

**ON EXTENDED CAMPOPIANO'S TYPE
BOUND FOR LPA CODES**

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Abstract

Linear partition Arihant (LPA) codes have been introduced by the author in [3] and a study of error correcting/detecting capabilities of these code was made with respect to the random block errors [2]. Also, the concept of P -bursts has been formulated by the author in [5] to study clustered block errors that occur during the process of communication. In this paper, we take up the problem of simultaneous correction of P -burst errors with the detection of random block errors in LPA codes and obtain an extended Campopiano's type bound for LPA codes.

Keywords: LPA codes, LPA distance, P -bursts

1. Introduction

P -burst error correcting/detecting codes [5] are suitable for correcting/detecting block errors which do not occur independently but are clustered over a particular block length. These type of errors occur in many situations. One important and practical situation is in which the transmitted/stored information is disturbed over a particular block length together with occasional disturbances, thus, creating simultaneously P -bursts as well as random block errors. Such a situation arise, e.g., in semiconductor memory systems where the memory is highly vulnerable to clustered block errors due to bombardment of strong radioactive particles such as cosmic particles on RAM chips and occasional/random block errors result from decay of RAM chips. However, P -burst error correcting LPA codes fail to detect even a few random block errors when these are not within P -bursts of specified block length. Under such type of situations, block codes capable of correcting P -burst errors and simultaneously detecting random block errors are very useful. Keeping this in view, in this paper, we obtain a construction upper bound on the number of parity check digits required for LPA codes correcting weighted P -burst block errors and simultaneously detecting random block errors of upto a specified Arihant weight. The bound obtained in this paper is an extended Campopiano's type bound [1] for LPA block space.

2. Preliminaries

Let q, n be positive integers with $q > 1$. Let \mathbf{Z}_q be the ring of integers modulo q . Let \mathbf{Z}_q^n be the set of all n -tuples over \mathbf{Z}_q . Then \mathbf{Z}_q^n is a module over \mathbf{Z}_q . For q prime, \mathbf{Z}_q becomes a field and \mathbf{Z}_q^n becomes a vector space over \mathbf{Z}_q . A partition P of the positive integer n is defined as

$$P : n = n_1 + n_2 + \cdots + n_s \quad \text{where} \\ 1 \leq n_1 \leq n_2 \leq \cdots \leq n_s, s \geq 1.$$

The partition P is denoted as

$$P : n = [n_1][n_2] \cdots [n_s].$$

In the case, when

$$P : n = \underbrace{[m_1] \cdots [m_1]}_{l_1\text{-copies}} \underbrace{[m_2] \cdots [m_2]}_{l_2\text{-copies}} \\ \cdots \underbrace{[m_r] \cdots [m_r]}_{l_r\text{-copies}},$$

we write

$$P : n = [m_1]^{l_1} [m_2]^{l_2} \cdots [m_r]^{l_r},$$

where $m_1 < m_2 < \cdots < m_r$.

Given a partition $P : n = [n_1][n_2] \cdots [n_s]$ of the positive integer n , the module space \mathbf{Z}_q^n over \mathbf{Z}_q can be viewed as a direct sum

$$\begin{aligned} \mathbf{Z}_q^n &= \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}, \\ \text{or} \\ V &= V_1 \oplus V_2 \oplus \cdots \oplus V_s, \end{aligned}$$

where $V = \mathbf{Z}_q^n$ and $V_i = \mathbf{Z}_q^{n_i}$ for all $1 \leq i \leq s$.

Consequently, each vector $v \in \mathbf{Z}_q^n$ can be uniquely written as $v = (v_1, v_2, \dots, v_s)$ where $v_i \in V_i = \mathbf{Z}_q^{n_i}$ for all $1 \leq i \leq s$.

Here $v_i (1 \leq i \leq s)$ is called the i^{th} block of block size n_i of the vector v .

