi_g^{0(9)} = 0 \quad (68)$$

$$\zeta\phi_g^{0(7)} + \beta\psi_g^{0(7)} + \gamma\Phi_g^{0(7)} + \vartheta\Psi_g^{0(7)} = \zeta M_{eg} + \beta M_{hg} + \gamma M_{gg} + \vartheta M_{gf} = M_5 \quad (69)$$

$$\xi\phi_g^{0(7)} + \lambda\psi_g^{0(7)} + \vartheta\Phi_g^{0(7)} + \eta\Psi_g^{0(7)} = \xi K_S\phi_g^{0(9)} + \lambda K_L\psi_g^{0(9)} + \vartheta K_T\Phi_g^{0(9)} + \eta K_F\Psi_g^{0(9)} = 0 \quad (70)$$

For the linear combinations of the components of the second G-eigenvector, it is natural to demonstrate the following obtained properties shown in the five expressions below:

$$\varepsilon\phi_g^{0(9)} + \alpha\psi_g^{0(9)} + \zeta\Phi_g^{0(9)} + \xi\Psi_g^{0(9)} = -\varepsilon K_E\phi_g^{0(7)} - \alpha K_A\psi_g^{0(7)} - \zeta K_Z\Phi_g^{0(7)} - \xi K_S\Psi_g^{0(7)} = \frac{e}{CK_{emgc}^2}\left(e\phi_g^{0(7)} + h\psi_g^{0(7)} + g\Phi_g^{0(7)} + f\Psi_g^{0(7)}\right) = \frac{eM_3}{CK_{emgc}^2} \quad (71)$$

$$\alpha\phi_g^{0(9)} + \mu\psi_g^{0(9)} + \beta\Phi_g^{0(9)} + \lambda\Psi_g^{0(9)} = -\alpha K_A\phi_g^{0(7)} - \mu K_M\psi_g^{0(7)} - \beta K_B\Phi_g^{0(7)} - \lambda K_L\Psi_g^{0(7)} = \frac{h}{CK_{emgc}^2}\left(e\phi_g^{0(7)} + h\psi_g^{0(7)} + g\Phi_g^{0(7)} + f\Psi_g^{0(7)}\right) = \frac{hM_3}{CK_{emgc}^2} \quad (72)$$

$$\zeta K_Z\phi_g^{0(9)} + \beta K_B\psi_g^{0(9)} + \gamma K_G\Phi_g^{0(9)} + \vartheta K_T\Psi_g^{0(9)} = 0 \quad (73)$$

Using equality (73), it is possible to obtain the following equalities:

$$\zeta\phi_g^{0(9)} + \beta\psi_g^{0(9)} + \gamma\Phi_g^{0(9)} + \vartheta\Psi_g^{0(9)} = \zeta\frac{K_\zeta^2}{K_{emgc}^2}\phi_g^{0(9)} + \beta\frac{K_\beta^2}{K_{emgc}^2}\psi_g^{0(9)} + \gamma\frac{K_g^2}{K_{emgc}^2}\Phi_g^{0(9)} + \vartheta\frac{K_\vartheta^2}{K_{emgc}^2}\Psi_g^{0(9)} = \frac{g}{CK_{emgc}^2}\left(e\phi_g^{0(9)} + h\psi_g^{0(9)} + g\Phi_g^{0(9)} + \right.$$

$$f\Psi_g^{0(9)}\Big) = \frac{g}{CK_{emgc}^2}\left(e\phi_g^{0(7)} + h\psi_g^{0(7)} + g\Phi_g^{0(7)} + f\Psi_g^{0(7)}\right) = \zeta\frac{K_\zeta^2}{K_{emgc}^2}\phi_g^{0(7)} + \beta\frac{K_\beta^2}{K_{emgc}^2}\psi_g^{0(7)} + \gamma\frac{K_g^2}{K_{emgc}^2}\Phi_g^{0(7)} + \vartheta\frac{K_\vartheta^2}{K_{emgc}^2}\Psi_g^{0(7)} = \frac{gM_3}{CK_{emgc}^2} \quad (74)$$

For the last linear combination, one can get the following equality:

$$\xi\phi_g^{0(9)} + \lambda\psi_g^{0(9)} + \vartheta\Phi_g^{0(9)} + \eta\Psi_g^{0(9)} = -\xi K_S\phi_g^{0(7)} - \lambda K_L\psi_g^{0(7)} - \vartheta K_T\Phi_g^{0(7)} - \eta K_F\Psi_g^{0(7)} = \frac{g}{CK_{emgc}^2}\left(e\phi_g^{0(7)} + h\psi_g^{0(7)} + g\Phi_g^{0(7)} + f\Psi_g^{0(7)}\right) = \frac{fM_3}{CK_{emgc}^2} \quad (75)$$

5 The *H*-eigenvectors and their properties

The third independent pair of the suitable eigenvectors is naturally called the *H*-eigenvectors. Their explicit forms are given below in expressions (76) and (77). For first eigenvector (76), the eigenvector components respectively represent the parameters M_{eh}, M_{hh}, M_{hg}, and M_{hf}. These parameters are defined by expressions (26), (23), (29), and (30), respectively, and contain the subscript "*h*" symbolizing the *H*-eigenvectors. Accordingly, the first and second *H*-eigenvectors are respectively written as follows:

$$\begin{pmatrix} U^{0(1)} \\ \phi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(2)} \\ \phi^{0(2)} \\ \psi^{0(2)} \\ \Phi^{0(2)} \\ \Psi^{0(2)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \phi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(4)} \\ \phi^{0(4)} \\ \psi^{0(4)} \\ \Phi^{0(4)} \\ \Psi^{0(4)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \phi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(6)} \\ \phi^{0(6)} \\ \psi^{0(6)} \\ \Phi^{0(6)} \\ \Psi^{0(6)} \end{pmatrix} = \begin{pmatrix} U^{0(7)} \\ \phi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix} = \begin{pmatrix} U^{0(8)} \\ \phi^{0(8)} \\ \psi^{0(8)} \\ \Phi^{0(8)} \\ \Psi^{0(8)} \end{pmatrix} = \begin{pmatrix} U_h^{0(7)} = 0 \\ \phi_h^{0(7)} = M_{eh} \\ \psi_h^{0(7)} = M_{hh} \\ \Phi_h^{0(7)} = M_{hg} \\ \Psi_h^{0(7)} = M_{hf} \end{pmatrix} \quad (76)$$

