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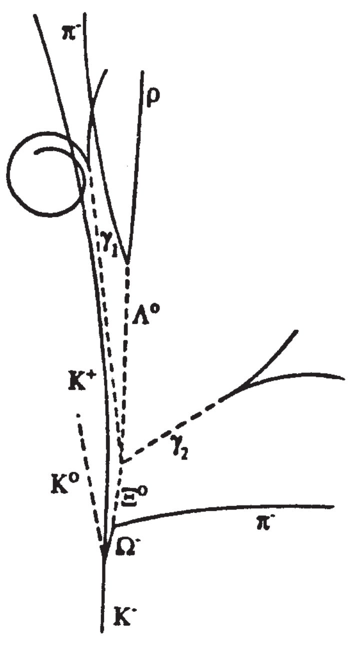

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UNIVERSE (PARTICLES) AND ANTI-UNIVERSE (ANTIPARTICLES) AS WELL AS VACUUM ENERGY DENSITY OR COSMOLOGICAL "CONSTANT" INCLUDING QUANTUM GRAVITY

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Abstract

We introduce the massless (however more exactly massive) universe and anti-universe for distances smaller or equal to the Planck length as well as the massive universe and anti-universe for scale factors equal to or greater than the Planck length. From the big bang, the momentary formation of the massless universe and anti-universe, which expand in opposite directions, leads to a separation of the virtual particle-antiparticle pairs of the quantum vacuum into real particle (matter) and antiparticle (antimatter) via the gravitational interaction, whereat these separated particles and antiparticles are in thermal equilibrium with the photons, so that at the Planck length all, real particles and antiparticles have the Planck energy as relativistic energy for their start in the correspondingly separated massive (Euclidian) universe and anti-universe. On this way, for these massless universes, we can derive a simple solution for the long-sought, four-dimensional quantum gravity, which via the transition from the final state of these massive universes to the big bang in the massless universes permits the estimation of the parameters of the big bang and the evaluation of the lifetime of the sterile neutrinos, so that we obtain a cyclic evolution of the total (massless and massive) universes because of the decay of the sterile neutrinos of the massive universes in photons by the gravitation. We show that for the massive universe all results (except the age of its final state), derived in works [1-3], are confirmed. For the first time, we demonstrate that all these results [1-3] are also valid for the massive anti-universe.

1 Introduction

In this work, we distinguish the massless ($R \leq R_{\mathrm{Pl}}$) and the massive ($R \geq R_{\mathrm{Pl}}$) universe and anti-universe, where R_{Pl} is the distance of the Planck length. We have introduced these massless universes because their massless photons which are in thermal equilibrium with the particles, define originally their radiation energy density, so that it must be identical with their particle energy density and vacuum energy density, i.e. these massless universes contain also correspondingly massive (anti)matter. In contrast to these massless universes, the massive universe and anti-universe are characterized by identical radiation density and particle density, which however dominate the vacuum energy density, so that it can be neglected at the description of the massive universes except at the present, accelerated expansion and the final state of the massive universes. The massless universes are described by the gravitation in this work, whereas the massive universes are defined by a new inflation model [1-3] and the Friedmann-Lemaitre Equations [1-10].

In work [1], we have derived the vacuum energy density for $R = R_{\mathrm{Pl}}$ via a new inflation model [1-3] defined by the supersymmetric grand unification particles (X and Y gauge bosons [3] as well as magnetic monopoles [3]) in the massive universe ($R \geq R_{\mathrm{Pl}}$). In the present work, for $R = R_{\mathrm{Pl}}$, we find the same value by a new expression for this vacuum energy density via a new, modified, quantum-statistical photon energy density. This new expression can be generalized for $R \geq R_{\mathrm{Pl}}$. Using this new expression at $R = R_{\mathrm{Pl}}$, for the massless universes ($R \leq R_{\mathrm{Pl}}$), we find a general vacuum energy density derived by aid of the gravitation. The works [1-3] show that the massive universe is completely described via the new inflation model [1-3] and the Friedmann-Lemaitre Equations [1-10] for Euclidian geometry [1-3] by the present, cosmological parameter values [1-10], which were also exactly estimated in Refs. [1, 2] via the light neutrino density parameters [1-3], multiplied by the different ratios of the relativistic energy and the rest energy of the supersymmetric grand unification particles [1-3]. On this way, because of the known, general properties of particles and antiparticles, in the present work, for the massive anti-universe ($R \geq R_{Pl}$), we can derive also the present, cosmological parameter values similar as at the massive universe if we use the antiparticles of the light neutrinos and the supersymmetric grand unification particles. Therefore, we assume that the massive anti-universe is also described completely as the massive universe [1-3] if we determine the necessary quantities of the Friedmann-Lemaitre Equations for the massive anti-universe on the basis of the general properties of particles and antiparticles. Thus, because of the time-symmetry of the solutions of the Friedmann-Lemaitre equations, these massive universes must expand in opposite directions, i.e. separation of matter and antimatter.

This result is supported by following considerations. From the big bang, the momentary formation of the massless universe and anti-universe, which expand in opposite directions, leads to a separation of the virtual particle-antiparticle pairs of the quantum vacuum into real particle (matter) and antiparticle (antimatter) via the gravitational interaction, whereat these separated particles and antiparticles are in thermal equilibrium with the photons, so that at the Planck length all, real particles and antiparticles have the Planck energy as relativistic energy for their start in the correspondingly separated massive (Euclidian) universe and anti-universe. On this way, for these massless universes, we can derive a simple solution for the long-sought, four-dimensional quantum gravity, which via the transition from the final state of these massive universes to the big bang permits the determination of the parameters of the big bang and the evaluation of the lifetime of the sterile neutrinos, i.e. we obtain a cyclic evolution of the total (massless and massive) universes as a result of the dark energy converted half into (massless) photons or (massive) relics by the decay of the sterile neutrinos via the gravitation.

We show that for the massive universe all results (except the age of its final state), derived in works [1-3], are confirmed. For the first time, we demonstrate that all these results [1-3] are also valid for the massive anti-universe.

For the massive universe, the critically analyzed, present-day, cosmological parameter values [5] are given in Table I. For better comparison of observed and estimated, present-day, cosmological parameter values, we mention here also their measured Planck 2013 data [10], summarized in Table II.

TABLE I. The most important, critically analyzed, (present-day), cosmological parameter values [5] of the massive universe for the Friedmann-Lemaitre Equations (see Sec. 2). [a] Two different fits [5]

Quantity	Symbol, equation	Value
present day CMB temperature	T_0	$2.7255(6)\,\mathrm{K}$
present day Hubble expansion rate	H_0	$67.3 \pm 1.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}} =$ $= h \times (9.777752\,\mathrm{Gyr})^{-1} =$ $= (2.181 \pm 0.039) \cdot 10^{-18}\,\mathrm{s^{-1}}$
scale factor for Hubble expansion rate	h	0.673 ± 0.012
critical present density of the universe	$\rho_{0C} = 3H_0^2 / 8\pi G_N$	$1.05375(13) \cdot 10^{-5} h^2 (\mathrm{GeV}/c^2)\,\mathrm{cm^{-3}} =$ $= 4.77(17) \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm^{-3}}$
baryon (proton) density of the universe	Ω_b	$0.02207(27)\,h^{-2} = 0.0487(23)$[a], $0.0499(22)$[a]
(cold) dark matter density of the universe	Ω_{dm}	$0.1198(26)\,h^{-2} = 0.265(15)$, $0.265(11)$[a]
dark energy density of the universe	Ω_Λ	$0.685^{+0.017}_{-0.016}$

TABLE I, continued.

Quantity	Symbol, equation	Value
pressureless matter density of the universe	$\Omega_{\mathrm{m}} = \Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}}$	$0.315^{+0.016}_{-0.017}$
CMB radiation density of the universe	Ω_γ	$2.473 \cdot 10^{-5} (T/2.7255\ \mathrm{K})^4 h^{-2} =$ $= 5.46(19) \cdot 10^{-5}$
curvature	$\Omega_{\mathrm{tot}} = \Omega_{\mathrm{m}} + \Omega_\Lambda + \cdots$	$0.96^{+0.4}_{-0.5}$ (95% CL); 1.000(7) (95% CL; CMB + BAO)
sum of neutrino masses	$\sum m_\nu c^2$	< 0.23 eV
neutrino density of the universe	Ω_ν	$< 0.0025\, h^{-2}$; < 0.0055
redshift of matter-radiation equality	z_{eq}	3360 ± 70
redshift of photon decoupling	z_{dec}	1090.2 ± 0.7
redshift at half reionization	z_{reion}	11.1 ± 1.1
age of the universe	t_0	13.81 ± 0.05 Gyr = $= (4.358 \pm 0.016) \cdot 10^{17}$ s

TABLE II. The measured, cosmological parameter values of Planck 2013 [10]. In column 1, the corresponding parameter symbols are given. The column 2 gives results for the Planck temperature data alone. Column 3, denoted as Planck+WP, combines these Planck data and the WMAP polarization data at low multipoles.

	Planck	**Planck+WP**
Parameter	**Best fit [68% limits]**	**Best fit [68% limits]**
H_0 [km/s Mpc]	67.11 [67.4 ± 1.4]	67.04 [67.3 ± 1.2]
Ω_Λ	0.6825 [0.686 ± 0.020]	0.6817 [$0.685^{+0.018}_{-0.016}$]
$100\,\Omega_b h^2$	2.2068 [2.207 ± 0.033]	2.2032 [2.205 ± 0.028]
$\Omega_{dm} h^2$	0.12029 [0.1196 ± 0.0031]	0.12038 [0.1199 ± 0.0027]
$\Omega_m h^2$	0.14300 [0.1423 ± 0.0029]	0.14305 [0.1426 ± 0.0025]
Age t_0 [Gyr]	13.819 [13.813 ± 0.058]	13.8242 [13.817 ± 0.048]
z_{eq}	3402 [3386 ± 69]	3403 [3391 ± 60]
z_{dec}	1090.43 [1090.37 ± 0.65]	1090.48 [1090.43 ± 0.54]
z_{reion}		11.37 [11.1 ± 1.1]

For the description of the massive anti-universe, the (present-day), cosmological parameter values of Tables I and II are also valid, except the values H_0, h and t_0, which are here negative. The reasons of this behavior are the known properties of particle and antiparticle according to Table III.

TABLE III. The known properties of particles and antiparticles.

Property	**Particle**	**Anti-particle**
energy	E	E
mass	M	M
spin	s	s
momentum	p	$-p$
velocity	v	$-v$
elementary electric charge	q	$-q$

Using these special properties of particles and antiparticles in Table III, we can also determine important quantities, which are necessary for the description of the massless and massive universe and anti-universe. These quantities are summarized in Table IV. However, the Friedmann-Lemaitre Equations and the new inflation model are only valid for scale factors $R \geq R_{\mathrm{Pl}}$.

At $R \leq R_{\mathrm{Pl}}$, for the massless universe and anti-universe, we must use the quantum cosmology or quantum gravity, for which the corresponding quantities are also given in Table IV. However, unfortunately, to this day, this quantum gravity is still highly incomplete and yields therefore no reliable predictions because the connection between cosmological "constant" and vacuum energy density is not clear [4].

Therefore, in this work, for the massless universes, by aid of the virtual matter of the quantum vacuum, we have derived a simple solution of the above-mentioned four-dimensional quantum gravity.

TABLE IV. The most important quantities of the massless and massive universe and anti-universe.

Quantity	Universe	Anti-universe
energy	E	E
temperature	T	T
Boltzmann constant	k	k
gravitational constant	G_N	G_N
scale factor	R	R
momentum	p	$-p$
velocity	v	$-v$
speed of light in vacuum	c	$-c$
reduced Planck constant	$\hbar$	$-\hbar$
time	t	$-t$
Hubble expansion rate	H	$-H$
scale factor of Hubble expansion rate	h	$-h$
inflationary phase	Δt	$-\Delta t$
ionized mass	q	$-q$

Under virtual matter, as a result of the uncertainty relation, we understand its formed particle-antiparticle (photon-photon) pairs, which have only an extremely short lifetime, so that they can be described however by a mean energy density and mean pressure. Because they are also formed in a radiation-free and matter-free space, the denotation "quantum vacuum" is usually applied. Taking the quantum field theory as a starting point, the total space-time continuum of the massless and the massive universe and anti-universe is penetrated with this quantum vacuum, which forms a non-eliminative background as ground state at absence of real matter.