Further, we define the modular value $|a|$ of an element $a \in \mathbf{Z}_q$ by

$$|a| = \begin{cases} a & \text{if } 0 \leq a \leq q/2 \\ q - a & \text{if } q/2 < a \leq q - 1. \end{cases}$$

We note that non-zero modular value $|a|$ can be obtained by two different elements viz. a and $q - a$ of \mathbf{Z}_q provided $\{q \text{ is odd}\}$ or $\{q \text{ is even and } a \neq [q/2]\}$ i.e.

$$|a| = |q - a| \quad \text{if} \quad \begin{cases} q \text{ is odd} \\ \text{or} \\ q \text{ is even and } a \neq q/2. \end{cases}$$

If q is even and $a = [q/2]$ or if $a = 0$, then $|a|$ is obtained in only one way viz. $|a| = a$. Thus there may be one or two equivalent values of $|a|$ which we shall refer to as repetitive equivalent values of a . The number of repetitive equivalent values of a will be denoted by e_a where

$$e_a = \begin{cases} 1 & \text{if } \{q \text{ is even and } a = [q/2]\} \text{ or } \{a = 0\} \\ 2 & \text{if } \{q \text{ is odd and } a \neq 0\} \text{ or } \{q \text{ is even, } a \neq 0 \text{ and } a \neq [q/2]\}. \end{cases}$$

3. Definitions and notations

We begin with the discussion of LPA codes [3]:

Let n, q be positive integers with $q > 1$. Let $P : n = [n_1][n_2] \cdots [n_s]$ be a partition of n . We define *Arihant metric* on \mathbf{Z}_q^n corresponding to the partition P as follows:

Let $v = (v_1, v_2, \dots, v_s) \in \mathbf{Z}_q^n = \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$. The Arihant weight of the i^{th} block $v_i \in \mathbf{Z}_q^{n_i}$ ($1 \leq i \leq s$) of the vector v corresponding to the partition P of n is defined as

$$w_A^P(v_i) = \max_{j=1}^{n_i} |v_j^{(i)}|$$

where

$$v_i = (v_1^{(i)}, v_2^{(i)}, \dots, v_{n_i}^{(i)}) \in \mathbf{Z}_q^{n_i}.$$

Thus the Arihant weight of a block is the maximum modular value amongst all its components. Then the Arihant weight of the vector $v = (v_1, v_2, \dots, v_s) \in \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$ corresponding to the partition P is defined as the sum of Arihant weights of all its blocks i.e.

$$W_A^P(v) = \sum_{i=1}^s w_A^P(v_i).$$

For any $u = (u_1, u_2, \dots, u_s)$ and $v = (v_1, v_2, \dots, v_s) \in \mathbf{Z}_q^n = \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$, we define the Arihant distance (or Arihant metric) $d_A^P(u, v)$ between u and v as

$$d_A^P(u, v) = w_A^P(u - v).$$

Then d_A^P is a metric on $\mathbf{Z}_q^n = \mathbf{Z}_q^{n_1} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$.

If the partition P is clear from the context, we shall denote Arihant weight by w_A and Arihant metric by d_A only.

Definition 3.1 [3]. A linear partition Arihant (LPA) code corresponding to the partition $P : n = [n_1] \cdots [n_s]$ is a \mathbf{Z}_q -submodule of $\mathbf{Z}_q^n = \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$ equipped with the Arihant metric and is denoted as $[n, k, d_A; P]$ or $[n, k; P]$ code where

$$k = \text{rank}_{\mathbf{Z}_q}(V),$$

and

$$\begin{aligned} d_A &= d_A(V) \\ &= \text{minimum Arihant distance of } V \\ &= \min\{d_A(u, u') \mid u, u' \in V, u \neq u'\}. \end{aligned}$$

Remark 3.2.

1. For $P : n = [1]^n$, the linear partition Arihant codes reduce to the classical Lee weight codes [7-9]. For this partition, the Arihant distance and Arihant weight reduce to classical Lee distance and Lee weight respectively.
2. For $q = 2, 3$, the linear partition Arihant codes reduce to the linear error-block codes [1] and the Arihant metric reduces to the π -metric introduced by Feng et al. [2].
3. In general, we have

$$\begin{aligned} \pi\text{-metric} &\leq \text{Arihant metric} \\ &\leq \text{Lee metric,} \end{aligned}$$

or equivalently

$$\begin{aligned} \pi\text{-weight} &\leq \text{Arihant weight} \\ &\leq \text{Lee weight.} \end{aligned}$$

We now define P -burst in $\mathbf{Z}_q^n = \mathbf{Z}_q^{n_1} \oplus \mathbf{Z}_q^{n_2} \oplus \cdots \oplus \mathbf{Z}_q^{n_s}$ as follows [5].