$$\begin{pmatrix} U^{0(9)} \\ \phi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^{0(10)} \\ \phi^{0(10)} \\ \psi^{0(10)} \\ \Phi^{0(10)} \\ \Psi^{0(10)} \end{pmatrix} = \begin{pmatrix} U_h^{0(9)} \\ \phi_h^{0(9)} \\ \psi_h^{0(9)} \\ \Phi_h^{0(9)} \\ \Psi_h^{0(9)} \end{pmatrix} \tag{77}$$

where

$$U_h^{0(9)} = \frac{e\phi_h^{0(9)} + h\psi_h^{0(9)} + g\Phi_h^{0(9)} + f\Psi_h^{0(9)}}{CK_{emgc}^2} = \frac{M_2}{CK_{emgc}^2} \tag{78}$$

$$\phi_h^{0(9)} = \alpha\vartheta^2 K_A K_T{}^2 + \eta\beta\zeta K_F K_B K_Z + \gamma\lambda\xi K_G K_L K_S - \gamma\eta\alpha K_G K_F K_A - \vartheta\lambda\zeta K_T K_L K_Z - \beta\vartheta\xi K_B K_T K_S = \phi_h^{0(7)} - \alpha\vartheta^2\left(\frac{K_\alpha^2}{K_{emgc}^2} + 2\frac{K_\vartheta^2}{K_{emgc}^2}\right) - \eta\beta\zeta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \gamma\lambda\xi\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \gamma\eta\alpha\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2}\right) + \vartheta\lambda\zeta\left(\frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \beta\vartheta\xi\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) \tag{79}$$

$$\psi_h^{0(9)} = 2\vartheta\xi\zeta K_T K_S K_Z + \varepsilon\gamma\eta K_E K_G K_F - \gamma\xi^2 K_G K_S{}^2 - \eta\zeta^2 K_F K_Z{}^2 - \varepsilon\vartheta^2 K_E K_T{}^2 = \psi_h^{0(7)} - 2\vartheta\xi\zeta\left(\frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \varepsilon\gamma\eta\left(\frac{K_e^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2}\right) + \gamma\xi^2\left(\frac{K_g^2}{K_{emgc}^2} + 2\frac{K_\xi^2}{K_{emgc}^2}\right) + \eta\zeta^2\left(\frac{K_f^2}{K_{emgc}^2} + 2\frac{K_\zeta^2}{K_{emgc}^2}\right) + \varepsilon\vartheta^2\left(\frac{K_e^2}{K_{emgc}^2} + 2\frac{K_\vartheta^2}{K_{emgc}^2}\right) \tag{80}$$

$$\Phi_h^{0(9)} = \eta\alpha\zeta K_F K_A K_Z + \beta\xi^2 K_B K_S{}^2 + \varepsilon\vartheta\lambda K_E K_T K_L - \lambda\xi\zeta K_L K_S K_Z - \alpha\vartheta\xi K_A K_T K_S - \varepsilon\eta\beta K_E K_F K_B = \Phi_h^{0(7)} - \eta\alpha\zeta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) -$$

$$\beta\xi^2\left(\frac{K_\beta^2}{K_{emgc}^2}+2\frac{K_\xi^2}{K_{emgc}^2}\right)-\varepsilon\vartheta\lambda\left(\frac{K_e^2}{K_{emgc}^2}+\frac{K_\vartheta^2}{K_{emgc}^2}+\frac{K_\lambda^2}{K_{emgc}^2}\right)+\lambda\xi\zeta\left(\frac{K_\lambda^2}{K_{emgc}^2}+\frac{K_\xi^2}{K_{emgc}^2}+\frac{K_\zeta^2}{K_{emgc}^2}\right)+\alpha\vartheta\xi\left(\frac{K_\alpha^2}{K_{emgc}^2}+\frac{K_\vartheta^2}{K_{emgc}^2}+\frac{K_\xi^2}{K_{emgc}^2}\right)+\varepsilon\eta\beta\left(\frac{K_e^2}{K_{emgc}^2}+\frac{K_f^2}{K_{emgc}^2}+\frac{K_\beta^2}{K_{emgc}^2}\right) \quad (81)$$

$$\Psi_h^{0(9)}=\gamma\alpha\xi K_G K_A K_S+\varepsilon\beta\vartheta K_E K_B K_T+\lambda\zeta^2 K_L {K_Z}^2-\alpha\vartheta\zeta K_A K_T K_Z-\beta\xi\zeta K_B K_S K_Z-\varepsilon\gamma\lambda K_E K_G K_L=\Psi_h^{0(7)}-\gamma\alpha\xi\left(\frac{K_g^2}{K_{emgc}^2}+\frac{K_\alpha^2}{K_{emgc}^2}+\frac{K_\xi^2}{K_{emgc}^2}\right)-\varepsilon\beta\vartheta\left(\frac{K_e^2}{K_{emgc}^2}+\frac{K_\beta^2}{K_{emgc}^2}+\frac{K_\vartheta^2}{K_{emgc}^2}\right)-\lambda\zeta^2\left(\frac{K_\lambda^2}{K_{emgc}^2}+2\frac{K_\zeta^2}{K_{emgc}^2}\right)+\alpha\vartheta\zeta\left(\frac{K_\alpha^2}{K_{emgc}^2}+\frac{K_\vartheta^2}{K_{emgc}^2}+\frac{K_\zeta^2}{K_{emgc}^2}\right)+\beta\xi\zeta\left(\frac{K_\beta^2}{K_{emgc}^2}+\frac{K_\xi^2}{K_{emgc}^2}+\frac{K_\zeta^2}{K_{emgc}^2}\right)+\varepsilon\gamma\lambda\left(\frac{K_e^2}{K_{emgc}^2}+\frac{K_g^2}{K_{emgc}^2}+\frac{K_\lambda^2}{K_{emgc}^2}\right) \quad (82)$$