In this work, we have above assumed that from the big bang (BB) at $R = R_{\text{BB}}$, the immediately formed, massless universe and anti-universe expand in opposite direction, so that we obtain directly and correspondingly the separation of the virtual matter [photon(particle)-photon(anti-particle) pairs] of the massless universe and antimatter [photon(anti-particle)-photon(particle) pairs] of the massless anti-universe into real matter [photon(particle)-photon(particle) pairs] and antimatter [photon(antiparticle)-photon(antiparticle) pairs] as "two" real radiation fields as well as real particle ($R_{\text{BB}} \le R \le R_{\text{Pl}}$) and antiparticle field ($R_{\text{BB}} \le R \le R_{\text{Pl}}$) of the massless universe and anti-universe, respectively. Because of the very high photon density of the massless universes, these processes are possible, whereat we must however consider that the photon is its own antiparticle.

Therefore, in this work, for example, the thermal energy of the photons in the new thermal equilibrium is directly equivalent to the rest energy of the particles or antiparticles for the limiting case, where now the kinetic energy of the particles or antiparticles is zero, i.e. we can simply solve the separation of matter and antimatter. In the old interpretation, where here for the universe the particles and antiparticles are produced simultaneously by photons with the thermal energy of the sum of their rest energy in the thermal equilibrium, so that this old interpretation cannot solve the problem of the separation of matter and antimatter.

Attention! In the next chapters. If for $R \ge R_{\text{Pl}}$ we apply the Friedmann-Lemaitre Equations and the new inflation model for the description of universe and anti-universe, we mean always the massive universe and anti-universe. This condition is also valid for the Tables I to IV. However, if for $R \le R_{\text{Pl}}$ we describe universe and anti-universe by the general vacuum energy density of the quantum gravity, we treat always the massless universe and anti-universe. This last condition is also valid for Tables I to IV.

Therefore, this work is organized as follows. In Sec. 2, we describe shortly the Friedmann-Lemaitre Equations as well as their most important, known solutions and problems, whereat we include SUSY GUTs. In Sec. 3, we derive a new inflationary cosmology, whereat we treat the neutrino data (sec. 3.1) the necessary properties of the X, Y gauge bosons (Sec. 3.2), the scale factors of absorption of light at the redshift condition before and after the inflation (Sec. 3.3), the necessary properties of the magnetic monopoles (Sec. 3.4), the definition of the new inflation model (Sec. 3.5) and the derivation of the vacuum energy density for universe and anti-universe by the new inflation model via quantities of the old inflation model at $R = R_{\mathrm{Pl}}$ (Sec. 3.6). In Sec. 4, we find a simple solution for the four-dimensional quantum gravity via the vacuum energy density, whereat we treat the vacuum energy as a result of the negative pressure of the photons by a "new", quantum-statistical expression for their energy density at $R = R_{\mathrm{Pl}}$ (Sec. 4.1), the vacuum energy density as a result of the negative pressure of the photons in thermal equilibrium with the particles or antiparticles via gravitation at $R = R_{\mathrm{Pl}}$ (Sec. 4.2), the particle formation or the general vacuum energy density as a result of the negative pressure of the photons in thermal equilibrium with the particles or antiparticles via the gravitation for $R \leq R_{\mathrm{Pl}}$ (Sec. 4.3), the general vacuum energy density as a result of the negative pressure of the photons by a "new" quantum-statistical expression of their energy density for $R \geq R_{\mathrm{Pl}}$ (Sec. 4.4) and some conclusions from the vacuum energy density or the cosmological "constant" (Sec. 4.5). In Sec. 5, we derive generally the curvature of the universe, whereat we show that this derivation must be valid also for the anti-universe. In Sec. 6, we define the most important, (present-day) cosmological parameter values on the basis of general properties of particles and antiparticles for the description of universe and anti-universe by the Friedmann Lemaitre Equations and the new inflation model. In Sec. 7, we describe shortly important, new results for the universe and the anti-universe as the final state and the end of the present, exponential expansion by a negative acceleration (Sec. 7.1), the astronomical unit changing (Sec. 7.2), the future by a slow, linear expansion after the present, accelerated expansion (Sec. 7.3) and the redshift values of the reionization (Sec. 7.4). In Sec. 8, we derive the heavy (anti)neutrino types. In Sec. 9, for the (anti-)universe, on the basis of observed sterile neutrino data, we introduce the sterile (anti)neutrino types for the dark (anti)matter (Sec. 9.1), the dark (anti-)energy (Sec. 9.2), the (anti)baryon mass (Sec. 9.3) and the photon decoupling [sterile CMB (anti)neutrinos] (Sec. 9.4) as well as explain semi-empirically the sterile (anti)neutrino calculations (Sec 9.5). In Sec. 10, we define special properties of all neutrino types for universe and anti-universe, whereat we treat the definition of the (present-day) cosmological parameter values by the rest energy of the heavy and the sterile (anti)neutrinos (Sec. 10.1) as well as the transformations of various types of the light and heavy as well as sterile (anti)neutrinos into one another (Sec. 10.2). In Sec. 11, we estimate the parameters of the big bang and evaluate the lifetime of the sterile neutrinos. In Sec. 12, we give a short summary. The values of the physical constants, used in this work, are given in works [5, 8].

2 The Friedmann-Lemaitre Equations with known solutions and problems of standard model as well as SUSY GUTs

In this chapter, the considerations of the Friedmann-Lemaitre Equations and their most important, known solutions are based on Refs. [1-10]. In agreement with all of those parts of the universe, which are accessible by observations, we can assume that at (very) early times and today on a sufficiently large scale the universe is very nearly homogeneous and isotropic. For such a space, the most general four-dimensional line element ds is determined by the Friedmann-Robertson-Walker metric in the form

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-\mathrm{k}r^2} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right], \qquad (2.1)$$

where c is the velocity of light in vacuum and t is the time, whereas the dimensionless Lagrange coordinates r, ϑ and φ are constant because of the expansion of the universe, so that $R(t)$ determines alone the time dependence of this metric, i.e. $R = R(t)$ is the scale factor for distances in comoving coordinates. By appropriate rescaling of the radial coordinate, the constant k can be chosen to be $+1$ (closed geometry with finite volume), -1 (open hyperbolic space) or 0 (Euclidian (spatially flat) geometry with infinite volume).

In such an expanding universe, the observed wavelength λ of the light emitted (e) from a distant source with λ_e, is shifted towards the red. Via the Friedmann-Robertson-Walker metric, this so-called redshift z is defined by

$$1 + z = \frac{\lambda}{\lambda_e} = \frac{R_0}{R}, \tag{2.2}$$

where R_0 and R are the scale factors of absorption and emission of light, respectively.

In the Friedmann-Robertson-Walker metric, Einstein's equations lead to the Friedmann equation

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = -\mathrm{k}\frac{c^2}{R^2} + \frac{8\pi G_N}{3}\rho + \frac{c^2}{3}\Lambda \tag{2.3}$$

as well as to the equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{2.4}$$

as the so-called Friedmann-Lemaitre Equations, where $H = H(t)$ is the Hubble parameter, G_N is the gravitational constant, ρ is the mean mass density, P is the isotropic pressure and Λ is the cosmological constant. From Eqs. (2.3) and (2.4), we can derive a third useful equation

$$\dot{\rho} = -3\frac{\dot{R}}{R}(\rho + \frac{P}{c^2}) = -3\frac{\dot{R}}{R}\rho(1+w), \tag{2.5}$$

where $w = P/\rho c^2$ is the so-called state parameter. Additional to the Hubble parameter, it is useful to define also several, other, measurable cosmological parameters. For this goal, at $\Lambda = 0$ (standard model) and $\mathrm{k} = 0$, via the Friedmann equation, a critical (C) density can be defined by

$$\rho_C = \frac{3H^2}{8\pi G_N} = 1.87847(23)\times 10^{-29} h^2 \,\mathrm{g\,cm^{-3}} =$$
$$= 1.05375(13)\times 10^{4} h^2 (\mathrm{eV}/c^2)\,\mathrm{cm^{-3}}\,, \tag{2.6}$$

where the scaled Hubble parameter h is defined by the expression

$$H = 100 h \,\mathrm{km\,s^{-1}\,Mpc^{-1}} = \frac{h}{9.777752\,\mathrm{Gyr}} =$$
$$= 3.240779\times 10^{-18} h \,\mathrm{s^{-1}}\,. \tag{2.7}$$

Then, we can rewrite the Friedmann equation as

$$\mathrm{k}\frac{c^2}{R^2} = H^2(\Omega - 1) \tag{2.8}$$

with

$$\Omega = \frac{\rho}{\rho_C}\,. \tag{2.9}$$

Because it is often necessary to distinguish between different contributions to the density ρ, it is usual to define present-day density parameters for constant dark energy (Ω_Λ), pressureless matter (Ω_m) and relativistic particles (Ω_r), so that the very important Friedmann equation (2.8) can be written in the form

$$\mathrm{k}\frac{c^2}{R_0^2} = H_0^2(\Omega_\Lambda + \Omega_\mathrm{m} + \Omega_\mathrm{r} - 1) = H_0^2(\Omega_\mathrm{tot} - 1)\,, \tag{2.10}$$

where the new subscript "0" characterizes the present-day values for $\Omega_\mathrm{tot} = \Omega_\Lambda + \Omega_\mathrm{m} + \Omega_\mathrm{r}$. In the expression (2.10), it is often used the quantity

$$\Omega_\mathrm{k} = -\mathrm{k}\frac{c^2}{R_0^2 H_0^2} = \Omega_\mathrm{tot} - 1\,. \tag{2.11}$$

In Eq. (2.10), Ω_r is defined by

$$\Omega_\mathrm{r} = \frac{1}{2} N(T)\,\Omega_\gamma\,, \tag{2.12}$$

where the statistical particle quantity $N(T)$, which has here the value

$N(T) = 3.362644$ for photons accompanied with massless or nearly massless neutrinos and antineutrinos of three different types, counts generally the massless degrees of freedom of bosons and fermions, whereat $\Omega_\gamma = 2.4728 \times 10^{-5} h^{-2}$ is defined by the present-day ratio of the photon density to the critical density (see Eq. (3.20)). For massive particles, the values $N(T)$ are given in the Table V.

For constant w, Eq. (2.5) can easily be integrated, so that we get

$$\rho \propto R^{-3(1+w)}. \tag{2.13}$$

In Eq. (2.3), at early times when R is small, the term $-\mathrm{k}c^2/R^2$ can be neglected for $w > -1/3$. Then, for $w \neq -1$, the Friedmann equation can be written as $(\dot{R}/R)^2 \propto R^{-3(1+w)}$ and is easily integrated to yield

$$R \propto t^{2/[3(1+w)]}. \tag{2.14}$$

For the radiation-dominated universe, in Eqs. (2.13) and (2.14), we can substitute $w = 1/3$, so that we obtain

$$\rho \propto R^{-4}; \qquad R \propto t^{1/2}; \qquad H = 1/2t. \tag{2.15}$$

At the matter-dominated universe, in Eqs. (2.13) and (2.14), we must substitute $w = 0$, so that we get

$$\rho \propto R^{-3}; \qquad R \propto t^{2/3} \quad (\text{if } \mathrm{k} = 0); \qquad H = 2/3t. \tag{2.16}$$

If there is an universe for $\Lambda \neq 0$ with a dominant source of constant vacuum (vac) mass density ρ_{vac}, we would observe this mass density as cosmological "constant" $\Lambda = 8\pi G_\mathrm{N} \rho_{\text{vac}} / c^2$ and state $w = -1$ (see Eq. (2.5) and (2.13)). In this case (denoted in this work as old inflation model), Eq. (2.3) yields an extremely rapid, exponential expansion of the universe according to

$$R = R_1\, \mathrm{e}^{\sqrt{8\pi G_N \rho_{\text{vac}}/3}\,\Delta t} = R_1\, \mathrm{e}^{\sqrt{c^2 \Lambda/3}\,\Delta t} = R_1\, \mathrm{e}^{H \Delta t}, \tag{2.17}$$

where Δt describes thus the period of the inflationary phase, whereas H denotes the short-time Hubble constant.

TABLE V. The important $N(T)$ values. [a] T_c corresponds to the confinement-deconfinement transition between quarks and hadrons. [b] For $T > m_{X,Y}$, the minimal SU(5) model (SUSY GUTs) yields $N(T) = 160.75$.