Definition 3.3 [5]. Let n be a positive integers and $P : n = [n_1][n_2] \cdots [n_s]$, $1 \leq n_1 \leq n_2 \leq \cdots \leq n_s$ be a partition of n . A P -burst of block length b ($1 \leq b \leq s$) is a vector $v = (v_1, v_2, \dots, v_s) \in \mathbf{Z}_q^n = \bigoplus_{i=1}^s \mathbf{Z}_q^{n_i}$ such that all the non-zero blocks in v are confined to some b consecutive block positions, the first and last of which are non-zero.

Definition 3.4 [5]. A P -burst of block length b or less ($1 \leq b \leq s$) is a P -burst of block length t where $1 \leq t \leq b \leq s$.

Throughout this paper, we shall use the following notations:

1. $[x]$ = The largest integer less than or equal to x .
2. $\lceil x \rceil$ = The smallest integer greater than or equal to x .
3. Q_i = The sum of repetitive equivalent values up to i i.e.,

$$Q_i = e_0 + e_1 + \cdots + e_i$$

where e_i denotes the repetitive equivalent value of i .

4. Extended Campopiano's type bound for LPA codes correcting P -burst errors and simultaneously detecting random block errors

In this section, we obtain an extended Campopiano's type bound which is infact a sufficient bound on the number of parity checks required for an LPA code that correct all random block errors of Arihant weight t or less ($t \geq 1$) and simultaneously detect all P burst errors of block length b or less ($b \geq 1$) with Arihant weight w or less ($w \geq t$).

To prove the result, we need the following.

Let $A_{t,q}^{(n_1, n_2, \dots, n_s)}$ [6] denote the number of all n -vectors corresponding to the partition $P : n = [n_1][n_2] \cdots [n_s]$ with $1 \leq n_1 \leq n_2 \leq \cdots \leq n_s$ having Arihant weight t over \mathbf{Z}_q . Then $A_{t,q}^{(n_1, n_2, \dots, n_s)}$ is given by :

$$A_{t,q}^{(n_1, n_2, \dots, n_s)} = \sum_{r=(r_{ij})} \left(\prod_{i=1}^s \prod_{j=0}^{\lfloor q/2 \rfloor} ((Q_j)^{n_i} - (Q_{j-1})^{n_i})^{r_{ij}} \right), \quad (1)$$

where $r = (r_{ij}) (1 \leq i \leq s, 0 \leq j \leq \lfloor q/2 \rfloor)$ satisfies

(i) for a fixed $i (1 \leq i \leq s)$, $r_{ij} = 1$ for exactly one value of $j (0 \leq j \leq \lfloor q/2 \rfloor)$ and 0 elsewhere; and

(ii)

$$\sum_{i=1}^s \sum_{j=0}^{\lfloor q/2 \rfloor} j r_{ij} = t. \quad (2)$$

Again if $V_{t,q}^{(n_1, n_2, \dots, n_s)}$ denote the number of all n -vector corresponding to the partition $P : n = [n_1][n_2] \cdots [n_s]$, $1 \leq n_1 \leq n_2 \leq \cdots \leq n_s$ having Arihant weight t or less over \mathbf{Z}_q . Then $V_{t,q}^{(n_1, n_2, \dots, n_s)}$ is given by

$$V_{t,q}^{(n_1, n_2, \dots, n_s)} = \sum_{j=0}^t A_{t,q}^{(n_1, n_2, \dots, n_s)} \quad (3)$$

We now give a definition for linear combination of vectors having Arihant weight w .

Definition 4.1. A linear combination of vectors u_1, u_2, \dots, u_r given by

$$\lambda_1.u_1 + \lambda_2.u_2 + \cdots + \lambda_r.u_r,$$

where $\lambda_i = (\lambda_1^{(i)}, \lambda_2^{(i)} \dots \lambda_{n_i}^{(i)})$, $u_i = (u_1^{(i)}, u_2^{(i)}, \dots, u_{n_i}^{(i)}) \in \mathbf{Z}_q^{n_i}$ for all $1 \leq i \leq r$ and (\cdot) denote the usual dot product of vectors, is called a linear combination of Arihant weight w if

$$\max_{a=1}^{n_1} |\lambda_a^{(1)}| + \max_{b=1}^{n_2} |\lambda_b^{(2)}| + \dots + \max_{l=1}^{n_r} |\lambda_l^{(r)}| = w.$$