The natural linear combinations of the corresponding *H*-eigenvectors' components disclose the properties obtained in all the expressions written below in this section, where the CEMGCMC coupling mechanisms M_2 and M_5 are defined by equations (18) and (21), respectively. For the linear combinations of the components of the first *H*-eigenvector, the following five properties can be obtained:

$$e\phi_h^{0(7)}+h\psi_h^{0(7)}+g\Phi_h^{0(7)}+f\Psi_h^{0(7)}=e\phi_h^{0(9)}+h\psi_h^{0(9)}+g\Phi_h^{0(9)}+f\Psi_h^{0(9)}=eM_{eh}+hM_{hh}+gM_{hg}+fM_{hf}=M_2 \quad (83)$$

$$\varepsilon\phi_h^{0(7)}+\alpha\psi_h^{0(7)}+\zeta\Phi_h^{0(7)}+\xi\Psi_h^{0(7)}=\varepsilon K_E\phi_h^{0(9)}+\alpha K_A\psi_h^{0(9)}+\zeta K_Z\Phi_h^{0(9)}+\xi K_S\Psi_h^{0(9)}=0 \quad (84)$$

$$\alpha\phi_h^{0(7)}+\mu\psi_h^{0(7)}+\beta\Phi_h^{0(7)}+\lambda\Psi_h^{0(7)}=\alpha M_{eh}+\mu M_{hh}+\beta M_{hg}+\lambda M_{hf}=M_5 \quad (85)$$

$$\zeta\phi_h^{0(7)}+\beta\psi_h^{0(7)}+\gamma\Phi_h^{0(7)}+\vartheta\Psi_h^{0(7)}=\zeta K_Z\phi_h^{0(9)}+\beta K_B\psi_h^{0(9)}+\gamma K_G\Phi_h^{0(9)}+\vartheta K_T\Psi_h^{0(9)}=0 \quad (86)$$

$$\xi\phi_h^{0(7)}+\lambda\psi_h^{0(7)}+\vartheta\Phi_h^{0(7)}+\eta\Psi_h^{0(7)}=\xi K_S\phi_h^{0(9)}+\lambda K_L\psi_h^{0(9)}+\vartheta K_T\Phi_h^{0(9)}+\eta K_F\Psi_h^{0(9)}=0 \quad (87)$$

For the linear combinations of the components of the second H-eigenvector there are also the five properties written down in the following last five expressions for this section:

$$\varepsilon\phi_h^{0(9)} + \alpha\psi_h^{0(9)} + \zeta\Phi_h^{0(9)} + \xi\Psi_h^{0(9)} = -\varepsilon K_E\phi_h^{0(7)} - \alpha K_A\psi_h^{0(7)} - \zeta K_Z\Phi_h^{0(7)} - \xi K_S\Psi_h^{0(7)} = \frac{e}{CK_{\bar{e}mgc}^2}\left(e\phi_h^{0(7)} + h\psi_h^{0(7)} + g\Phi_h^{0(7)} + f\Psi_h^{0(7)}\right) = \frac{eM_2}{CK_{\bar{e}mgc}^2} \quad (88)$$

$$\alpha K_A\Phi_h^{0(9)} + \mu K_M\psi_h^{0(9)} + \beta K_B\Phi_h^{0(9)} + \lambda K_L\Psi_h^{0(9)} = 0 \quad (89)$$

Utilizing expression (89), the following equalities in expression (90) can be obtained:

$$\alpha\phi_h^{0(9)} + \mu\psi_h^{0(9)} + \beta\Phi_h^{0(9)} + \lambda\Psi_h^{0(9)} = \alpha\frac{K_{\bar{\alpha}}^2}{K_{\bar{e}mgc}^2}\phi_h^{0(9)} + \mu\frac{K_{\bar{m}}^2}{K_{\bar{e}mgc}^2}\psi_h^{0(9)} + \beta\frac{K_{\bar{\beta}}^2}{K_{\bar{e}mgc}^2}\Phi_h^{0(9)} + \lambda\frac{K_{\bar{\lambda}}^2}{K_{\bar{e}mgc}^2}\Psi_h^{0(9)} = \frac{h}{CK_{\bar{e}mgc}^2}\left(e\phi_h^{0(9)} + h\psi_h^{0(9)} + g\Phi_h^{0(9)} + f\Psi_h^{0(9)}\right) = \frac{h}{CK_{\bar{e}mgc}^2}\left(e\phi_h^{0(7)} + h\psi_h^{0(7)} + g\Phi_h^{0(7)} + f\Psi_h^{0(7)}\right) = \alpha\frac{K_{\bar{\alpha}}^2}{K_{\bar{e}mgc}^2}\phi_h^{0(7)} + \mu\frac{K_{\bar{m}}^2}{K_{\bar{e}mgc}^2}\psi_h^{0(7)} + \beta\frac{K_{\bar{\beta}}^2}{K_{\bar{e}mgc}^2}\Phi_h^{0(7)} + \lambda\frac{K_{\bar{\lambda}}^2}{K_{\bar{e}mgc}^2}\Psi_h^{0(7)} = \frac{hM_2}{CK_{\bar{e}mgc}^2} \quad (90)$$

For this section, the last two natural linear combinations demonstrate the following properties:

$$\zeta\phi_h^{0(9)} + \beta\psi_h^{0(9)} + \gamma\Phi_h^{0(9)} + \vartheta\Psi_h^{0(9)} = -\zeta K_Z\phi_h^{0(7)} - \beta K_B\psi_h^{0(7)} - \gamma K_G\Phi_h^{0(7)} - \vartheta K_T\Psi_h^{0(7)} = \frac{g}{CK_{\bar{e}mgc}^2}\left(e\phi_h^{0(7)} + h\psi_h^{0(7)} + g\Phi_h^{0(7)} + f\Psi_h^{0(7)}\right) = \frac{gM_2}{CK_{\bar{e}mgc}^2} \quad (91)$$

$$\xi\phi_h^{0(9)} + \lambda\psi_h^{0(9)} + \vartheta\Phi_h^{0(9)} + \eta\Psi_h^{0(9)} = -\xi K_S\phi_h^{0(7)} - \lambda K_L\psi_h^{0(7)} - \vartheta K_T\Phi_h^{0(7)} - \eta K_F\Psi_h^{0(7)} = \frac{f}{CK_{\bar{e}mgc}^2}\left(e\phi_h^{0(7)} + h\psi_h^{0(7)} + g\Phi_h^{0(7)} + f\Psi_h^{0(7)}\right) = \frac{fM_2}{CK_{\bar{e}mgc}^2} \quad (92)$$