Temperature	**New particles**	$4N(T)$ [b]
$T < m_e$	photons (γ)+neutrinos (ν)	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c$ [a]	$\pi^\pm$, π^0	69
$T_c < T < m_{\text{strange}}$	$\pi^\pm$, $\pi^0 + u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\text{charm}}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\text{bottom}}$	$\tau^\pm$	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\text{Higgs}}$	$W^\pm$, Z^0	381
$m_H < T < m_{\text{top}}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

More generally, in the standard model, via the present-day critical density (see Eqs. (2.6) and (2.10)), we can now introduce the density

$$\rho = \rho_\Lambda + \rho_m + \rho_r = \left[\Omega_\Lambda + \Omega_m (R_0/R)^3 + \Omega_r (R_0/R)^4\right]\frac{3H_0^2}{8\pi G_N} =$$

$$= \left[\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_r (1+z)^4\right]\frac{3H_0^2}{8\pi G_N}. \tag{2.18}$$

Then, the relativistic formalism (2.3) yields

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_m\left(\frac{R_0}{R}\right)^3 + \Omega_r\left(\frac{R_0}{R}\right)^4\right] - k\frac{c^2}{R^2} \tag{2.19}$$

with the general solution

$$t = t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\,(\Omega_\Lambda + \Omega_k x^{-2} + \Omega_m x^{-3} + \Omega_r x^{-4})^{1/2}}, \tag{2.20}$$

where $x \equiv (R/R_0) = (1/[1+z])$. In Eq. (2.20), for a flat universe ($\Omega_k = 0$), at $z \to 0$, because of the very small value Ω_r compared with Ω_Λ and Ω_m, the term $\Omega_r x^{-4}$ can be neglected, since at present most of the integral (2.20) comes from an epoch when the expansion is dominated by the matter and the dark energy.

Then, by Eq. (2.20), for $z = 0$, the age of the present universe (see below) is defined by

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{dx}{x\,(\Omega_\Lambda + \Omega_m x^{-3})^{1/2}}. \tag{2.21}$$

For an universe, which expands adiabatically, the entropy per comoving volume is constant, so that we can describe the mass density of the radiation-dominated universe for $T > 3\cdot 10^5$ K by

$$\rho = \rho_r = \{\pi^2/30(\hbar^3 c^5)\}(kT)^4 N(T), \tag{2.22}$$

where $\hbar$ is the Planck constant and k is the Boltzmann constant. Comparing ρ_r from Eqs. (2.18) and (2.22), for the redshift of all relativistic particles, we can also write

$$1 + z = \frac{R_0}{R} = \frac{T}{T_0}, \tag{2.23}$$

where T_0 is the present-day, mean temperature of the cosmic microwave background (CMB) [see Table I].

For Eq. (2.22), the values of $N(T)$, taken from Ref. [8], are given in Table V. At higher temperatures, these values depend on the particle physics models. Because of Table III, we assume that they are also valid for anti-particles.

Using the expressions (2.15) and (2.22), the Friedmann equation (2.3) yields the time for the radiation-dominated universe ($z > 10^5$) as a function of the thermal energy kT, the temperature T or the redshift z by the expressions

$$t = t(z) = \frac{1}{2}\left(\frac{90\hbar^3 c^5}{8\pi^3 G_N}\right)^{1/2} \frac{1}{\sqrt{N(T)}\,(kT)^2} =$$

$$= \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\text{MeV}}{kT}\right)^2 \text{s} =$$

$$= \frac{(3.25936 \pm 0.00021)}{\sqrt{N(T)}}\left(\frac{10^{10}\ \text{K}}{T}\right)^2 \text{s} =$$

$$= \frac{(0.438773 \pm 0.000028)}{\sqrt{N(T)}}\left(\frac{10^{10}}{1+z}\right)^2 \text{s}. \tag{2.24}$$

According to Eqs. (2.2) and (2.18), for the matter-dominated universe, at $z_{\text{eq}} = 3360 >> z >> 1$ (see Tables I and II) and $\text{k} = 0$, we observe as dominant density

$$\rho = \rho_{\text{m}} = \left[\Omega_{\text{m}} (R_0/R)^3\right]\frac{3H_0^2}{8\pi G_N} = \left[\Omega_{\text{m}} (1+z)^3\right]\frac{3H_0^2}{8\pi G_N}, \tag{2.25}$$

whereas for $z_{\text{eq}} >> z$ we cannot more neglect Ω_Λ, so that here we have

$$\rho = \rho_\Lambda + \rho_{\text{m}} = \left[\Omega_\Lambda + \Omega_{\text{m}} (R_0/R)^3\right]\frac{3H_0^2}{8\pi G_N} =$$

$$= \left[\Omega_\Lambda + \Omega_{\text{m}} (1+z)^3\right]\frac{3H_0^2}{8\pi G_N}. \tag{2.26}$$

In Sec. 7.1, these cases (2.25) and (2.26) are treated.

At $k = 0$, for the dominant term $\Omega_\Lambda H_0^2$, Eq. (2.19) yields the (present-day) exponential expansion

$$R = R_0\, e^{\Omega_\Lambda^{1/2} H_0\,(t-t_0)} = R_0\, e^{H(t-t_0)} \quad \text{at} \quad H = \Omega_\Lambda^{1/2} H_0\,, \qquad (2.27)$$

where t_0 is the age of the present-day universe (see Table I).

Consequently, the standard cosmology ($\Lambda = 0$) [1-4] provides a very successful framework for the explanation of the cosmic observations. However, there are a number of problems, which cannot simply be solved within the standard cosmology. These problems are the horizon problem, the monopole problem, the flatness problem and the initial condition problem. A way out of these problems is the inflation [1-4, 6], which in the early universe enlarges the scale factor by a factor of about 10^{30} [1-3, 6, 7, 9]. However, this inflationary cosmology and its transition to the standard model are quantitatively still an open problem [1-3, 7].

For a better understanding, we explain now shortly the above-mentioned problems of the standard model as follows.

The temperature of the cosmic microwave background (CMB) is the same in all directions of the universe with an accuracy better than about 10^{-5}. The question is how all these disconnected and non-interacted regions can have this extremely high temperature isotropy. This problem is the so-called horizon problem of the standard cosmology. By the inflation, this isotropy of the CMB can be explained because prior to the inflation the universe occupied such a small volume, so that after the inflation these very small regions rapidly attain gigantic dimensions and cannot longer interact [1-3, 7].

Using the SUSY GUT transition [5] for the early universe, the formation of magnetic monopoles is expected with a density of about one per horizon volume at that epoch [4]. This enormous density dominates the present cosmic matter density in a fatal conflict with the measurements [1-4]. This is the so-called monopole problem of the standard cosmology. At the end of the inflation this monopole density would be reduced by a factor of about $(10^{-30})^3$, so that the contribution of the monopoles to the present cosmic density would be completely negligible in accordance with the observations [1- 4]. However, this inflation is only qualitatively solved (see above).

In spite of the Planck 2013 results [10], which favor $\Omega_k = 0$ (see Eq. (2.11)), the assumption that Ω_{tot} (see Eqs. (2.8) and (2.10)) differs mildly from unity, leads to the so-called flatness problem of the standard cosmology, which at the Planck time requires an extreme "fine-tuning" of Ω_{tot}. A trivial way out of this problem is to postulate $\Omega_{tot} \equiv 1$ throughout the history of the universe [1-4].

All above-mentioned problems do not falsify the standard model. However, they can be interpreted into the standard cosmology as initial conditions. Therefore, the standard cosmology provides only a consistent model to explain the observable universe with some assumed initial conditions without to explain their origin [1, 3, 4]. For many years it was assumed that the initial conditions for the standard cosmology could arise from the quantum cosmology at very early times, where the universe was so small that the classical cosmology model is no longer valid. Unfortunately, the quantum cosmology cannot present up to now reliable predictions. However, the situation changed in the early 1980s by the new inflation concept, which solves qualitatively these most pressing problems within the classical cosmology because the inflation operates at an energy scale that is much lower than the Planck energy, so that the cosmology can be treated classically [1-4].

For the early universe, the supersymmetric grand unification theories (SUSY GUTs) predict that X, Y gauge bosons and magnetic monopoles play an important role as a result of the unification of the strong and electroweak interaction at energies $E_G \approx 3 \cdot 10^{16}\,\mathrm{GeV}$ [5]. In these SUSY GUTs, the baryon number is violated, i.e. we must expect a proton decay [5]. The rest energy of the X and Y gauge bosons is equivalent to $E_G \approx 3 \cdot 10^{16}\,\mathrm{GeV}$, whereas the rest energy of the magnetic monopoles (M) is given by $E_0(\mathrm{M}) = E_G / \alpha_{GUT}$ [5, 11], where the SUSY coupling constant is $\alpha_{GUT} \approx 0.04$ [12-16].

In the next chapters, we see that the considerations, treated in this Sec. 2 for the massive universe, are also valid for the massive anti-universe, using the time-symmetry of the solutions of the Friedmann-Lemaitre Equations.

3 A new inflationary cosmology

3.1 The light neutrino data

There are three light (left-handed) neutrinos (tauon neutrino ν_τ, muon neutrino ν_μ and electron neutrino ν_e).

According to Beringer et al. [8], the best values of the 3-neutrino oscillation parameters, derived from global neutrino oscillation data, yield for the square mass differences of the neutrino rest energies [1-3]:

$$\Delta m_{\tau\mu}^2 c^4 = E_0^2(\nu_\tau) - E_0^2(\nu_\mu) = (2.35^{+0.12}_{-0.09}) \cdot 10^{-3}\,\mathrm{eV}^2, \tag{3.1}$$

$$\Delta m_{\mu e}^2 c^4 = E_0^2(\nu_\mu) - E_0^2(\nu_e) = (7.58^{+0.22}_{-0.26}) \cdot 10^{-5}\,\mathrm{eV}^2. \tag{3.2}$$

From Eqs. (3.1) and (3.2), we can form the equations

$$E_0(\nu_\tau) = \left(\Delta m_{\tau\mu}^2 c^4 + E_0^2(\nu_\mu)\right)^{1/2} = \\ = \left(\Delta m_{\tau\mu}^2 c^4 + \Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \tag{3.3}$$

and

$$E_0(\nu_\mu) = \left(\Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2}. \tag{3.4}$$

By Beringer et al., for the normal hierarchy $E_0(\nu_\tau) > E_0(\nu_\mu) > E_0(\nu_e)$, from Eqs. (3.1) to (3.4), the neutrino rest energies $E_0(\nu_\tau)$ and $E_0(\nu_\mu)$ were estimated to

$$E_0(\nu_\tau) \approx \left|\Delta m_{\tau\mu}^2 c^4\right|^{1/2} \approx 4.8 \cdot 10^{-2}\,\mathrm{eV} \tag{3.5}$$

and

$$E_0(\nu_\mu) \approx \left|\Delta m_{\mu e}^2 c^4\right|^{1/2} \approx 8.6 \cdot 10^{-3}\,\mathrm{eV}. \tag{3.6}$$

Then, for normal hierarchy, because of transformations of various types of neutrinos into one another (neutrino oscillations), their rest energies must be connected [1-3], i.e. we can assume as new approach

$$E_0(\nu_e)/E_0(\nu_\mu) = E_0(\nu_\mu)/E_0(\nu_\tau) \quad \Rightarrow \quad E_0(\nu_e) = \frac{E_0^2(\nu_\mu)}{E_0(\nu_\tau)}, \tag{3.7}$$

so that by Eqs. (3.3) and (3.4) via Eq. (3.7) we obtain

$$E_0(\nu_e) = \frac{\Delta m_{\mu e}^2 c^4}{\left[\Delta m_{\tau\mu}^2 c^4 - \Delta m_{\mu e}^2 c^4\right]^{1/2}} = (1.589^{+0.078}_{-0.098}) \cdot 10^{-3}\,\text{eV}. \tag{3.8}$$

Consequently, by Eq. (3.8), the expressions (3.3) and (3.4) yield

$$E_0(\nu_\tau) = \left(\Delta m_{\tau\mu}^2 c^4 + \Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \cong (4.93^{+0.12}_{-0.10}) \cdot 10^{-2}\,\text{eV} \tag{3.9}$$

and

$$E_0(\nu_\mu) = \left(\Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \cong (8.85^{+0.14}_{-0.16}) \cdot 10^{-3}\,\text{eV} \tag{3.10}$$

in accordance with Eqs. (3.5) and (3.6). Then, the sum of the neutrino rest energies (see Eqs. (3.8) to (3.10)) gives the approximate solution

$$\sum_i E_0(\nu_i) = E_0(\nu_\tau) + E_0(\nu_\mu) + E_0(\nu_e) \cong (5.97^{+0.14}_{-0.13}) \cdot 10^{-2}\,\text{eV}, \tag{3.11}$$

where the subscript $i = \text{e}, \mu, \tau$ characterizes the e, μ and τ neutrino. The sum (3.11) does not contradict the observed limit $\sum_i E_0(\nu_i) < 0.23\,\text{eV}$ [5, 10].