The following lemma enumerates the number of Arihant weighted P -bursts in block coding:

Lemma 4.2. The number of P bursts of block length b or less with Arihant weight $g \geq 1$ in the space of all $(n_1 + n_2 + \dots + n_{j-1})$ -block vectors over the ring \mathbf{Z}_q is given by

$$\begin{aligned} C_q(b, \sum_{i=1}^{j-1} n_i, g) &= C_q(1, \sum_{i=1}^{j-1} n_i, g) + \sum_{m=2}^b \sum_{r=1}^{j-m} \left(\sum_{\substack{2 \leq \lambda_1 + \lambda_2 \leq g \\ \lambda_1, \lambda_2 \geq 1}} ((Q_{\lambda_1})^{n_r} - (Q_{\lambda_1-1})^{n_r}) \times \right. \\ &\quad \left. ((Q_{\lambda_2})^{n_{r+m-1}} - (Q_{\lambda_2-1})^{n_{r+m-1}}) A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})} \right), \end{aligned} \quad (4)$$

where

$$C_q(1, \sum_{i=1}^{j-1} n_i, g) = \begin{cases} \sum_{i=1}^{j-1} ((Q_g)^{n_i} - (Q_{g-1})^{n_i}) & \text{if } g \leq [q/2], \\ 0 & \text{if } g > [q/2], \end{cases}$$

and $A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})}$ is given by (1) satisfying (2).

Proof. There are two cases:

Case 1. When $b = 1$.

In this case, the number of P -bursts of block length 1 with Arihant weight g in the space of all $n_1 + n_2 + \dots + n_{j-1}$ -tuples over \mathbf{Z}_q is given by

$$C_q(1, \sum_{i=1}^{j-1} n_i, g) = \begin{cases} \sum_{i=1}^{j-1} ((Q_g)^{n_i} - (Q_{g-1})^{n_i}) & \text{if } g \leq [q/2], \\ 0 & \text{if } g > [q/2]. \end{cases} \quad (5)$$

Case 2. When $b \geq 2$.

Consider a P -burst of block length m and Arihant weight g where $2 \leq m \leq b \leq j-1$ and $g \geq 1$. Such a P -burst can have first $(j-m)$ block positions as the starting block positions. Suppose the P -burst starts at the r^{th} block ($1 \leq r \leq j-m$) and suppose that the Arihant weights of the starting and ending blocks are $\lambda_1 (\neq 0)$ and $\lambda_2 (\neq 0)$ resp.

Then the number of choices for the starting r^{th} block and ending $(r + m - 1)^{th}$ block together is given by

$$((Q_{\lambda_1})^{n_r} - (Q_{\lambda_1-1})^{n_r})((Q_{\lambda_2})^{n_{r+m-1}} - (Q_{\lambda_2-1})^{n_{r+m-1}}). \quad (6)$$

The remaining $(m - 2)$ blocks viz $(r + 1)^{th}, (r + 2)^{th}, \dots, (r + m - 2)^{th}$ blocks of the P -burst should make up a sum of Arihant weight $g - \lambda_1 - \lambda_2$ so that the total Arihant weight of the P -burst becomes equal to g . The number of ways in which these $(m - 2)$ blocks can be filled is given by

$$A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})}, \quad (7)$$

where $A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})}$ is given by (1) satisfying (2).

The total number of P -bursts of Arihant weight g and block length m starting from the r^{th} block is obtained by multiplying (6) and (7) and then summing the resulting product for different values of λ_1 's and λ_2 's satisfying $\lambda_1 \geq 1, \lambda_2 \geq 1, 2 \leq \lambda_1 + \lambda_2 \leq g$ and is given by

$$\sum_{\substack{2 \leq \lambda_1 + \lambda_2 \leq g \\ \lambda_1, \lambda_2 \geq 1}} ((Q_{\lambda_1})^{n_r} - (Q_{\lambda_1-1})^{n_r})((Q_{\lambda_2})^{n_{r+m-1}} - (Q_{\lambda_2-1})^{n_{r+m-1}}) \times A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})}. \quad (8)$$