6 The *E*-eigenvectors and their properties

The last independent pair of the eigenvectors is called the *E*-eigenvectors. Expressions (93) and (94) written below stand for the first and second *E*-eigenvectors, respectively. The corresponding components of the first *E*-eigenvector are naturally equal to parameters M_{ee} (22), M_{eh} (26), M_{eg} (27), and M_{ef} (28). Here, subscript "*e*" symbolizes the *E*-eigenvectors. Therefore, the *E*-eigenvectors are

$$\begin{pmatrix} U^{0(1)} \\ \phi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(2)} \\ \phi^{0(2)} \\ \psi^{0(2)} \\ \Phi^{0(2)} \\ \Psi^{0(2)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \phi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(4)} \\ \phi^{0(4)} \\ \psi^{0(4)} \\ \Phi^{0(4)} \\ \Psi^{0(4)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \phi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(6)} \\ \phi^{0(6)} \\ \psi^{0(6)} \\ \Phi^{0(6)} \\ \Psi^{0(6)} \end{pmatrix} =$$

$$\begin{pmatrix} U^{0(7)} \\ \phi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix} = \begin{pmatrix} U^{0(8)} \\ \phi^{0(8)} \\ \psi^{0(8)} \\ \Phi^{0(8)} \\ \Psi^{0(8)} \end{pmatrix} = \begin{pmatrix} U_e^{0(7)} = 0 \\ \phi_e^{0(7)} = M_{ee} \\ \psi_e^{0(7)} = M_{eh} \\ \Phi_e^{0(7)} = M_{eg} \\ \Psi_e^{0(7)} = M_{ef} \end{pmatrix} \tag{93}$$

$$\begin{pmatrix} U^{0(9)} \\ \phi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^{0(10)} \\ \phi^{0(10)} \\ \psi^{0(10)} \\ \Phi^{0(10)} \\ \Psi^{0(10)} \end{pmatrix} = \begin{pmatrix} U_e^{0(9)} \\ \phi_e^{0(9)} \\ \psi_e^{0(9)} \\ \Phi_e^{0(9)} \\ \Psi_e^{0(9)} \end{pmatrix} \tag{94}$$

where

$$U_e^{0(9)} = \frac{e\phi_e^{0(9)} + h\psi_e^{0(9)} + g\Phi_e^{0(9)} + f\Psi_e^{0(9)}}{CK_{emgc}^2} = \frac{M_1}{CK_{emgc}^2} \tag{95}$$

$$\phi_e^{0(9)} = 2\beta\vartheta\lambda K_B K_T K_L + \mu\gamma\eta K_M K_G K_F - \gamma\lambda^2 K_G {K_L}^2 - \eta\beta^2 K_F {K_B}^2 - \mu\vartheta^2 K_M {K_T}^2 = \phi_e^{0(7)} - 2\beta\vartheta\lambda\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) - \mu\gamma\eta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2}\right) + \gamma\lambda^2\left(\frac{K_g^2}{K_{emgc}^2} + 2\frac{K_\lambda^2}{K_{emgc}^2}\right) + \eta\beta^2\left(\frac{K_f^2}{K_{emgc}^2} + 2\frac{K_\beta^2}{K_{emgc}^2}\right) + \mu\vartheta^2\left(\frac{K_m^2}{K_{emgc}^2} + 2\frac{K_\vartheta^2}{K_{emgc}^2}\right) \quad (96)$$

$$\psi_e^{0(9)} = \alpha\vartheta^2 K_A {K_T}^2 + \eta\beta\zeta K_F K_B K_Z + \gamma\lambda\xi K_G K_L K_S - \gamma\eta\alpha K_G K_F K_A - \beta\vartheta\xi K_B K_T K_S - \vartheta\lambda\zeta K_T K_L K_Z = \psi_e^{0(7)} - \alpha\vartheta^2\left(\frac{K_\alpha^2}{K_{emgc}^2} + 2\frac{K_\vartheta^2}{K_{emgc}^2}\right) - \eta\beta\zeta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \gamma\lambda\xi\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \gamma\eta\alpha\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2}\right) + \beta\vartheta\xi\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \vartheta\lambda\zeta\left(\frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) \quad (97)$$

$$\Phi_e^{0(9)} = \eta\alpha\beta K_F K_A K_B + \lambda^2\zeta {K_L}^2 K_Z + \mu\vartheta\xi K_M K_T K_S - \beta\lambda\xi K_B K_L K_S - \alpha\vartheta\lambda K_A K_T K_L - \mu\eta\zeta K_M K_F K_Z = \Phi_e^{0(7)} - \eta\alpha\beta\left(\frac{K_f^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2}\right) - \lambda^2\zeta\left(2\frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \mu\vartheta\xi\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \beta\lambda\xi\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \alpha\vartheta\lambda\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) + \mu\eta\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_f^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) \quad (98)$$

$$\Psi_e^{0(9)} = \gamma\alpha\lambda K_G K_A K_L + \mu\vartheta\zeta K_M K_T K_Z + \beta^2\xi {K_B}^2 K_S - \alpha\beta\vartheta K_A K_B K_T - \beta\lambda\zeta K_B K_L K_Z - \mu\gamma\xi K_M K_G K_S = \Psi_e^{0(7)} - \gamma\alpha\lambda\left(\frac{K_g^2}{K_{emgc}^2} + \frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2}\right) - \mu\vartheta\zeta\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) - \beta^2\xi\left(2\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) + \alpha\beta\vartheta\left(\frac{K_\alpha^2}{K_{emgc}^2} + \frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\vartheta^2}{K_{emgc}^2}\right) + \beta\lambda\zeta\left(\frac{K_\beta^2}{K_{emgc}^2} + \frac{K_\lambda^2}{K_{emgc}^2} + \frac{K_\zeta^2}{K_{emgc}^2}\right) + \mu\gamma\xi\left(\frac{K_m^2}{K_{emgc}^2} + \frac{K_g^2}{K_{emgc}^2} + \frac{K_\xi^2}{K_{emgc}^2}\right) \quad (99)$$

With the natural linear combinations of the suitable *E*-eigenvectors' components, this pair of the *E*-eigenvectors possesses the properties demonstrated in the rest ten expressions written below, where M_1 in expression

(100) stands for CEMGCMC coupling mechanism (17). The following first five expressions relate to the linear combinations of the components of the first E-eigenvector:

$$e\phi_e^{0(7)} + h\psi_e^{0(7)} + g\Phi_e^{0(7)} + f\Psi_e^{0(7)} = e\phi_e^{0(9)} + h\psi_e^{0(9)} + g\Phi_e^{0(9)} + f\Psi_e^{0(9)} = eM_{ee} + hM_{eh} + gM_{eg} + fM_{ef} = M_1 \quad (100)$$

$$\varepsilon\phi_e^{0(7)} + \alpha\psi_e^{0(7)} + \zeta\Phi_e^{0(7)} + \xi\Psi_e^{0(7)} = \varepsilon M_{ee} + \alpha M_{eh} + \zeta M_{eg} + \xi M_{ef} = M_5 \quad (101)$$

$$\alpha\phi_e^{0(7)} + \mu\psi_e^{0(7)} + \beta\Phi_e^{0(7)} + \lambda\Psi_e^{0(7)} = \alpha K_A\phi_e^{0(9)} + \mu K_M\psi_e^{0(9)} + \beta K_B\Phi_e^{0(9)} + \lambda K_L\Psi_e^{0(9)} = 0 \quad (102)$$

$$\zeta\phi_e^{0(7)} + \beta\psi_e^{0(7)} + \gamma\Phi_e^{0(7)} + \vartheta\Psi_e^{0(7)} = \zeta K_Z\phi_e^{0(9)} + \beta K_B\psi_e^{0(9)} + \gamma K_G\Phi_e^{0(9)} + \vartheta K_T\Psi_e^{0(9)} = 0 \quad (103)$$

$$\xi\phi_e^{0(7)} + \lambda\psi_e^{0(7)} + \vartheta\Phi_e^{0(7)} + \eta\Psi_e^{0(7)} = \xi K_S\phi_e^{0(9)} + \lambda K_L\psi_e^{0(9)} + \vartheta K_T\Phi_e^{0(9)} + \eta K_F\Psi_e^{0(9)} = 0 \quad (104)$$