Finally, we derive still the neutrino densities of the universe. Prior to the derivation of these light neutrino density parameters, we must explain the connection between the radiation and the critical density of the present universe. These considerations are based on Refs. [1-3, 6, 8, 9]. The cosmic microwave (radiation) background (CMB) is observed as a black-body spectrum with the mean temperature

$$T = T_0 = 2.7255 \pm 0.0006\,\text{K}. \tag{3.12}$$

Consequently, this observation is consistent with a Planck distribution at the temperature (3.12). The Planck distribution of photons is quantum statistically characterized by the energy density

$$\rho(\gamma)c^2 = \frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3} = $$

$$= (4.722158 \pm 0.000017) \cdot 10^{-3} \left(\frac{T}{\text{K}}\right)^4 \text{eV/cm}^3 \tag{3.13}$$

and the number density

$$n(\gamma) = \frac{2.404113806}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 =$$

$$= (20.286843 \pm 0.000057)\left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3}, \qquad (3.14)$$

where the value 2.404113806 corresponds to the integral $\int_0^\infty [x^2/(e^x - 1)]\mathrm{dx}$.

The radiation density of the universe is defined by

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0\mathrm{C}}}, \qquad (3.15)$$

where the present photon density $\rho_0(\gamma)$ and the critical present density $\rho_{0\mathrm{C}}$ are given by

$$\rho_0(\gamma) = \frac{\pi^2}{15}\frac{(kT_0)^4}{\hbar^3 c^5} \qquad \text{(see Eq. (3.13))}, \qquad (3.16)$$

$$\rho_{0\mathrm{C}} = \frac{3H_0^2}{8\pi G_N} \qquad \text{(see Eq. (2.6))}. \qquad (3.17)$$

Thus, Eq. (3.16) gives

$$\rho_0(\gamma)\,c^2 = \frac{\pi^2}{15}\frac{(kT_0)^4}{(\hbar c)^3} = (0.26057 \pm 0.00023)\ \mathrm{eV\,cm^{-3}}. \qquad (3.18)$$

For Eq. (3.17), according to Table I, we get

$$\rho_{0\mathrm{C}}\,c^2 = \frac{3c^2 H_0^2}{8\pi G_N} = (1.05375 \pm 0.00013)\cdot 10^4 h^2\ \mathrm{eV\,cm^{-3}}. \qquad (3.19)$$

Then, via Eqs. (3.18) and (3.19), Eq. (3.15) yields the present radiation density of the universe to

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0\mathrm{C}}} = (2.4728 \pm 0.0025)\cdot 10^{-5} h^{-2}. \qquad (3.20)$$

By Eqs. (3.13) and (3.14), the kinetic (K) energy of the photons is given by

$$E_\mathrm{K}(\gamma) = \frac{\rho(\gamma)\,c^2}{n(\gamma)} = 2.701178\,kT. \qquad (3.21)$$

Now, analogously to the present baryon number density [1-3, 5, 6], we can introduce the present neutrino number densities

$$n_0(\nu_i) = \Omega_\nu(\nu_i)\frac{\rho_{0C}c^2}{E_0(\nu_i)}, \tag{3.22}$$

so that the neutrino densities of the universe are determined by

$$\Omega_\nu(\nu_i) = E_0(\nu_i)\frac{n_0(\nu_i)}{\rho_{0C}c^2}. \tag{3.23}$$

For the neutrino temperature $T_0(\nu_i) = (4/11)^{1/3} T_0 = (1.94537 \pm 0.00043)\,\mathrm{K}$ [6, 8], we find the present neutrino number densities (for every subscript $i = \mathrm{e}, \mu, \tau$) to

$$n_0(\nu_i) = \frac{3}{4}\frac{2.404113806}{\pi^2}\frac{(k[(4/11)^{1/3} T_0])^3}{(\hbar c)^3} =$$

$$= (112.016 \pm 0.074)/\mathrm{cm}^3. \tag{3.24}$$

Analogous to Refs. [1-3, 5], the expressions (3.15) as well as (3.23) and (3.24) yield

$$\Omega_\nu(\nu_i) = E_0(\nu_i)\frac{n_0(\nu_i)}{\rho_0(\gamma)c^2}\Omega_\gamma = \frac{45}{11}\frac{2.4041138}{\pi^4}\frac{E_0(\nu_i)}{kT_0}\Omega_\gamma =$$

$$= (429.889 \pm 0.095)\,\mathrm{eV}^{-1} E_0(\nu_i)\,\Omega_\gamma =$$

$$= (0.010630 \pm 0.000011)\,\mathrm{eV}^{-1} E_0(\nu_i)\,h^{-2}. \tag{3.25}$$

Using the rest energies (3.8) to (3.10) together with the radiation density $\Omega_\gamma = (5.46 \pm 0.19) \cdot 10^{-5}$ (see Table I), by Eq. (3.25), we get (see Refs. [1-3]):

$$\Omega_\nu(\nu_\mathrm{e}) \cong (0.683^{+0.034}_{-0.042})\Omega_\gamma =$$

$$= (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} h^{-2} = (3.73^{+0.32}_{-0.36}) \cdot 10^{-5}, \tag{3.26}$$

$$\Omega_\nu(\nu_\mu) \cong (3.805^{+0.061}_{-0.070})\Omega_\gamma =$$

$$= (9.41^{+0.16}_{-0.18}) \cdot 10^{-5} h^{-2} = (2.08 \pm 0.11) \cdot 10^{-4}, \tag{3.27}$$

$$\Omega_\nu(\nu_\tau) \cong (21.19^{+0.52}_{-0.44})\Omega_\gamma =$$
$$= (5.24^{+0.13}_{-0.11}) \cdot 10^{-4} h^{-2} = (1.157^{+0.069}_{-0.064}) \cdot 10^{-3} . \tag{3.28}$$

The sum of Eqs. (3.26) to (3.28) gives

$$\Omega_\nu = \sum_i \Omega_\nu(\nu_i) \cong (25.68^{+0.62}_{-0.55})\Omega_\gamma =$$
$$= (6.35^{+0.16}_{-0.14}) \cdot 10^{-4} h^{-2} = (1.402^{+0.083}_{-0.079}) \cdot 10^{-3} . \tag{3.29}$$

The sum (3.29) does not contradict the observed limiting values $\Omega_\nu < 0.0055$ and $\Omega_\nu < 0.0025 h^{-2}$ (see Table I). Then, the deviations amount to only a factor of 4.

Using the data of Tables III and IV, the results, derived in this Sec. 3.1, are also valid for the anti-universe.

3.2 The necessary properties of the X and Y gauge bosons

For the Standard $\mathrm{SU(2)} \times \mathrm{U(1)}$ electroweak theory, Fujikawa and Shrock [17] (see also Ref. [8]) have derived the magnetic moment of massive neutrinos to

$$\mu_\nu(\nu_i) = \frac{3}{8\pi^2\sqrt{2}} e[G_F/(\hbar c)^3]\hbar c^2 E_0(\nu_i) =$$
$$= 3.203 \cdot 10^{-19} (E_0(\nu_i)/\mathrm{eV}) \mu_B , \tag{3.30}$$

where $[G_F/(\hbar c)^3] = 1.1663787 \cdot 10^{-23} \mathrm{eV}^{-2}$ describes the Fermi coupling constant [8]. The Bohr magneton μ_B [8] is defined by

$$\mu_B = \frac{e\hbar c^2}{2 E_0(\mathrm{e})} = (5.7883818066(38) \cdot 10^{-11} \mathrm{MeV/T} , \tag{3.31}$$

where $E_0(\mathrm{e})$ is the rest energy of the electron (e) [8]. The magnetic moment of the electron is given by $\mu_\mathrm{e} = \mathrm{C_e}\mu_B$, where $\mathrm{C_e} \cong 1.00116$ [8]. Similar to the nucleon magneton $\mu_N = (E_0(\mathrm{e})/E_0(p))\mu_B$ [8] with the proton (p) rest energy $E_0(\mathrm{p})$ [8], the magneton of the X gauge boson (SUSY grand unification magneton [1-3]) is assumed to

$$\mu_X = \frac{E_0(e)}{E_0(X)}\mu_B \,, \tag{3.32}$$

where $E_0(X) = E_0(Y)$ is the rest energy of the X, Y gauge bosons, so that analogous to the magnetic moment of the electron (see above) the magnetic moment of the neutrinos, using reasonably $1 \le C_\nu \le C_e$, may be assumed to

$$\mu_\nu(\nu_i) \cong C_\nu \frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)}\mu_X = C_\nu \frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)} \frac{E_0(e)}{E_0(X)}\mu_B \,. \tag{3.33}$$

Then, the derivation of Eq. (3.33) is clear because the expression $\sum_i \mu_\nu(\nu_i)/C_\nu$ gives again the supersymmetric grand unification magneton μ_X (see Eq. (3.32)). By Eqs. (3.30) and (3.33), if we assume as the maximum value $C_\nu = C_e$ in Eq. (3.33), we may estimate the rest energy of the X gauge boson to

$$E_0(X) \cong C_e \frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)} \frac{E_0(e)}{\mu_\nu(\nu_i)}\mu_B =$$

$$= C_e \frac{E_0(e)}{\sum_i E_0(\nu_i)} \frac{\text{eV}}{3.203 \cdot 10^{-19}} = (2.675^{+0.058}_{-0.063}) \cdot 10^{16}\,\text{GeV} \,. \tag{3.34}$$

For the early universe, the value (3.34) agrees excellently with the gauge coupling unification $E_G \approx 3 \times 10^{16}\,\text{GeV}$ of SUSY GUTs [5], which is identical with $E_0(X) = E_G$ and $E_0(Y) = E_G$ [5, 11].

Because of the new interpretation $kT = E_0(X)$ of the thermal equilibrium (see the paragraph after Eq. (3.36)), by Eqs. (2.24) and (3.34) as well as $N(T) = N_{X,Y}(T) = 160.75$ (see Table V, which is also valid for the anti-universe because of Tables III and IV), we obtain the time $t_{X,Y}$ and the temperature $T_{X,Y}$ for these X and Y gauge bosons of SUSY GUTs [1-3]:

$$t = t_{X,Y} = \frac{2.42035}{\sqrt{N_{X,Y}(T)}} \left[\frac{\text{MeV}}{E_0(X)}\right]^2 \text{s} = (2.67^{+0.13}_{-0.12}) \cdot 10^{-40}\,\text{s} \tag{3.35}$$

and $$T = T_{X,Y} = E_0(X)/k = (3.104^{+0.067}_{-0.073}) \cdot 10^{29}\,\text{K} \propto t_{X,Y}^{-1/2} \,. \tag{3.36}$$

At the beginning of the massive universe ($R \geq R_{\text{Pl}}$), all real particles (P) have the Planck energy as relativistic energy in the new thermal equilibrium (see Sec. 4.4) with the photons ($kT \geq E_0(\text{P})$). In this particle mixture, only the events, which are determined by the reaction laws of nuclear and elementary physics, can take place, i.e. we have no complete mixture of particles and antiparticles in the universe, so that the small number of particles and antiparticles, produced by these reaction laws, must again disappear by annihilation. Therefore, in this work, we can also use the statistical weight $N(T)$, given in Sec. 2, since they were obtained by consideration of these reaction laws of nuclear and elementary physics.

Taking Eqs. (2.23) and (3.36), we find the redshift of the X and Y gauge bosons to

$$1 + z_{\text{X,Y}} = \frac{T_{\text{X,Y}}}{T_0} = (1.139^{+0.025}_{-0.027}) \cdot 10^{29}\,. \tag{3.37}$$

Using the data of Tables III and IV, the results, derived in this Sec. 3.2, are also valid for the anti-universe.

3.3 The scale factors of absorption of light at the redshift condition before and after the inflation

Using $H_0 = (2.181 \pm 0.039) \cdot 10^{-18}\,\text{s}^{-1}$ (see Table I), for the present-day critical density $\rho_{0\text{C}}$ (see Eq. (2.6)), we can introduce (see Ref. [1, 3]):

$$\rho_{0\text{C}} = \frac{3 H_0^2}{8\pi G_N}\left[\frac{R_{\text{Pl}}^2}{R_{\text{Pl}}^2}\right] = \frac{3 H_0^2}{8\pi G_N}\left[\frac{\hbar\, G_N/c^3}{R_{\text{Pl}}^2}\right] =$$

$$= \frac{3\hbar}{8\pi c (R_{\text{Pl}}\, c/H_0)^2} = \frac{3\hbar}{8\pi c \widetilde{R}_0^4}\,, \tag{3.38}$$

where the Planck length $R_{\text{Pl}} = 1.616200 \cdot 10^{-35}\,\text{m}$ is defined by Eq. (7.1), so that the scale factor $\widetilde{R}_0$ is given by

$$\widetilde{R}_0 = (R_{\text{Pl}}\, c/H_0)^{\frac{1}{2}} = (4.713 \pm 0.042) \cdot 10^{-5}\,\text{m}\,. \tag{3.39}$$

Similar to Eq. (3.38), we can also introduce (see Ref. [1, 3]):

$$\rho_{0\text{C}} = \frac{3H_0^2}{8\pi G_N}\left[\frac{c^2}{c^2}\right] = \frac{3}{8\pi G_N}\frac{c^2}{[c/H_0]^2} = \frac{3c^2}{8\pi G_N R_0^2} \tag{3.40}$$

with

$$R_0 = c/H_0 = (1.375 \pm 0.025) \cdot 10^{26}\,\text{m}\,. \tag{3.41}$$

Between Eqs. (3.39) and (3.41), we have trivially the connection $\widetilde{R}_0 = (R_{\text{Pl}}\, R_0)^{\frac{1}{2}}$. Because of $\widetilde{R}_0 << R_0$, we assume that Eqs. (3.39) and (3.41) represent the "two" unknown scale factors of the absorption of light at the redshift (see Eq. (2.2)) prior to and after the inflation [1-3], respectively.