Since r can take values from 1 to $j - m$ and m can take values from 2 to b , therefore, summing (8) for different values for m and r gives the number of P -bursts of block length varying from 2 to b and having Arihant weight g and is given by

$$\sum_{m=2}^b \sum_{r=1}^{j-m} \left(\sum_{\substack{2 \leq \lambda_1 + \lambda_2 \leq g \\ \lambda_1, \lambda_2 \geq 1}} ((Q_{\lambda_1})^{n_r} - (Q_{\lambda_1-1})^{n_r}) \times ((Q_{\lambda_2})^{n_{r+m-1}} - (Q_{\lambda_2-1})^{n_{r+m-1}}) A_{g-\lambda_1-\lambda_2, q}^{(n_{r+1}, n_{r+2}, \dots, n_{r+m-2})} \right). \quad (9)$$

The result now follows by adding (5) and (9). \square

Remark 4.3.(i) The number of all P -bursts in $\mathbf{Z}_q^n = \bigoplus_{i=1}^s \mathbf{Z}_q^{n_i}$ of block length b or less ($b \leq s$) with Arihant weight w or less including the pattern of all zeros is given by

$$C_q^*(b, \sum_{i=1}^s n_i, w) = 1 + \sum_{g=1}^w C_q(b, \sum_{i=1}^s n_i, g).$$

(ii) The number of all P -bursts in $\mathbf{Z}_q^n = \bigoplus_{i=1}^s \mathbf{Z}_q^{n_i}$ of block length b or less ($b \leq s$) having Arihant weight lying between w_1 and w_2 is given by

$$C_q^*(b, \sum_{i=1}^s n_i, w_1, w_2) = \sum_{g=w_1}^{w_2} C_q(b, \sum_{i=1}^s n_i, g).$$

We now prove the extended Campopiano's type bound for LPA codes correcting P -burst errors and simultaneously detecting random block errors.

Theorem 4.4. *Let n be a positive integer and $P : n = [n_1][n_2] \cdots [n_s], 1 \leq n_1 \leq n_2 \leq \cdots \leq n_s$ be a partition of n . Let t, w and b be positive integers such that $1 \leq t \leq 2w \leq 2b[q/2]$ and $s \geq 2b$. Then a sufficient condition for the existence of an $[n, k; P]$ LPA code with minimum Arihant weight at least t that correct all P -bursts of block length b or less having Arihant weight w or less is given by*

$$q^{n-k} \geq \left(\sum_{\lambda=1}^{[q/2]} ((Q_\lambda)^{n_s} - (Q_{\lambda-1})^{n_s}) \right) \times D + E, \quad (10)$$

where

$$\begin{aligned} D = & V_{t-1-\lambda, q}^{(n_1, n_2, \dots, n_{s-1})} + C_q^*(b-1, \sum_{i=s-b+1}^{s-1} n_i, 2t-\lambda, 2w-\lambda) + \\ & \sum_{k=1}^{b-1} \sum_{\theta=1}^{\min([q/2], w-1)} \sum_{r_{1\theta}, r_{2\theta}, r_{3\theta}} ((Q_\theta)^{n_{s-2b+k+1}} - (Q_{\theta-1})^{n_{s-2b+k+1}}) \times \\ & A_{r_{1\theta}, q}^{(n_{s-2b+k+1}, \dots, n_{s-b})} A_{r_{2\theta}, q}^{(n_{s-b+1}, \dots, n_{s-b+k})} A_{r_{3\theta}, q}^{(n_{s-b+k+1}, \dots, n_{s-1})}, \end{aligned}$$

and

$$E = \sum_{\substack{p_1, p_2: \\ p_1+p_2=t}}^{2w} \sum_{\lambda=1}^{[q/2]} ((Q_\lambda)^{n_s} - (Q_{\lambda-1})^{n_s}) A_{p_1-\lambda, q}^{(n_{s-b+1}, \dots, n_{s-1})} C_q(b, \sum_{i=1}^{s-b} n_i, p_2),$$

subject to

$$\begin{aligned} 1 &\leq \theta + r_{1\theta} \leq w-1; \\ 1 &\leq +r_{2\theta} \leq 2w-1-\lambda; \\ 0 &\leq +r_{3\theta} \leq w-\lambda; \\ r_{2\theta} + r_{3\theta} &\geq t-\lambda; \\ \theta + r_{1\theta} + r_{2\theta} &\geq t; \\ 2t-\lambda &\leq \theta + r_{1\theta} + r_{2\theta} + r_{3\theta} \leq 2w-\lambda; \\ 0 &\leq p_1, p_2 \leq w. \end{aligned} \quad (11)$$