Concerning the properties of the second E-eigenvector, the following equality is useful:

$$\varepsilon K_E\phi_e^{0(9)} + \alpha K_A\psi_e^{0(9)} + \zeta K_Z\Phi_e^{0(9)} + \xi K_S\Psi_e^{0(9)} = 0 \quad (105)$$

With expressions (105), (32), (36), (39), and (40), the following property can be obtained:

$$\varepsilon\phi_e^{0(9)} + \alpha\psi_e^{0(9)} + \zeta\Phi_e^{0(9)} + \xi\Psi_e^{0(9)} = \varepsilon\frac{K_\varepsilon^2}{K_{emgc}^2}\phi_e^{0(9)} + \alpha\frac{K_\alpha^2}{K_{emgc}^2}\psi_e^{0(9)} + \zeta\frac{K_\zeta^2}{K_{emgc}^2}\Phi_e^{0(9)} + \xi\frac{K_\xi^2}{K_{emgc}^2}\Psi_e^{0(9)} = \frac{e}{CK_{emgc}^2}\Big(e\phi_e^{0(9)} + h\psi_e^{0(9)} + g\Phi_e^{0(9)} +$$

$$f\Psi_e^{0(9)}\Big) = \frac{e}{CK_{emgc}^2}\left(e\phi_e^{0(7)} + h\psi_e^{0(7)} + g\Phi_e^{0(7)} + f\Psi_e^{0(7)}\right) = \varepsilon\frac{K_\varepsilon^2}{K_{emgc}^2}\phi_e^{0(7)} + \alpha\frac{K_\alpha^2}{K_{emgc}^2}\psi_e^{0(7)} + \zeta\frac{K_\zeta^2}{K_{emgc}^2}\Phi_e^{0(7)} + \xi\frac{K_\xi^2}{K_{emgc}^2}\Psi_e^{0(7)} = \frac{eM_1}{CK_{emgc}^2} \quad (106)$$

The final three expressions for the linear combinations of the components of the second E-eigenvector are

$$\alpha\phi_e^{0(9)} + \mu\psi_e^{0(9)} + \beta\Phi_e^{0(9)} + \lambda\Psi_e^{0(9)} = -\alpha K_A\phi_e^{0(7)} - \mu K_M\psi_e^{0(7)} - \beta K_B\Phi_e^{0(7)} - \lambda K_L\Psi_e^{0(7)} = \frac{h}{CK_{emgc}^2}\left(e\phi_e^{0(7)} + h\psi_e^{0(7)} + g\Phi_e^{0(7)} + f\Psi_e^{0(7)}\right) = \frac{hM_1}{CK_{emgc}^2} \quad (107)$$

$$\zeta\phi_e^{0(9)} + \beta\psi_e^{0(9)} + \gamma\Phi_e^{0(9)} + \vartheta\Psi_e^{0(9)} = -\zeta K_Z\phi_e^{0(7)} - \beta K_B\psi_e^{0(7)} - \gamma K_G\Phi_e^{0(7)} - \vartheta K_T\Psi_e^{0(7)} = \frac{g}{CK_{emgc}^2}\left(e\phi_e^{0(7)} + h\psi_e^{0(7)} + g\Phi_e^{0(7)} + f\Psi_e^{0(7)}\right) = \frac{gM_1}{CK_{emgc}^2} \quad (108)$$

$$\xi\phi_e^{0(9)} + \lambda\psi_e^{0(9)} + \vartheta\Phi_e^{0(9)} + \eta\Psi_e^{0(9)} = -\xi K_S\phi_e^{0(7)} - \lambda K_L\psi_e^{0(7)} - \vartheta K_T\Phi_e^{0(7)} - \eta K_F\Psi_e^{0(7)} = \frac{f}{CK_{emgc}^2}\left(e\phi_e^{0(7)} + h\psi_e^{0(7)} + g\Phi_e^{0(7)} + f\Psi_e^{0(7)}\right) = \frac{fM_1}{CK_{emgc}^2} \quad (109)$$

So, this paper has obtained the four pairs of the main independent eigenvectors called the F-eigenvectors (third section), G-eigenvectors (fourth section), H-eigenvectors (fifth section), and E-eigenvectors (this sixth section). Their properties are described in the corresponding section below each pair of the corresponding eigenvectors. All the four pairs of the main independent eigenvectors are written above in convenient explicit forms. However, the reader can find that their forms are quite complicated and to verify their properties is not easy to perform. Indeed, it was not easy to find that all the twenty-four possible eigenvectors (paper [24] has obtained only half of them) reduce to one of the four pairs of the main independent eigenvectors and then to demonstrate the properties of the F-, G-, H-, and E-eigenvectors.

All the properties obtained in this and the previous three sections are the main results of these theoretical investigations. In this section, property (100) demonstrates that the E-eigenvectors relate to the coupling mechanism M_1 (17) of the CEMGCMC K_{emgc}^2 (16). Corresponding properties (49), (66), and (83) in the previous three sections manifest that the *F*-, *G*-, and *H*-eigenvectors relate to the CEMGCMC coupling mechanisms M_4, M_3, and M_2 defined by formulae (20), (19), and (18), respectively. Moreover, all of these four independent eigenvectors also relate to the fifth coupling mechanisms M_5 (21). Properties (53), (69), (85), and (101) support this statement. It is necessary to state that each of these five coupling mechanisms can release propagation velocities of the 4P-SH acoustic waves guided by the surface of the solid, the interface between two dissimilar solids, or plate (thin film) waveguide. All the properties of the four eigenvectors obtained in this and the previous three sections can be actually useful for analysis of various boundary conditions' determinants to obtain suitable propagation velocities. Indeed, suitable properties can be used for representing final results of propagation speeds in compact and convenient forms because many linear combinations of the eigenvector components (i.e. properties) are equal to zero. This will be useful even in numerical calculations because it is obvious that many possible final results of propagation speeds will be very complicated due to the existence of a lot of material parameters listed in table 1. Also, for the calculations it is preferable to use measured values of the parameters listed in the table. Note that measured values of the material parameters listed in the last several rows of table 1 are currently absent because their measurements require creation of perfect experimental tools.