To these values (3.39) and (3.41), we will return in Sec. 3.5, where they are used generally for the determination of the scale factors $R = R(t)$ by the redshift condition (2.2).

Using the data of Table IV, the results, derived in this Sec. 3.3, are also valid for the anti-universe.

3.4 The necessary properties of the magnetic monopoles

Using the two scale factors of the absorption of light at the redshift before (see Eq. (3.39)) and after (see Eq. (3.41)) the inflation [1-3] (see also Sec. 3.5), because of Eqs. (2.2) and (2.23), we can define

$$1 + z_{\text{M}} = \frac{T_{\text{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\text{Pl}}} \doteq \frac{R_0}{\widetilde{R}_0} = (2.916 \pm 0.026) \cdot 10^{30}\,. \tag{3.42}$$

In Eq. (3.42), in contrast to Eq. (3.37), because of $E_0(\text{M}) > E_0(\text{X})$ (see the next to last paragraph of Sec. 2), the subscript M is assumed to define the magnetic monopoles, so that via Eqs. (2.23) and (3.12) we can determine their temperature [1-3] by the new thermal equilibrium (see the paragraph after Eq. (3.36) in Sec. 3.2) as follows:

$$T_{\rm M} = (1+z_{\rm M})T_0 = (7.948 \pm 0.073)\cdot 10^{30}\,{\rm K}\,. \qquad (3.43)$$

Consequently, the rest energy of the magnetic monopoles [1-3] is defined by the redshift condition $1+z_{\rm M} = T_{\rm M}/T_0$ (see Eq. (3.42)) to

$$E_0({\rm M}) = kT_{\rm M} = (1+z_{\rm M})kT_0 = (6.849 \pm 0.063)\cdot 10^{17}\,{\rm GeV}\,, \qquad (3.44)$$

if we use the new interpretation of the thermal equilibrium (see above).

Then, by Eqs. (3.34) and (3.44), the coupling constant of the SUSY GUTs [1-3, 5] is found to

$$\alpha_{\rm GUT} = \frac{E_0({\rm X})}{E_0({\rm M})} = 0.0391^{+0.0012}_{-0.0013} \qquad (3.45)$$

in excellent agreement with the value $\alpha_{\rm GUT} \approx 0.04$ [12-16], which was obtained by a completely other way [16].

Therefore, the results (3.34) and (3.45) confirm the assumption (3.43) and the estimation (3.11) as well as the solutions (3.39) and (3.41) as the unknown scale factor of the absorption of light at the redshift condition (see Eq. (2.2)) before and after the inflation [1-3].

In SUSY GUTs the baryon number is violated, so that they predict a proton decay. By Eqs. (3.34) and (3.45), the proton lifetime [1, 3, 5] is estimated to

$$\tau_{\rm p} = \frac{\hbar}{2\alpha_{\rm GUT}^2\,[E_0({\rm p})]^5/[E_0({\rm X})]^4} = 4.803\cdot 10^{36}\,{\rm yr} \qquad (3.46)$$

in accordance with the experimental limiting value of the proton lifetime $\tau_{\rm p}({\rm p}\to\pi^0{\rm e}^+) \geq 1.0\cdot 10^{34}\,{\rm yr}$ [8].

With that, non-SUSY GUTs (e.g., non-SUSY SU(5)), which predict a proton lifetime $\tau_{\rm p} \approx 10^{31}\,{\rm yr}$ at $E_{\rm G} \approx 10^{15}\,{\rm GeV}$ [16], are ruled out by the non-observation of the proton decay in the allowed range [5].

Using Eqs. (2.24) and (3.44), for the magnetic monopoles, we can determine the time [1-3]:

$$t_{\rm M} = \frac{2.42035}{\sqrt{N_{\rm M}(T)}}\left[\frac{\rm MeV}{kT_{\rm M}}\right]^2 {\rm s} = \frac{(5.160 \pm 0.095)\cdot 10^{-42}}{\sqrt{N_{\rm M}(T)}}\,{\rm s}\,. \qquad (3.47)$$

Using the data of Tables III and IV, the results, derived in this Sec. 3.4, are also valid for the anti-universe.

3.5 A new inflation model for universe and anti-universe

In this chapter, we analyse the connections (3.42) [1, 3]:

$$1+z_{\mathrm{M}} = \frac{T_{\mathrm{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\mathrm{Pl}}} = \mathrm{e}^{H\Delta t} = (2.916 \pm 0.026) \cdot 10^{30} \tag{3.48}$$

and

$$1+z_{\mathrm{M}} = \frac{T_{\mathrm{M}}}{T_0} = \frac{R_0}{\widetilde{R}_0} = \mathrm{e}^{H\Delta t} = (2.916 \pm 0.026) \cdot 10^{30}\,, \tag{3.49}$$

whereat the exponential term $\mathrm{e}^{H\Delta t}$ is considered in the next Sec. 3.6.

For this goal, we use the radiation-dominated universe ($z > 10^5$). Then, by Eqs. (2.2) and (2.22) as well as (2.23) and (3.15), we can write

$$\rho_{\mathrm{r}} = \{\pi^2/30(\hbar^3 c^5)\}(kT)^4 N(T) = \frac{1}{2} N(T)\, \rho_0(\gamma) \left(\frac{T}{T_0}\right)^4 =$$

$$= \frac{1}{2} N(T)\, \Omega_\gamma\, \rho_{0\mathrm{C}} \left(\frac{T}{T_0}\right)^4 =$$

$$= \frac{1}{2} N(T)\, \Omega_\gamma\, \rho_{0\mathrm{C}} \left(\frac{R_0}{R}\right)^4 = \frac{1}{2} N(T)\, \Omega_\gamma\, \rho_{0\mathrm{C}}\, (1+z)^4 . \tag{3.50}$$

Assuming instead of $1+z = R_0/R$ (see Eq. (3.41)) the ratio $\widetilde{R}_0/\widetilde{R} = 1+z$ (see Eq. (3.39)) and $\rho_{0\mathrm{C}}$ (see Eq. (3.17)) in Eq. (3.50), Eq. (2.3) yields

$$\widetilde{R} = (2\, N(T)\, \Omega_\gamma)^{1/4} (R_{\mathrm{Pl}}\, c\, t)^{1/2} \propto t^{1/2}\,, \tag{3.51}$$

whereas for $1+z = R_0/R$ in Eq. (3.50) the Friedmann equation (see Eq. (2.3)) gives

$$R = (2\, N(T)\, \Omega_\gamma)^{1/4} (R_0\, c\, t)^{1/2} \propto t^{1/2}\,, \tag{3.52}$$

so that for the radiation-dominated universe ($z > 10^5$) by Eqs. (3.51) and (3.52) we can derive

$$t = t(z) = \frac{1/H_0}{(2\,N(T)\;\Omega_\gamma)^{1/2}\,(1+z)^2} \quad (z > 10^5)\,. \tag{3.53}$$

At the matter-dominated universe, for Eqs. (2.25) and (2.26), the time $t = t(z)$ is determined by Eqs. (7.9) and (7.11), respectively.

Then, using $t = t_{\mathrm{M}}$ (see Eq. (3.47)) and $t = t_{\mathrm{X,Y}}$ (see Eq. (3.35)) in Eqs. (3.51) and (3.52), we obtain

$$\widetilde{R} = \widetilde{R}_{\mathrm{M}} = (2\,N_{\mathrm{M}}(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c t_{\mathrm{M}})^{1/2} = R_{\mathrm{Pl}} = \frac{\widetilde{R}_0}{1+z_{\mathrm{M}}}\,, \tag{3.54}$$

$$R = R_{\mathrm{M}} = (2\,N_{\mathrm{M}}(T)\,\Omega_\gamma)^{1/4}(R_0\,c t_{\mathrm{M}})^{1/2} = \widetilde{R}_0 = \frac{R_0}{1+z_{\mathrm{M}}}\,, \tag{3.55}$$

$$\widetilde{R} = \widetilde{R}_{\mathrm{X,Y}} = (2\,N_{\mathrm{X,Y}}(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c t_{\mathrm{X,Y}})^{1/2} =$$

$$= \frac{\widetilde{R}_0}{1+z_{\mathrm{X,Y}}} = 4.138\cdot 10^{-34}\ \mathrm{m} \tag{3.56}$$

and

$$R = R_{\mathrm{X,Y}} = (2\,N_{\mathrm{X,Y}}(T)\,\Omega_\gamma)^{1/4}(R_0\,c t_{\mathrm{X,Y}})^{1/2} =$$

$$= \frac{R_0}{1+z_{\mathrm{X,Y}}} = 1.207\cdot 10^{-3}\ \mathrm{m}\,. \tag{3.57}$$

Now, we can interpret Eqs. (3.48) and (3.49), using Eqs. (3.54) to (3.57). Then, Eq. (3.48) means that by the inflation the dimension R_{Pl} of the beginning of the early universe is enlarged by the enormous factor of $1 + z_{\mathrm{M}} = 2.916\cdot 10^{30}$ to the scale factor $\widetilde{R}_0 = R_{\mathrm{Pl}} \times (1 + z_{\mathrm{M}})$ of absorption of light at the redshift before the inflation, if we assume $\widetilde{R} = \widetilde{R}_0/(1+z)$ for every z value (see Eqs. (3.54) and (3.56)). More generally, we have the connection $R = \widetilde{R}(1+z_{\mathrm{M}}) = R_0/(1+z)$ with the scale factor $R_0 = \widetilde{R}_0(1+z_{\mathrm{M}})$ of absorption of light at the redshift after the inflation (see Eqs. (3.49), (3.55) and

(3.57)). Thus, indeed, because of Eqs. (3.54) to (3.57), the two scale factors $\widetilde{R}_0$ and R_0 are characteristical quantities (see Sec. 3.3). Attention! The scale factor $\widetilde{R}_0$ has a double role because it determines the end of the early universe and the beginning of the late universe.

Therefore, for the redshift evolution $1+z$, we can generally introduce (see Refs. [1-3]):

$$\widetilde{R} = \frac{R}{1+z_{\mathrm{M}}} = \frac{\widetilde{R}_0}{1+z} \tag{3.58}$$

and

$$R = (1+z_{\mathrm{M}})\widetilde{R} = \frac{R_0}{1+z}, \tag{3.59}$$

where $\widetilde{R}$ and R characterize again the two scale factors for the emission of light at the redshift in the FRW geometry prior to and after the inflation, respectively.

The redshift condition (3.59) of the photon gas must depend on the light ray coming to the observer along the radial direction [4, 6, 9]. Therefore, if the light ray leaves a source at the comoving coordinate r' at the time t', it is observed at the origin $r = 0$ to a later time t_0. Because the light ray moves on a null geodesic line ($ds^2 = 0$ in Eq. (2.1)), for $\mathrm{k} = 0$ (see Sec. 5), the time t_0 is given by

$$\int_{t'}^{t_0} \frac{c\,dt}{R(t)} = \int_0^{r'} \frac{dr}{\sqrt{1-\mathrm{k}r^2}} = r' + \frac{\mathrm{k}(r')^3}{6} + \cdots = r' . \tag{3.59 a}$$

Taking Eq. (3.59 a), the comoving coordinate r' of the luminous object is defined by

$$\frac{c(t_0 - t')}{R(t_0)} + \frac{c H_0 (t_0 - t')^2}{2R(t_0)} + \cdots = r' . \tag{3.59 b}$$

Using the differential $c\,dt/R(t)$ of Eq. (3.59 a), where the radial coordinate r' of the comoving source is time-independent, we can see that the time interval $\delta t'$ between the emission of subsequent light signals is related to the time interval δt_0 of the observations of these light signals by

$$\frac{c\delta t'}{R(t')} = \frac{c\delta t_0}{R(t_0)}. \tag{3.59c}$$

If these signals are subsequent wave crests, the emitted (e) wavelength is $\lambda_e = c\,\delta t'$ and the observed or absorbed wavelength is $\lambda = c\,\delta t_0$, so that we get as usually the factor $1+z$:

$$1+z = \frac{\lambda}{\lambda_e} = \frac{R(t_0)}{R(t')}. \tag{3.59d}$$

Introducing for the scale factor of observation or absorption $R(t_0) = R_0$ and for the scale factor of emission $R(t') = R$, we obtain again the relation (3.59), i.e. this simple formula describes the connection between the cosmological redshift (blueshift) z and the time expansion of $R(t)$ in a power series.