Proof. The existence of such a code is shown by constructing an appropriate $(n - k) \times n$ (where $n = n_1 + n_2 + \dots + n_s$) parity check matrix H for the desired LPA code. The constructed block matrix H will be of the form

$$H = [H_1, H_2, \dots, H_s],$$

where $H_i = (H_1^{(i)}, H_2^{(i)}, \dots, H_{n_i}^{(i)})$ is the i^{th} block of H of size n_i .

We construct the block matrix H as follows:

Suppose we have chosen the first $(j - 1)$ blocks suitably. The j^{th} block H_j of size n_j can be added to H provided the following two conditions are satisfied:

1. There should not exist a linear dependence relation between blocks of columns of H of Arihant weight $(t - 1)$ or less i.e. every linear combination of Arihant weight $(t - 1)$ or less of the blocks of columns of H must be linearly independent;
2. Any two linear combinations, each of Arihant weight w or less, of blocks of columns of H from amongst b or fewer consecutive blocks should be different.

In other words, we can say

$$\lambda_1.H_1 + \lambda_2.H_2 + \dots + \lambda_{j-1}.H_{j-1} + \lambda_j.H_j \neq 0, \quad (12)$$

and

$$\begin{aligned} &(\alpha_j.H_j + \alpha_{j-1}.H_{j-1} + \dots + \alpha_{j-b+1}.H_{j-b+1}) + \\ &(\beta_{i_1}.H_{i_1} + \beta_{i_2}.H_{i_2} + \dots + \beta_{i_b}.H_{i_b}) \neq 0, \end{aligned} \quad (13)$$

where

- (i) $\lambda_1.H_1 + \lambda_2.H_2 + \dots + \lambda_{j-1}.H_{j-1} + \lambda_j.H_j$ ($\lambda_i \in \mathbb{Z}_q^{n_i}$ for all $1 \leq i \leq j-1$, $\lambda_j \in \mathbb{Z}_q^{n_j} \setminus \{0\}$) is any linear combination of the j^{th} block H_j to be added and the first $(j - 1)$ previously chosen blocks such that the Arihant weight of the linear combination is atmost $t - 1$;
- (ii) $\alpha_j.H_j + \alpha_{j-1}.H_{j-1} + \dots + \alpha_{j-b+1}.H_{j-b+1}$ ($\alpha_i \in \mathbb{Z}_q^{n_i}$ for all $j - b + 1 \leq i \leq j - 1$ and $\alpha_j \in \mathbb{Z}_q^{n_j} \setminus \{0\}$) is any linear combination of Arihant weight w or less of the blocks of columns from amongst the j^{th} block H_j to be added and the immediately preceding $(b - 1)$ blocks;
- (iii) $\beta_{i_1}.H_{i_1} + \beta_{i_2}.H_{i_2} + \dots + \beta_{i_b}.H_{i_b}$ ($\beta_l \in \mathbb{Z}_q^{n_l}$ for all $i_1 \leq l \leq i_b$) is any linear combination of Arihant weight w or less of the columns from amongst any b (or fewer) consecutive blocks of columns out of the previously chosen $(j - 1)$ blocks.

The number of ways in which the coefficients λ_i can be chosen is given by

$$\sum_{\lambda=1}^{[q/2]} ((Q_\lambda)^{n_j} - (Q_{\lambda-1})^{n_j}) V_{t-1-\lambda, q}^{(n_1, n_2, \dots, n_{j-1})}, \quad (14)$$

where

$$1 \leq \lambda = w_A(\lambda_j) \leq [q/2].$$

Since all possible linear combinations of Arihant weight $(t-1)$ or less are included in (14), therefore, the coefficients α_i ($j-b+1 \leq i \leq j$) and β_l ($i_1 \leq l \leq i_b$) should be chosen so that the sum of their Arihant weights is at least t .

To obtain the number of all possible distinct linear combinations, we analyze three different cases:

Case 1. When the blocks H_l ($i_1 \leq l \leq i_b$) are taken from the first $(j-b)$ blocks.