It is necessary to mention that the acoustic waves are very slow in comparison with the speed of light, C_L, and can propagate in solids at the speeds of $\sim 10^3$ m/s. For the mechanical subsystem, the treatment of additional subsystems such as the electrical, magnetic, gravitational, and cogravitational can result in the fact that the propagation speeds of the acoustic waves can be slightly higher. However, the presence of these additional subsystems allows one to deal with smart materials because any change in one subsystem can result in some response changes in the other subsystems. This fact allows the constitution of various smart technical devices.

7 Discussion

It is necessary to mention also that this work belongs to the case of investigations of the piezogravitocogravitoelectromagnetic materials [1, 31] possessing the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems. Namely this paper further develops the results obtained in pioneer work [1] and paper [24]. It is now possible to compare the obtained results in this work with the results obtained in book [12] for the case of acoustic wave propagation in piezoelectromagnetics (PEMs) possessing only the mechanical, electrical, and magnetic subsystems, i.e. without taking into account the gravitational phenomena. For the case of the PEM materials there are only two possible pairs of the main independent eigenvectors and they are quite simple compared with the case described in this paper above. However, the investigations of the PEM materials also have peculiarities described in papers [32] and [33]. To ignore these peculiarities can result in obtaining incorrect final solution for the acoustic wave propagation velocities in the PEM materials. The reader can also find single review [14] on the subject of the acoustic wave propagation in the PEM materials. The common problem representing the peculiarity of this study and study [12] is that any eigenvector component does not depend on the propagation velocity. It is worth noting that the cubic PEM materials do not possess such peculiarity because the eigenvectors depend on the propagation velocity in the cubic case treated in book [13]. However, the cubic PEM materials have the other peculiarities. Concerning some investigations of the piezogravitocogravitoelectromagnetic materials with the cubic symmetry, the development of theoretical backgrounds is also possible in the future.

It is now possible to briefly discuss the most exciting thing for the reader concerning the possibility of the instant interplanetary (tele)communication. It was mentioned in the introduction that in the solids, the slow speed of the new 4P-SH-SAW [1] can naturally depend on the speed of the electromagnetic wave, speed of the gravitational wave, and the speeds $\Lambda_{01} = (\zeta_0\lambda_0)^{-1/2} \rightarrow 10^{13}C_L$ and $\Lambda_{02} = (\xi_0\beta_0)^{-1/2} \rightarrow 10^{13}C_L$ of the new fast waves that can be thirteen orders faster than the speed of light, C_L. Also, it is necessary to state that any acoustic wave cannot propagate in a vacuum. So, the transition of information in a vacuum is only possible with one of the following speeds: C_L, Λ_{01}, and Λ_{02}. It is obvious that the speed C_L is too slow for the instant interplanetary communication. On the other hand, the speeds Λ_{01}, and Λ_{02} that thirteen orders faster than C_L are already enough for the instant interplanetary (and even intergalactic) communication. Therefore, it is necessary to develop technologies and infrastructure in order to have the instant interplanetary communication. Indeed, it is not easy and fast. First of all, it is necessary to measure the vacuum parameters ζ_0, λ_0, ξ_0, and β_0 called the gravitoelectric, cogravitomagnetic, cogravitoelectric, and gravitomagnetic constants, respectively. This will allow the human civilization to know the complete set of the vacuum parameters and the exact values of the speeds Λ_{01} and Λ_{02}.

The C_L-communication due to the well-developed C_L-infrastructure is well-developed on the Earth. This successful development lasted during the last several centuries and involved millions of researchers and engineers on this planet. This means that trillions of the United States dollars were accumulated to have the success in the development of the C_L-communication that is now used even for the slow interplanetary communication. The C_L-communication simultaneously incorporates only two subsystems (electrical and magnetic) to transfer signals through a vacuum. For the mobile C_L-communication, many kinds of mobile devices are commercially available today, for instance, mobile phones even for children. For the Λ_{01}- and Λ_{02}-communication (say Λ-communication below) it is possible to use some interplanetary infrastructure schematically shown and discussed in paper [10]. It is assumed that the development of some mobile Λ-communication can be the final aim after the successful development of the voice and video Λ-communications. For this purpose, it is useful to know the complete set of the material constants for solids listed in table 1.

There are millions of different solids (monocrystals, alloys, composites, etc.) and their wave characteristics can be very important to constitute various (smart) technical devices. However, it is necessary to state that the contributions from both the piezoelectric and piezomagnetic effects to the speed of the acoustic wave are in general small but well-measurable today. Even the contribution from the magnetoelectric effect for the smart PEM materials is well-measurable today in spite of its weakness. The magnetoelectric effect is characterized by the electromagnetic constant α listed in table 1. This constant symbolizes some exchange between the electrical and magnetic subsystems with the following evaluated exchange speed: $V_\alpha = 1/\alpha$. The exchange speed V_α must be faster than the speed of the electromagnetic wave V_{EM} in a PEM solid, $V_\alpha > V_{EM} = (\varepsilon\mu)^{-1/2}$ because the following condition of thermodynamic stability $\alpha^2 < \varepsilon\mu$ [34, 35] must hold. Practically there is always $\alpha^2 << \varepsilon\mu$ ($V_\alpha >> V_{EM}$) for the magnetoelectric solids. Even for strong magnetoelectric composites the speed V_α is several orders faster than the speed V_{EM}. For majority of the magnetoelectric solids, it is assumed that the difference between the speeds V_α and V_{EM} can even reach ten orders. Indeed, this is a huge difference. The thermodynamic stability condition $V_\alpha >> V_{EM}$ for the system representing the electromagnetic wave in a solid or a vacuum means that the exchange speed V_α between the electrical and magnetic subsystems should be significantly higher the speed V_{EM} of the whole system. This allows the whole system to thermodynamically exist and propagate for long distances in continuous media such as a vacuum and solids.