Therefore, if $R(t)$ is increasing, we have a redshift ($z > 0$), i.e. an increase in wavelength by a factor $1+z = R_0/R$. Alternatively, if $R(t)$ is decreasing, we observe a blueshift ($z < 0$), i.e. a decrease in wavelength by the factor $1+z = R_0/R$. This behaviour is interpreted as Doppler effect, i.e. in an expanding, massive universe, we observe $R_0 > R$, so that $z > 0$. On the other hand, in a contracting, massive universe, we have $R_0 < R$, so that $z < 0$.

Via the Planck time $t_{Pl} = R_{Pl}/c = (5.39106 \pm 0.00032)\cdot 10^{-44}\,\mathrm{s}$ (see Eq. (7.2)), the time $t_* = t_{Pl}\,(1+z_M) = \widetilde{R}_0/c = (R_{Pl}/cH_0)^{1/2} = 1.572\cdot 10^{-13}\,\mathrm{s}$ (see Eqs. (3.39) and (3.41)) is a reduced Hubble time because of $1/H_0 = (1+z_M)t_*$. The age of the early, massive universe is given by $t_0^* = t_0/(1+z_M) = 1.495\cdot 10^{-13}\,\mathrm{s}$, where we have the end of this early, massive universe at the scale factor $\widetilde{R}_0$.

Then, using Eq. (3.58), we can also assume that because of $1+z = T/T_0$ in the early space region $R_{Pl} \le \widetilde{R} \le \widetilde{R}_0$ (see Eq. (3.48)) these redshift conditions for $T \le T_M$ determine the expansion of the early massive universe ($\widetilde{R} \ge R_{Pl}$)), which is converted into the late universe by the inflation according to Eq. (3.59). In other words, the inflation enlarges the scale factors $\widetilde{R}$ of the early universe at $R_{Pl} \le \widetilde{R} \le \widetilde{R}_0$ (see Eq. (3.58)) by the enormous factor $1+z_M$ (see

Eq. (3.48)) in the scale factors R of the late universe for $\widetilde{R}_0 \le R \le R_0$ (see Eq. (3.59)), which has today the redshift $z = 0$ at the scale factor $R = R_0$.

By this above-introduced, new inflation model (see also [1-3]), we have also confirmed the solution of the pressing problems of the standard model as the horizon problem and the monopole problem, whereat because of the rest energy of the magnetic monopoles $E_0(\mathrm{M}) << E_{\mathrm{Pl}}$ (see Eqs. (3.44) and (7.3)) the initial condition problem can be treated by the classical cosmology. To the flatness problem, we will return in Sec. 5.

Using the data of Tables III and IV, the results, derived in this Sec. 3.5, are also valid for the anti-universe.

3.6 Determination of the vacuum energy density by the new inflation model via quantities of the old inflation model for $R = R_{\mathrm{Pl}}$

Now, we consider the exponential term $\mathrm{e}^{H\Delta t}$ of the connections (3.48) and (3.49). The old inflation model (2.17) can be conserved into the new inflation model (see Sec. 3.5) if instead of the integration constant R_1 we assume formally the transformation $R_1 \to \widetilde{R}$, so that we obtain

$$R = \widetilde{R}\,\mathrm{e}^{H\Delta t} = \widetilde{R}\,(1 + z_{\mathrm{M}}) \quad \text{(see Eqs. (3.48) and (3.59))}, \qquad (3.60)$$

where Δt describes again the inflationary phase (seen from $t = t_{\mathrm{Pl}}$), which is attributed to the era of the supersymmetric grand unification, whereas H is the short-time Hubble constant [1-3] at $R = R_{\mathrm{Pl}}$:

$$H = \left(\frac{8\pi G_N}{3}\rho_{\mathrm{vac}}\right)^{1/2} = \left(\frac{c^2\Lambda}{3}\right)^{1/2} \quad \text{(see Eq. (2.17))}. \qquad (3.61)$$

In Eq. (3.60), the integration constant $\widetilde{R}$ must be taken into account for any, constant redshift $z \le z_{\mathrm{M}}$ (see Eqs. (3.48) and (3.59)), which is determined by the thermal energy $kT \le kT_{\mathrm{M}}$ of the particles (see Eq. (2.23) and Sec. 3.5).

Because of Eq. (3.60), we have the condition

$$H\Delta t = H\,(t_{\mathrm{X,Y}} - t_{\mathrm{M}}) = \ln(1 + z_{\mathrm{M}}) = 70.148\,, \tag{3.62}$$

where $t_{\mathrm{X,Y}}$ and t_{M} are the times (3.35) and (3.47), respectively.

From the statistical particle quantities is only known $N_{\mathrm{X,Y}}(T) = 160.75$ (see Eq. (3.35) and Table V), whereas $N_{\mathrm{M}}(T)$ of Eq. (3.47) must be determined via the dependence (3.54), i.e. $t_{\mathrm{M}} = t_{\mathrm{Pl}}$, so that we obtain

$$N_{\mathrm{M}}(T) = \left(\frac{5.160 \cdot 10^{-42}\ \mathrm{s}}{t_{\mathrm{Pl}}}\right)^2 = 9161 \pm 337\,. \tag{3.63}$$

Then, for $t_{\mathrm{M}} = t_{\mathrm{Pl}}$, the corresponding scale factors (see Eqs. (3.54) and (3.55)) are defined by

$$\widetilde{R} = R_{\mathrm{Pl}} = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_{\mathrm{Pl}}\,c\,t_{\mathrm{Pl}})^{1/2} \tag{3.64}$$

and

$$R = \widetilde{R}_0 = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_0\,c\,t_{\mathrm{Pl}})^{1/2}\,, \tag{3.65}$$

so that we must assume (see also Eqs. (3.51) and (3.52)):

$$N_{\mathrm{M}}(T) = \frac{1}{2\Omega_{\gamma}}\,. \tag{3.66}$$

Using the expressions (3.63) and (3.66), in accordance with Table I, we find

$$\Omega_{\gamma} = (5.46 \pm 0.20) \cdot 10^{-5}\,. \tag{3.67}$$

By Eq. (3.62), because of $t_{\mathrm{M}} = t_{\mathrm{Pl}}$ (see above) and $t_{\mathrm{X,Y}} \gg t_{\mathrm{Pl}}$, we find the short-time Hubble H constant and the cosmological constant Λ for $R = R_{\mathrm{Pl}}$ (see Refs. [1-3]) to

$$H = \frac{\ln(1 + z_{\mathrm{M}})}{t_{\mathrm{X,Y}} - t_{\mathrm{M}}} \cong \frac{1}{t_{\mathrm{X,Y}}}\ln(1 + z_{\mathrm{M}}) = (2.63^{+0.14}_{-0.15}) \cdot 10^{41}\ \mathrm{s}^{-1} \tag{3.68}$$

and

$$\Lambda = \frac{3H^2}{c^2} \cong (2.31^{+0.25}_{-0.26}) \cdot 10^{66}\ \mathrm{m}^{-2}\,. \tag{3.69}$$

Thus, at $t = t_{\mathrm{X,Y}} = 2.67 \cdot 10^{-40}\mathrm{s}$ (see Eq. (3.35)), the supersymmetric grand unification separates into the strong and electroweak interaction.

Consequently, by Eq. (2.17), for the boundary $R = R_{\text{Pl}}$ of the massive universe, the Planck vacuum energy density, which is connected with the cosmological constant (3.69), can be estimated to

$$\rho_{\text{vac}}(R_{\text{Pl}})c^2 = \frac{c^4}{8\pi G_N}\Lambda = (6.94^{+0.75}_{-0.78}) \cdot 10^{121}\,\text{eV}\,\text{cm}^{-3}. \tag{3.70}$$

Therefore, for the very short time of $\Delta t = 2.67 \cdot 10^{-41}\,\text{s}$, the inflation field acts as positive $\Lambda = 2.31 \cdot 10^{66}\,\text{m}^{-2}$ at $R = R_{\text{Pl}}$ and yields the kinetic energy for an extremely rapid, exponential expansion of the early universe (see Sec. 3.5).

For example, at the smallest possible, early universe [$R = R_{\text{Pl}}$ for $r' = 1$ (see Eq. (3.59 b))], because of Euclidian geometry (see Sec. 5), this kinetic energy E_{K} for the extremely rapid, exponential expansion can be estimated by

$$E_{\text{K}} \cong \rho_{\text{vac}} c^2\, \tfrac{4}{3}\pi (R_{\text{Pl}}\, e^{H\Delta t}\, r')^3 \cong$$
$$\cong \rho_{\text{vac}} c^2\, \tfrac{4}{3}\pi \left(\left[\widetilde{R}_0/(1+z_{\text{M}})\right] e^{H\Delta t}\, r'\right)^3 = 3.05 \cdot 10^{115}\,\text{eV}. \tag{3.71}$$

These estimations are possible because the inflation (via the rest energy of the magnetic monopoles) operates at an energy that is much lower than the Planck energy, so that we can treat classically the gravitation. Consequently, the expression $R_{\text{Pl}} = \widetilde{R}_0/(1+z_{\text{M}})$ (see Eqs. (3.48) and (3.54)) and the assumption $t_{\text{M}} = t_{\text{Pl}}$ (see Eqs. (3.64) and (3.66)) mean that via the reduced Compton wavelength $R_{\text{Pl}} = \hbar c/E_{\text{Pl}}$ because of $R_{\text{Pl}} = ct_{\text{Pl}}$ the relativistic energy of the magnetic monopoles is given by

$$E = \hbar/t_{\text{Pl}} = \hbar c/R_{\text{Pl}} = E_{\text{Pl}}. \tag{3.72}$$

Then, if the cooling $T_{\text{Pl}} \to T_{\text{M}}$ in the universe has taken place, for the relativistic energy of the X and Y gauge bosons [2, 3], because of the conditions $E_0(\text{X}) < E_0(\text{M})$ and $E_0(\text{Y}) < E_0(\text{M})$ [see Eqs. (3.34) and (3.44)], the sum of their relativistic maximum energies is limited by

$$E(\text{X}) + E(\text{Y}) = E_0(\text{M}), \tag{3.73}$$

so that the condition $E(\text{X}) = E(\text{Y})$ determines their relativistic maximum energies to

$$E(\mathrm{X}) = \tfrac{1}{2} E_0(\mathrm{M}) \tag{3.74}$$

and

$$E(\mathrm{Y}) = \tfrac{1}{2} E_0(\mathrm{M}) . \tag{3.75}$$

Using the data of Tables III and IV, the vacuum energy density of the anti-universe for $R = R_{\mathrm{Pl}}$ has the same value as that of the universe (see Eqs. (3.68) to (3.70)), i.e. we have again

$$\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2 = \frac{c^4}{8\pi G_N}\Lambda = 6.94 \cdot 10^{121}\,\mathrm{eV\,cm^{-3}} . \tag{3.76}$$

Finally, we show still that the cosmological "constant" is a variable. Then, from Eq. (2.18), for $w = -1$ (see Eq. (2.5)), the constant dark mass density term (see also Eq. (3.40)) can be introduced as usually by

$$\rho = \rho_\Lambda = \Omega_\Lambda \rho_{0C} = \Omega_\Lambda \frac{3H_0^2}{8\pi G_N} = \Omega_\Lambda \frac{3c^2}{8\pi G_N R_0^2} . \tag{3.77}$$

This dark mass density is connected with the vacuum mass density

$$\rho_{\mathrm{vac}} = \frac{c^2}{8\pi G_N}\Lambda = \rho_\Lambda = \Omega_\Lambda \frac{3c^2}{8\pi G_N R_0^2} , \tag{3.78}$$

which determines the present, accelerated expansion (see Eq. (2.27)), so that for $\Omega_\Lambda = 0.685^{+0.017}_{-0.016}$ (see Table I) the present cosmological "constant" Λ at $R = R_0 = (1.375 \pm 0.025) \cdot 10^{26}\,\mathrm{m}$ (see Eq. (3.41)) is found to

$$\Lambda = \frac{3\Omega_\Lambda}{R_0^2} = (1.087^{+0.067}_{-0.065}) \cdot 10^{-52}\,\mathrm{m^{-2}} \tag{3.79}$$

at a value of the vacuum mass density

$$\rho_{\mathrm{vac}} = \Omega_\Lambda \frac{3H_0^2}{8\pi G_N} = (3.27 \pm 0.20) \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm^{-3}} . \tag{3.80}$$

Considering Eqs. (3.77) to (3.80), we see that the vacuum energy density is again represented by the cosmological "constant", which has the value (3.79). Besides, it is connected with the dark energy [1-3], which is responsible for the present-day, accelerated expansion (see Eqs. (4.52) to (4.67)), so that the cosmological "constant" has here a new value, i.e. it is a variable.