In this case, choose α_i ($1 \leq i \leq j$) such that the sum of their Arihant weights is equal to a number p_1 and β_l ($i_1 \leq l \leq i_b$) having sum of their Arihant weights equal to p_2 such that $p_1 + p_2 \geq t$. The largest value which p_1 and p_2 can attain is w .

Now α_i ($1 \leq i \leq j$) with sum of their Arihant weights equal to p_1 can be chosen in

$$\sum_{\lambda=1}^{[q/2]} ((Q_\lambda)^{n_j} - (Q_{\lambda-1})^{n_j}) A_{p_1-\lambda, q}^{(n_{j-b+1}, \dots, n_{j-1})} \quad (15)$$

To choose β_l ($i_1 \leq l \leq i_b$) with sum of their Arihant weights equal to p_2 is equivalent to evaluating the number of P -bursts of block length b or less with Arihant weight p_2 in a block vector of length $n_1 + n_2 + \dots + n_{j-b}$. This can be done (by Lemma 4.2) in

$$C_q \left(b, \sum_{i=1}^{j-b} n_i, p_2 \right) \text{ ways.} \quad (16)$$

Case 2. When the blocks H_l ($i_1 \leq l \leq i_b$) are taken from the immediately preceding $(b-1)$ blocks.

The additional number of ways in which the coefficients α_i ($j-b+1 \leq i \leq j$) and β_i ($i_1 \leq l \leq i_b$) can be chosen are given by

$$\sum_{\lambda=1}^{[q/2]} ((Q_\lambda)^{n_j} - (Q_{\lambda-1})^{n_j}) C_q^*(b-1, \sum_{i=j-b+1}^{j-1} n_i, 2t-\lambda, 2w-\lambda). \quad (17)$$

Case 3. When the blocks H_l ($i_1 \leq l \leq i_b$) are selected from $H_{j-2b+2}, H_{j-2b+3}, \dots, H_{j-1}$ such that all are neither taken from $H_{j-2b+2}, \dots, H_{j-b}$ nor from $H_{j-b+1}, H_{j-b+2}, \dots, H_{j-1}$.

Let us suppose that the P -burst starts from the $(j - 2b + k + 1)^{th}$ block position which may continue upto $(j - b + k)^{th}$ block position ($1 \leq k \leq b - 1$). Also, let the Arihant weight of the $(j - 2b + k + 1)^{th}$ block be θ where $1 \leq \theta \leq \min([q/2], w - 1)$. The total number of choices for selecting the components of $(j - 2b + k + 1)^{th}$ block is given by

$$(Q_\theta)^{n_{j-2b+k+1}} - (Q_{\theta-1})^{n_{j-2b+k+1}}.$$

Our objective is to select non-zero blocks from the $(j - 2b + k + 1)^{th}, (j - 2b + k + 2)^{th}, \dots, (j - b)^{th}, (j - b + 1)^{th}, \dots, (j - b + k)^{th}, (j - b + k + 1)^{th}, \dots, (j - b)^{th}$ blocks having sum of their Arihant weight w or less or this, let us have linear combinations of Arihant weight $r_{1\theta}$ of blocks of columns from the $(j - 2b + k + 2)^{th}, \dots, (j - b)^{th}$ blocks; linear combination of Arihant weight $r_{2\theta}$ of columns from the $(j - b + 1)^{th} \dots, (j - b + k)^{th}$ blocks and linear combinations of Arihant weight $r_{3\theta}$ of columns from the $(j - b + k + 1)^{th}, \dots, (j - b)^{th}$ blocks. Then the total number of choices for the coefficients α_i ($1 \leq i \leq j$) and β_l ($i_1 \leq l \leq l_b$) in Case 3 is given by

$$\begin{aligned} & \sum_{\lambda=1}^{[q/2]} \left((Q_\lambda)^{n_j} - (Q_{\lambda-1})^{n_j} \right) \\ & \left(\sum_{k=1}^{b-1} \sum_{\theta=1}^{\min([q/2], w-1)} \sum_{r_{1\theta}, r_{2\theta}, r_{3\theta}} \right. \\ & \left. ((Q_\theta)^{n_{j-2b+k+1}} - (Q_{\theta-1})^{n_{j-2b+k+1}}) A_{r_{1\theta}, q}^{(n_{j-2b+k+1}, \dots, n_{j-b})} \times \right. \\ & \left. A_{r_{2\theta}, q}^{(n_{j-b+1}, \dots, n_{j-b+k})} A_{r_{3\theta}, q}^{(n_{j-b+k+1}, \dots, n_{j-1})} \right), \end{aligned} \quad (18)$$