For the gravitational and cogravitational subsystems there is the gravitocogravitic constant ϑ listed in table 1 and the evaluated exchange speed is $V_\vartheta = 1/\vartheta$. The corresponding thermodynamic stability condition can be written as follows: $\vartheta^2 < \gamma\eta$ and $V_\vartheta > V_{GC} = (\gamma\eta)^{-1/2}$ (even $\vartheta^2 << \gamma\eta$ and $V_\vartheta >> V_{GC}$) where the gravitic constant γ and the cogravitic constant η are also listed in table 1. It is natural to expect that minimum values for the exchange speeds V_α and V_ϑ can be comparable. Some exchange possibilities must exist among the gravitational, cogravitational, electrical, and magnetic subsystems. The possible exchange processes between each pair of the four subsystems can be characterized by the material constants ζ, λ, ξ, and β listed in the last four rows of table 1. These constants introduce the following exchange speeds: $\Lambda_1 = (\zeta\lambda)^{-1/2} \rightarrow 10^{13}C_L$ and $\Lambda_2 = (\xi\beta)^{-1/2} \rightarrow 10^{13}C_L$. The thermodynamic stability conditions can be written here as follows: $\Lambda_1 > V_\alpha > V_{EM}$, $\Lambda_2 > V_\alpha > V_{EM}$, $\Lambda_1 > V_\vartheta > V_{GC}$, $\Lambda_2 > V_\vartheta > V_{GC}$ or even $\Lambda_1 >> V_\alpha >> V_{EM}$, $\Lambda_2 >> V_\alpha >> V_{EM}$, $\Lambda_1 >> V_\vartheta >> V_{GC}$, $\Lambda_2 >> V_\vartheta >> V_{GC}$. The contributions from all of these speeds to the speed of the acoustic wave defined by formula (12) are complicatedly hybridized in the CEMGCMC K^2_{emgc} (13) that possesses the five different coupling mechanisms in form (16).

For a vacuum, it is also possible to assume that there are the following thermodynamic stability conditions: $\Lambda_{01} > V_{\alpha0} > C_L$, $\Lambda_{02} > V_{\alpha0} > C_L$, $\Lambda_{01} > V_{\vartheta0} > C_L$, $\Lambda_{02} > V_{\vartheta0} > C_L$ or even $\Lambda_{01} >> V_{\alpha0} >> C_L$, $\Lambda_{02} >> V_{\alpha0} >> C_L$, $\Lambda_{01} >> V_{\vartheta0} >> C_L$, $\Lambda_{02} >> V_{\vartheta0} >> C_L$, where $V_{\alpha0} = 1/\alpha_0$ and $V_{\vartheta0} = 1/\vartheta_0$. The most popular case of interactions among the gravitational, cogravitational, electrical, and magnetic subsystems in a vacuum is the captivation of an electromagnetic wave propagating close enough to a gigantic black hole. This case was also discussed in paper [31]. In this case, the black hole gravitation can change the propagation direction of the electromagnetic wave and the wave will be finally directed towards the black hole. It is obvious that these interactions must be significantly faster than the speed of light, C_L, i.e. at the fast speeds Λ_{01} and Λ_{02}. This allows one to treat the system (the electromagnetic wave) as a stationary system at an equilibrium. Each interaction is negligibly small that allows the system to relax after each exchange among the gravitational, cogravitational, electrical, and magnetic fields. However, an enormous number of these interactions can result in significant change in the propagation direction of the electromagnetic wave while the propagation speed C_L can stay the same because a vacuum represents an isotropic continuum, i.e. the value of the speed of light is the same in any propagation direction. Also, it is assumed that some change in energy of the electromagnetic wave can exist.

8 Conclusion

This theoretical research has obtained the *E*-, *H*-, *G*-, and *F*-eigenvectors. They are the four pairs of the main independent eigenvectors. The properties of the obtained eigenvectors were also demonstrated. It was found that some suitable linear combinations of the eigenvector components can be equal to one of the CEMGCMC coupling mechanisms M_1, M_2, M_3, M_4, and M_5 defined by expressions (17), (18), (19), (20), and (21), respectively. These eigenvectors (namely the CEMGCMC coupling mechanisms) can define the propagation velocities in the theory of the acoustic wave propagation in the transversely isotropic materials of the symmetry class 6 *mm*. The acoustic waves propagate in the directions when there is some coupling among the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems. It is expected that the coupling with some gravitational phenomena will allow the human civilization to develop gravitoelectromagnetic technologies for instant interplanetary (interstellar, intergalactic) communications. Indeed, the instant intergalactic Internet is necessary that can be based on the new fast four-potential gravitocogravitoelectromagnetic waves instead of the conventional two-potential electromagnetic waves propagating at the speed of light. The cooperative development of the electromagnetic and gravitational technologies can be feasible already to the end of this century because the four-potential waves can propagate in a vacuum at the speed thirteen orders faster than the speed of light.

References

[1] A.A. Zakharenko, On piezogravitocogravitoelectromagnetic shear-horizontal acoustic waves. Canadian Journal of Pure and Applied Sciences, 10 (3) (2016), 4011-4028; DOI: https://doi.org/10.5281/zenodo.1301184.

[2] A. Einstein, Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik, 354 (7) (1916), 769-822. Version of Record online: 14 MAR 2006, DOI: https://doi.org/10.1002/andp.19163540702.

[3] B.P. Abbott, R. Abbott, T.D. Abbott, M.R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R.X. Adhikari *et al.*, Observation of gravitational waves from a binary black hole merger. Physical Review Letters, 116 (6) (2016), 061102, 16 pages.
[4] B.M. Yavorsky, A.A. Detlaf and A.K. Lebedev, *The Physics Reference Book for Engineers and Student of the Higher Education*, 8th edition, ONICS Publishers, Moscow (2006), 1054 pages.
[5] O. Heaviside, A gravitational and electromagnetic analogy. The Electrician, 31 (Part I) (1893), 281-282 and 359.
[6] O.D. Jefimenko, *Gravitation and Cogravitation. Developing Newton's Theory of Gravitation to its Physical and Mathematical Conclusion*, Electret Scientific Publishing, USA (2006), 367 pages.
[7] N.A. Kozyrev, *Collected papers*, Leningrad University Publishing, Leningrad, USSR (1991), 448 pages; ISBN 5-288-00626-1, in Russian.
[8] P.S. Laplace, *Exposition du Systeme du Monde*, the fifth book, Nauka Publishers, Leningrad, USSR (1982), 309 pages, in Russian; the English translation by J. Pound, *The System of the World*, two volumes, Kessinger (2007) ISBN 1432653679 [J. Pound, *The System of the World*, two volumes, Richard Phillips, London, UK (1809)].
[9] G.L. Le Sage, *Essai de Chymie Mechanique*, (1761), 121 pages.
[10] A.A. Zakharenko, On necessity of development of instant interplanetary telecommunication based on some gravitational phenomena for remote medical diagnostics and treatment. Canadian Journal of Pure and Applied Sciences, 12 (2) (2018), 4481-4487; DOI: https://doi.org/10.5281/zenodo.1301289.
[11] A.A. Zakharenko, Acoustic wave propagation in (layered) magnetoelectroelastic (composite) materials and influence of gravitational subsystems. The 1st International Symposium on Mechanics, Aberdeen, Scotland, UK, 9-12 July 2018, Jones session: Mechanics of composite materials, DOI: https://doi.org/10.5281/zenodo.1307174. It is available online at https://mechanics.nscj.co.uk/sessions/Jones.html.
[12] A.A. Zakharenko, *Propagation of Seven New SH-SAWs in Piezoelectromagnetics of Class 6 mm*, LAP LAMBERT Academic Publishing GmbH & Co. KG, Saarbruecken-Krasnoyarsk, Germany (2010), 84 pages, ISBN: 978-3-8433-6403-4. DOI: https://doi.org/10.13140/2.1.4000.8645.
[13] A.A. Zakharenko, *Seven New SH-SAWs in Cubic Piezoelectromagnetics*, LAP LAMBERT Academic Publishing GmbH & Co. KG, Saarbruecken-