Consequently, by the considerations of this chapter, we can generally say that the variable vacuum energy density is represented correspondingly by a cosmological "constant" and a dark energy. More generally, we treat the vacuum energy density in Sec 4.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 3.6, are also valid for the anti-universe.

4 A simple solution for the four-dimensional quantum gravity via the vacuum energy density

In this chapter, where we derive the four-dimensional quantum gravity (quantum cosmology) via the vacuum energy density, we use the fact that for a particle gas in a cube with rigid walls and the particle density n between the gas pressure P and the mean momentum changing Δp of the particles according to the kinetic theory of gases [9] we have the relation

$$P = \frac{1}{6} n \langle v \Delta p \rangle , \tag{4.1}$$

where v is the velocity of the single particle, i.e. the positive momentum changing $\Delta p = 2p$ is the momentum transfer of the single particle on the rigid wall, if p is the perpendicular component of the single particle momentum on the rigid wall, so that we obtain a positive pressure

$$P = \frac{1}{3} n \langle v\, p \rangle . \tag{4.2}$$

However, if the particle itself is connected with the momentum changing and the imaginary wall is not related with the momentum changing, we have a negative momentum changing $\Delta p = -2p$, i.e. we get a negative pressure

$$P = -\frac{1}{3} n \langle v\, p \rangle , \tag{4.3}$$

For photons, we have $\langle v \rangle = c$ and $\langle p \rangle = E/c$, so that Eq. (4.2) gives

$$P = \frac{1}{3} n \langle E \rangle = \frac{1}{3} \rho_\gamma c^2 , \tag{4.4}$$

whereas Eq. (4.3) yields

$$P = -\frac{1}{3} n \langle E \rangle = -\frac{1}{3} \rho_\gamma c^2 , \tag{4.5}$$

For the expansion of the massive universe, Eq. (4.4) means as usually $w = 1/3$, i.e. according to Eq. (4.1) the positive, imaginary wall loading at the scale factor R is $\langle \Delta p \rangle = 2E/c$ as the mean momentum changing by the photons.

In contrast to Eq. (4.4), for the negative pressure (4.5), we must assume $w = -1/3$. For the massive universe this assumption is possible for $\mathrm{k} = 0$ (see Secs. 2 and 5). Consequently, according to Eq. (4.1) the imaginary wall loading at the scale factor R is zero, since the photon itself is related to the negative momentum changing $\langle \Delta p \rangle = -2E/c$.

According to quantum field theory, the negative pressure (4.5) must be connected with the vacuum energy density, since it is valid

$$\rho_{\text{vac}} c^2 = -P = \frac{1}{3} \rho_\gamma c^2 . \tag{4.6}$$

Therefore, Eqs. (4.5) and (4.6) mean a new, additional possibility at the description of the massless and the massive universe by photons. To this problem, we will return in Sec. 4.1 to 4.4.

4.1 The vacuum energy density directly by photons at $R = R_{\text{Pl}}$

At $R = R_{\text{Pl}}$, for the derivation of the vacuum energy density, by photons $E = kT_{\text{Pl}}$, according to Eqs. (4.5) and (4.6), we must find the expression

$$\rho_{\text{vac}}(R_{\text{Pl}}) c^2 = \rho_{\text{vac}}(T_{\text{Pl}}) c^2 = \frac{1}{3} \rho_\gamma (T_{\text{Pl}}) c^2 . \tag{4.7}$$

Using the quantum statistical photon energy density (3.13) for $\rho_\gamma(T_{\rm Pl})c^2$ at the real radiation field temperature $T_{\rm Pl}$ (see Sec. I), we suggest

$$\rho_{\rm vac}(T_{\rm Pl})c^2 \propto \tfrac{1}{3}\frac{\pi^2}{15}\frac{(kT_{\rm Pl})^4}{(\hbar c)^3}. \qquad (4.8)$$

Then, via Eqs. (3.51) and (3.64), we have the additional condition

$$\widetilde{R} = R_{\rm Pl} = (2N(T)\,\Omega_\gamma)^{1/4}(R_{\rm Pl}\,c\,t_{\rm Pl})^{1/2}, \qquad (4.9)$$

so that by Eq. (4.9) we can write

$$\frac{R_{\rm Pl}}{(2\Omega_\gamma)^{1/4}} = [N(T)]^{1/4}R_{\rm Pl}, \qquad (4.10)$$

i.e. by $R_{\rm Pl} = \hbar c/kT_{\rm Pl}$ we find

$$(2\Omega_\gamma)^{1/4}T_{\rm Pl} = \frac{T_{\rm Pl}}{[N(T)]^{1/4}}. \qquad (4.11)$$

Consequently, because of Eq. (4.11), we assume that the right term of Eq. (4.8) must be multiplied by $1/N(T)$ or $2\Omega_\gamma$ ($\Omega_\gamma = 5.46\cdot 10^{-5}$ see Table I) to get an equation, so that we find

$$\rho_{\rm vac}(T_{\rm Pl})c^2 = \tfrac{1}{3}\frac{\pi^2}{15}\frac{(kT_{\rm Pl})^4}{(\hbar c)^3}\frac{1}{N(T)}. \qquad (4.12)$$

and

$$\rho_{\rm vac}(T_{\rm Pl})c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{(kT_{\rm Pl})^4}{(\hbar c)^3} = (6.93\pm 0.24)\cdot 10^{121}\,{\rm eV/cm^3} \qquad (4.13)$$

in excellent agreement with Eq. (3.70), i.e. by this result the assumptions of Eqs. (4.5) and (4.6) are confirmed for photons with the thermal energy $E = kT_{\rm Pl}$ and $1/N(T) = 2\Omega_\gamma$.

This excellent agreement confirms also the correctness of the estimations of the rest energy of the neutrinos (see Eqs. (3.8) to (3.11)) and the supersymmetric grand unification particles [X and Y gauge bosons (see Eq. (3.34)) as well as magnetic monopoles (see Eq. (3.44))].

Using Eqs. (7.4) and (4.13) as well as $2\Omega_\gamma = 1/N(T)$ [see above] at $R = R_{\mathrm{Pl}}$, we can also write

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} = \frac{1}{3}\frac{\pi^2}{15}\frac{c^7}{G_N^2\hbar}\frac{1}{N(T)}, \tag{4.14}$$

where the factor $c^7/G_N^2\hbar$ is the Planck energy density $\rho_{\mathrm{Pl}}c^2$, which has the value

$$\rho_{\mathrm{Pl}}c^2 = \frac{c^7}{G_N^2\hbar} = 2.89206(69)\cdot 10^{126}\ \mathrm{eV\,cm^{-3}}, \tag{4.15}$$

i.e. the vacuum energy density (4.14) of the massless and massive universe (because of their joint boundary $R = R_{\mathrm{Pl}}$) is smaller than the Planck energy density (4.15) by a factor β of

$$\beta = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} = 2.395\cdot 10^{-5}, \tag{4.16}$$

so that we have

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} = \beta\,\frac{c^7}{G_N^2\hbar} = \beta\,\rho_{\mathrm{Pl}}c^2\,. \tag{4.17}$$

Between cosmological "constant" Λ and the vacuum energy density $\rho_{\mathrm{vac}}c^2$, the quantum field theory gives the general connection

$$\Lambda = \frac{8\pi G_N}{c^2}\rho_{\mathrm{vac}}, \tag{4.18}$$

so that by Eqs. (4.14) or (4.17) we get

$$\Lambda = \frac{16}{45}\pi^3\Omega_\gamma \frac{(kT_{\mathrm{Pl}})^2}{(\hbar c)^2} = \frac{16}{45}\pi^3\Omega_\gamma \frac{c^3}{G_N\hbar} =$$

$$= \frac{16}{45}\pi^3\Omega_\gamma \frac{1}{R_{\mathrm{Pl}}^2} = (2.30\pm 0.08)\cdot 10^{66}\ \mathrm{m^{-2}} \tag{4.19}$$

in excellent agreement with the result (3.69).

Generally, using the data of Tables III, IV and VI, all results, derived in this Sec. 4.1, are also valid for the massless and massive anti-universe because of their joint boundary $R = R_{\rm Pl}$.

4.2 The vacuum energy density directly via the gravitation by photons and particles at $R = R_{\rm Pl}$

According to Tables III and IV, for the massless universe and anti-universe, in their real radiation fields (see Sec. I), at $R = R_{\rm Pl}$, the gravitational (gr) potential $V_{\rm gr}$ of the massless photons, for which we assume here the thermal energy $kT_{\rm Pl} = E_{\rm Pl}$, yields

$$V_{\rm gr} = \frac{G_N}{c^4}\frac{(kT_{\rm Pl})^2}{R_{\rm Pl}} = \frac{G_N}{c^4}\frac{E_{\rm Pl}^2}{R_{\rm Pl}} = \frac{\hbar c}{R_{\rm Pl}} = \sqrt{\frac{\hbar c^5}{G_N}} = E_{\rm Pl}, \tag{4.20}$$

i.e. because of the new thermal equilibrium we can interpret also this potential energy (4.20) as the relativistic energy $E_{\rm Pl}$ of all particles and antiparticles in the massless and massive universe at their joint boundary $R = R_{\rm Pl}$. Then, Eq. (4.20), divided by the quantum volume $R_{\rm Pl}^3$, yields the Planck energy density

$$\rho_{\rm Pl} c^2 = \frac{E_{\rm Pl}}{R_{\rm Pl}^3} = \frac{V_{\rm gr}}{R_{\rm Pl}^3} = \frac{G_N}{c^4}\frac{(kT_{\rm Pl})^2}{R_{\rm Pl}^4} = \frac{\hbar c}{R_{\rm Pl}^4} = \frac{c^7}{G_N^2 \hbar}, \tag{4.21}$$

so that via the results (4.14) to (4.17) at the joint boundary $R = R_{\rm Pl}$ the vacuum energy density of the massless and massive universe is also found for particles and antiparticles to

$$\rho_{\rm vac}(E_{\rm Pl}) c^2 = \rho_{\rm vac}(T_{\rm Pl}) c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{V_{\rm gr}}{R_{\rm Pl}^3} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\rho_{\rm Pl} c^2 =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{c^7}{G_N^2 \hbar}\frac{1}{N(T)} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(E_{\rm Pl})^4}{(\hbar c)^3} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\rm Pl})^4}{(\hbar c)^3}. \tag{4.22}$$

Consequently, the cosmological "constant" is again defined by Eq. (4.19).

Thus, at the joint boundary $R = R_{\mathrm{Pl}}$, the energy $kT_{\mathrm{Pl}} = E_{\mathrm{Pl}}$ has twofold meaning as massless photons or massive particles. This fact characterizes the identity of the end of the massless universe or the beginning of the massive universe.

Generally, using the data of Tables III, IV and VI, all results, derived in this Sec. 4.2, are also valid for the massless and massive anti-universe because of their joint boundary $R = R_{\mathrm{Pl}}$.