where

$$\begin{aligned} 1 & \leq \theta + r_{1\theta} \leq w - 1; \\ 1 & \leq r_{2\theta} \leq 2w - 1 - \lambda; \\ 0 & \leq r_{3\theta} \leq w - \lambda; \\ r_{2\theta} + r_{3\theta} & \geq t - \lambda; \\ \theta + r_{1\theta} + r_{2\theta} & \geq t \\ 2t - \lambda & \leq \theta + r_{1\theta} + r_{2\theta} + r_{3\theta} \leq 2w - \lambda. \end{aligned}$$

Thus the total number of linear combinations occurring in all the three cases is given by

$$(14) + \left(\sum_{\substack{p_1 + p_2 = 2w \\ p_1, p_2: \\ p_1 + p_2 = t}} (15) \times (16) \right) + (17) + (18)$$

$$\begin{aligned}
&= \sum_{\lambda=1}^{\lfloor q/2 \rfloor} ((Q_{\lambda})^{n_j} - (Q_{\lambda-1})^{n_j}) V_{t-1-\lambda, q}^{(n_1, n_2, \dots, n_{j-1})} + \\
&\quad \left(\sum_{\substack{p_1, p_2: \\ p_1 + p_2 = t}}^{p_1 + p_2 = 2w \lfloor q/2 \rfloor} \sum_{\lambda=1}^{\lfloor q/2 \rfloor} ((Q_{\lambda})^{n_j} - (Q_{\lambda-1})^{n_j}) A_{p_1-\lambda, q}^{(n_{j-b+1}, \dots, n_{j-1})} \times \right. \\
&\quad \left. C_q(b, \sum_{i=1}^{j-b} n_i, p_2) \right) + \sum_{\lambda=1}^{\lfloor q/2 \rfloor} ((Q_{\lambda})^{n_j} - (Q_{\lambda-1})^{n_j}) \times \\
&\quad C_q^*(b-1, \sum_{i=j-b+1}^{j-1} n_i, 2t-\lambda, 2w-\lambda) + \sum_{\lambda=1}^{\lfloor q/2 \rfloor} ((Q_{\lambda})^{n_j} - (Q_{\lambda-1})^{n_j}) \times \\
&\quad \left(\sum_{k=1}^{b-1} \sum_{\theta=1}^{\min\{\lfloor q/2 \rfloor, w-1\}} \sum_{r_{1\theta}, r_{2\theta}, r_{3\theta}} ((Q_{\theta})^{n_{j-2b+k+1}} - (Q_{\theta-1})^{n_{j-2b+k+1}}) A_{r_{1\theta}, q}^{(n_{j-2b+k+1}, \dots, n_{j-b})} \times \right. \\
&\quad \left. A_{r_{2\theta}, q}^{(n_{j-b+1}, \dots, n_{j-b+k})} A_{r_{3\theta}, q}^{(n_{j-b+k+1}, \dots, n_{j-1})} \right), \\
&= L \quad (\text{say}).
\end{aligned}$$

Hence the j^{th} block H_j of block size n_j can be added to the parity check matrix H if the number of available $(n-k)$ -vectors which is q^{n-k} in number is at least as large as L i.e. if

$$q^{n-k} \geq L. \quad (19)$$

For the existence of an $[n, k; P]$ LPA code where $n = n_1 + n_2 + \dots + n_s$, the inequality (19) must hold for $j = s$ so that it is possible to add upto s^{th} block H_s to form an $(n-k) \times n$ block matrix H and we get (10). \square

Corollary 4.5. *Taking $t = 1$ in Theorem 4.4., we get a sufficient bound for weighted P -burst error correction in LPA codes analogous to classical Campopiano bound [1].*

5. Conclusion.

In this paper, we have obtained a sufficient bound which is infact a construction upper bound for LPA codes correcting weighted P -burst errors and simultaneously detecting random block errors. The bound obtained is an extended Campopiano's type bound for LPA codes.

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