Krasnoyarsk, Germany (2011), 172 pages, ISBN: 978-3-8473-3485-9. DOI: https://doi.org/10.13140/2.1.2428.0001.
[14] A.A. Zakharenko, Piezoelectromagnetic SH-SAWs: A review. Canadian Journal of Pure and Applied Sciences, 7 (1) (2013), 2227-2240; DOI: https://doi.org/10.5281/zenodo.1300699.
[15] K. Durdag, Wireless surface acoustic wave sensors. Sensors and Transducers Journal, 106 (7) (2009), 1-5.
[16] A. Nechibvute, A. Chawanda, and P. Luhanga, Piezoelectric energy harvesting devices: An alternative energy source for wireless sensors. Smart Materials Research (Hindawi Publishing Corporation), Volume 2012 (2012), Article ID 853481, 13 pages, DOI: http://dx.doi.org/10.1155/2012/853481.
[17] Sh. Priya, H.-Ch. Song, Yu. Zhou, R. Varghese, A. Chopra, S.-G. Kim, I. Kanno, L. Wu, D.S. Ha, J. Ryu, R.G. Polcawich, A review on piezoelectric energy harvesting: Materials, methods, and circuits. Energy Harvesting and Systems, 4 (1) (2017), 3-39, DOI: http://doi.org/10.1515/ehs-2016-0028.
[18] P.K. Sharma and P.V. Baredar, Analysis on piezoelectric energy harvesting small scale device – a review. Journal of King Saud University - Science (Production and hosting by Elsevier B.V.), 31 (4) (2019), 9 pages, in press, DOI: https://doi.org/10.1016/j.jksus.2017.11.002.
[19] J. Iannacci, Microsystem based Energy Harvesting (EH-MEMS): Powering pervasivity of the Internet of Things (IoT) – A review with focus on mechanical vibrations. Journal of King Saud University - Science, 31 (1) (2019), 66-74.
[20] R.B. Thompson, Physical Principles of Measurements with EMAT Transducers, in *Physical Acoustics*, W.P. Mason and R.N. Thurston, Eds., Academic Press, New York, USA (1990), Volume 19, pp. 157-200.
[21] M. Hirao and H. Ogi, *EMATs for Science and Industry: Non-contacting Ultrasonic Measurements*, Kluwer Academic, Boston, USA (2003).
[22] R. Ribichini, F. Cegla, P.B. Nagy, P. Cawley, Quantitative modeling of the transduction of electromagnetic acoustic transducers operating on ferromagnetic media. The IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 57 (12) (2010), 2808-2817.
[23] A. Füzfa, How current loops and solenoids curve space-time. Physical Review D, 93 (2) (2016), 024014.
[24] A.A. Zakharenko, The problem of finding of eigenvectors for 4P-SH-SAW propagation in 6 *mm* media. Canadian Journal of Pure and Applied Sciences, 11 (1) (2017), 4103-4119; DOI: https://doi.org/10.5281/zenodo.1301202.

[25] E. Dieulesaint and D. Royer, *Elastic Waves in Solids: Applications to Signal Processing*, (translated by A. Bastin and M. Motz, Chichester [English]), J. Wiley, New York (1980), 511 pages.
[26] A.A. Zakharenko, On new interfacial four-potential acoustic SH-wave in dissimilar media pertaining to transversely isotropic class 6 *mm*. Canadian Journal of Pure and Applied Sciences, 11 (3) (2017), 4321-4328; DOI: https://doi.org/10.5281/zenodo.1301215.
[27] A.A. Zakharenko, On existence of new dispersive four-potential SH-waves in 6 *mm* plates for new communication era based on gravitational phenomena. Canadian Journal of Pure and Applied Sciences, 12 (3) (2018), 4585-4591; DOI: http://doi.org/10.5281/zenodo.1471100.
[28] S.I. Khmelnik, *Gravitomagnetism: Nature's Phenomena, Experiments, Mathematical Models*, published by "MiC" - Mathematics in Computer Comp., Printed in the United States of America, Lulu Inc. (2017), 257 pages; DOI: http://doi.org/10.5281/zenodo.1403669.
[29] B.A. Auld, *Acoustic Fields and Waves in Solids*, Volumes I and II (set of two volumes), 2nd edition, Krieger Publishing Company (1990), 878 pages.
[30] Yu.V. Gulyaev, Review of shear surface acoustic waves in solids. The IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 45 (4) (1998), 935-938.
[31] A.A. Zakharenko, On separation of exchange terms for four-potential acoustic SH-wave case with dependence on gravitational parameters. Hadronic Journal, 41 (4) (2018), 349-370; DOI: http://doi.org/10.5281/zenodo.2842082.
[32] A.A. Zakharenko, Some problems of finding of eigenvalues and eigenvectors for SH-wave propagation in transversely isotropic piezoelectromagnetics. Canadian Journal of Pure and Applied Sciences, 8 (1) (2014), 2783-2787; DOI: https://doi.org/10.5281/zenodo.1300213.
[33] A.A. Zakharenko, Peculiarities study of acoustic waves' propagation in piezoelectromagnetic (composite) materials. Canadian Journal of Pure and Applied Sciences, 7 (2) (2013), 2459-2461; DOI: https://doi.org/10.5281/zenodo.1300926.
[34] Ü. Özgür, Ya. Alivov and H. Morkoç, Microwave ferrites, part 2: Passive components and electrical tuning. Journal of Materials Science: Materials in Electronics, 20 (10) (2009), 911-952.
[35] M. Fiebig, Revival of the magnetoelectric effect. Journal of Physics D: Applied Physics, 38 (8) (2005), R123-R152.