4.3 The particle formation or the general vacuum energy density directly via the gravitation by photons for $R \le R_{\mathrm{Pl}}$

Taking the data of Tables III and IV as well as the result (4.20), in the massless universe ($R \le R_{\mathrm{Pl}}$) and anti-universe ($R \le R_{\mathrm{Pl}}$), which have the distance R' prior to their separation by their expansion in opposite directions (see Sec. I), the rest energy of the virtual particle(P)-antiparticle($\overline{\mathrm{P}}$) pairs of the quantum vacuum can be described by the gravitational potential energy

$$V_{\mathrm{gr}}^{*}(R') = \frac{G_N}{c^4} \frac{(kT_{\mathrm{vac}})(kT_{\mathrm{vac}})}{R'}, \tag{4.23}$$

if we introduce

$$kT_{\mathrm{vac}} = kT + kT = E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}}) \tag{4.24}$$

and

$$V_{\mathrm{gr}}^{*}(R') = kT_{\mathrm{vac}}, \tag{4.25}$$

so that for $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$ we obtain the Schwarzschild radius (see Sec. 7.1)

$$R' = \frac{G_N}{c^4} kT_{\mathrm{vac}} = \frac{G_N}{c^4}\left[E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}})\right]. \tag{4.26}$$

Because of the very high photon density of these massless universes, after their separation into the massless universe and anti-universe, the description of these rest energies is possible by

$$kT = E_0(\mathrm{P}) \tag{4.27}$$

and

$$kT = E_0(\overline{\mathrm{P}}), \tag{4.28}$$

where kT are correspondingly the thermal energy of the photons in the separated, massless universe and anti-universe. Now, we explain this effect. For this goal, we use the condition (4.24) in the expression (4.23), so that because of $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$ we get

$$\begin{aligned} V^*_{\mathrm{gr}}(R') &= 2\,E_0(\mathrm{P}) = 2\,kT = \frac{G_N}{c^4}\frac{(kT)^2}{R'} + \frac{G_N}{c^4}\frac{(kT)(kT)}{R'} + \\ &+ \frac{G_N}{c^4}\frac{(kT)(kT)}{R'} + \frac{G_N}{c^4}\frac{(kT)^2}{R'} = 4V_{\mathrm{gr}}(R') = 4\times\frac{1}{2}kT = \\ &= \frac{G_N}{c^4}\frac{E_0^2(\mathrm{P})}{R'} + \frac{G_N}{c^4}\frac{E_0(\overline{\mathrm{P}})\times E_0(\mathrm{P})}{R'} + \\ &+ \frac{G_N}{c^4}\frac{E_0(\mathrm{P})\times E_0(\overline{\mathrm{P}})}{R'} + \frac{G_N}{c^4}\frac{E_0^2(\overline{\mathrm{P}})}{R'} = 4V_{\mathrm{gr}}(R') = 4\times\frac{1}{2}E_0(\mathrm{P})\,. \end{aligned} \tag{4.29}$$

Now, we can use the relation $2V^*_{\mathrm{gr}}(R') = 2\times 4V_{\mathrm{gr}}(R') = V^*_{\mathrm{gr}}(R) = 4V_{\mathrm{gr}}(R)$, where at $V^*_{\mathrm{gr}}(R')$ and $4V_{\mathrm{gr}}(R')$ the factor 2 represents the massless universe and anti-universe, so that we can write

$$V_{\mathrm{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)^2}{R} = \frac{G_N}{c^4}\frac{E_0^2(\mathrm{P})}{R}, \tag{4.30}$$

$$V_{\mathrm{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)(kT)}{R} = \frac{G_N}{c^4}\frac{E_0(\overline{\mathrm{P}})\times E_0(\mathrm{P})}{R}, \tag{4.31}$$

$$V_{\mathrm{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)(kT)}{R} = \frac{G_N}{c^4}\frac{E_0(\mathrm{P})\times E_0(\overline{\mathrm{P}})}{R} \tag{4.32}$$

and

$$V_{\mathrm{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)^2}{R} = \frac{G_N}{c^4}\frac{E_0^2(\overline{\mathrm{P}})}{R}, \tag{4.33}$$

if we introduce $V_{\mathrm{gr}}(R) = kT = E_0(\mathrm{P})$, for example, by Eq. (4.30), i.e. we obtain

$$R = \frac{G_N}{c^4} kT = \frac{\hbar c}{E_{\rm Pl}^2} kT = \frac{G_N}{c^4} E_0(\mathrm{P}) = \frac{R'}{2}. \qquad (4.34)$$

Because of $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$, the value R is again the same for Eqs. (4.31) to (4.33). Therefore, the expressions (4.30) and (4.33) can be considered as a separation into matter and antimatter by the expansion of the massless universe and anti-universe in opposite directions. We can assume that the terms (4.30) and (4.33) must be proportional to the vacuum energy density [vacuum energy (dark energy)], which determines the expansion dynamics of the massless universe and anti-universe, where the mixed terms (4.31) and (4.32) must be proportional to the field energy of the quantum vacuum because they disappear by annihilation after an extremely short lifetime, so that their mean energy density must represent this field energy.

Now, we demonstrate this behaviour, for example, via the massless universe. This demonstration is possible because the terms ((4.32) and (4.33) of the massless anti-universe possess a completely analogous behaviuor as the corresponding expressions (4.31) and (4.30) of the massless universe. Then, for the massless universe, we introduce a new energy density via Eqs. (4.30) and (4.31), divided by the quantum volume R^3, i.e. because of Eq. (4.34) we get

$$\rho_{\rm vac}(R)\, c^2 = \mathrm{const}\frac{\hbar c}{E_{\rm Pl}^2}\frac{(kT)^2}{R^4} = \mathrm{const}\frac{E_{\rm Pl}^2}{\hbar c\, R^2} \qquad (4.35)$$

and

$$"\rho_{\rm vac}(R)\, c^2" = \mathrm{const}\frac{\hbar c}{E_{\rm Pl}^2}\frac{(kT)(kT)}{R^4} = \mathrm{const}\frac{\overline{E}_{\rm Pl}\times E_{\rm Pl}}{\hbar c\, R^2}. \qquad (4.36)$$

For $R = R_{\rm Pl}$, the expression (4.35) must yield the Planck energy density, so that according to Eqs. (4.14) to (4.17) as well as (4.22) we find the vacuum energy density (vacuum energy) of the massless universe by Eq. (4.35), if we assume $\mathrm{const} = 1/N(T)$ and $N(T) = 1/2\Omega_\gamma$ for $R \le R_{\rm Pl}$, so that we obtain

$$\rho_{\rm vac}(R)\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\hbar c}{E_{\rm Pl}^2}\frac{(kT)^2}{R^4} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\rm Pl}^2}{\hbar c\, R^2}. \qquad (4.37)$$

Analogous to Eq. (4.37), the mixed expressions (4.36) is also defined by

$$"\rho_{\text{vac}}(R)c^2" = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\hbar c}{E_{\text{Pl}}^2}\frac{(kT)^2}{R^4} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\overline{E}_{\text{Pl}} \times E_{\text{Pl}}}{\hbar c R^2}, \tag{4.38}$$

which contains anti-energy as well as energy and disappears therefore after an extremely short lifetime by annihilation, so that it can form a mean energy density. Because of the energy conservation, this mean energy density provides the field energy for the quantum vacuum. Consequently, this field penetrates completely the space-time continuum of the massless universe.

In Eq. (4.37), the limiting values $kT = E_{\text{Pl}}$ and $R = R_{\text{Pl}}$ mean also that all particles have the relativistic energy E_{Pl} in the massless and massive universe because of their joint boundary $R = R_{\text{Pl}}$.

The vacuum energy of the massless universe, which is connected with the dark energy is responsible for its expansion dynamics.

Using the connection between cosmological constant and vacuum energy density (see Eq. (4.18)), we obtain the following expression for the cosmological "constant"

$$\Lambda = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R^2}. \tag{4.39}$$

For photons, in accordance with $R_{\text{Pl}} = ct_{\text{Pl}}$, we can assume the connection

$$R = ct. \tag{4.40}$$

Because of $kT = E_0(\text{P})$ [see Eq. (4.27)] and $kT = E_{\text{Pl}}^2 R/\hbar c$ (see Eq. (4.34)), we get for the thermal photon (particle) number density

$$n_{\text{P}}(R) = \frac{\rho_{\text{vac}}(R)c^2}{kT} = \frac{\rho_{\text{vac}}(R)c^2}{E_0(\text{P})} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R^3}. \tag{4.41}$$

For $kT = E_{\text{Pl}}^2 R/\hbar c$ (see Eq. (4.34), Eq. (3.21) provides the kinetic energy of the photons to

$$E_{\text{K}}(\gamma) = 2.701178\, kT = 2.701178 \frac{E_{\text{Pl}}^2}{\hbar c} R, \tag{4.42}$$

so that the thermal photon number density is given by

$$n_\gamma(R) = 2.701178 \frac{\rho_{\text{vac}}(R)c^2}{E_{\text{K}}(\gamma)} = \frac{\rho_{\text{vac}}(R)c^2}{kT} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R^3}, \tag{4.43}$$

i.e. we have $n_{\mathrm{P}}(R) = n_{\gamma}(R)$.

Consequently, in the four-dimensional quantum gravity, the radiation energy density and the vacuum energy density as well as the thermal photon number density and the particle number density are equivalent.

This four-dimensional quantum gravity, which is explicitly determined completely by the expressions (4.34) and (4.37) as well as (4.39) to (4.43), yields uniquely the general vacuum energy density (4.37) or the cosmological "constant" (4.39) in the space range $R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$ of the massless universe and anti-universe according to the data of Tables III, IV and VI, i.e. we have found a simple quantum-theoretical, four-dimensional description for the gravitation and antigravitation.

In Sec. 11, the usefulness of this four-dimensional is shown by its application at the transition from the final state of the massive universes to the big bang as the beginning of the massless universes.

4.4 The general vacuum energy density directly by particles and antiparticles in the thermal equilibrium with photons for $R \geq R_{\mathrm{Pl}}$

In contrast to the old inflation model $R = R_1\,\mathrm{e}^{H\Delta t}$ (see Eq. (2.17)), in the new inflation model (see Sec. 3.5 and the beginning of Sec. 3.6), the inflation is characterized by $R = \widetilde{R}\,\mathrm{e}^{H\Delta t} = \widetilde{R} \times (1 + z_{\mathrm{M}})$ according to Eqs. (3.58) and (3.59), i.e. because of the redshift condition $1 + z$ instead of the integration constant R_1 we have assumed the transformations

$$R_1 \rightarrow \widetilde{R} = \widetilde{R}_1(z) = \frac{\widetilde{R}_0}{1+z} \tag{4.44}$$

and

$$R_1 \rightarrow \widetilde{R} \times (1 + z_{\mathrm{M}}) = R = \widetilde{R}_1(z) \times (1 + z_{\mathrm{M}}) = \\ = \frac{\widetilde{R}_0}{1+z} \times (1 + z_{\mathrm{M}}) = \frac{R_0}{1+z}. \tag{4.45}$$

For $z = 0$, we have the real integration constants

$$R_1 \rightarrow \widetilde{R}_1(z=0) = \widetilde{R}_0 \tag{4.46}$$

and

$$R_1 \rightarrow \widetilde{R}_1(z=0) \times (1+z_M) = \widetilde{R}_0 \times (1+z_M) = R_0 \,. \tag{4.47}$$

Using for the massive universe ($R \geq R_{Pl}$) the constant vacuum mass density (see Eqs. (3.77) and (3.78)):

$$\rho_{vac} = \Omega_\Lambda \rho_{0C} = \Omega_\Lambda \frac{3H_0^2}{8\pi G_N} = 3.27 \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm}^{-3} \tag{4.48}$$

as well as the constants (4.46) and (4.47), for the pressure $P = -\rho_{vac} c^2$ ($w = -1$) and $\Lambda = 0$, the Friedmann-Lemaitre Equations (see Eqs. (2.3) and (2.4)) provide the present, exponential expansions (see also Eq. (2.27)):

$$\widetilde{R} = \widetilde{R}_0\, \mathrm{e}^{\Omega_\Lambda^{1/2} H_0 (t-t_0^*)} = \widetilde{R}_0\, \mathrm{e}^{(8\pi G_N \rho_{vac}/3)^{1/2}(t-t_0^*)} = \widetilde{R}_0\, \mathrm{e}^{(c^2\Lambda/3)^{1/2}(t-t_0^*)} \tag{4.49}$$

and

$$R = R_0\, \mathrm{e}^{\Omega_\Lambda^{1/2} H_0 (t-t_0)} = R_0\, \mathrm{e}^{(8\pi G_N \rho_{vac}/3)^{1/2}(t-t_0)} = R_0\, \mathrm{e}^{(c^2\Lambda/3)^{1/2}(t-t_0)}, \tag{4.50}$$

where $t_0^* = t_0/(1+z_M) = 1.495 \cdot 10^{-13}\,\mathrm{s}$ (see Sec. 3.5). Then, by Eq. (3.79), for the massive universe, we have the cosmological "constant"

$$\Lambda = \frac{3\Omega_\Lambda}{R_0^2} = 1.087 \cdot 10^{-52}\,\mathrm{m}^{-2} \,. \tag{4.51}$$

Then, by generalization, via the transformation $T_{Pl} \rightarrow T$ in Eq. (4.12), for the massive universe $(R \geq R_{Pl})$, we obtain the general vacuum energy density

$$\rho_{vac}(T)c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \,. \tag{4.52}$$

Consequently, for the massive universe, the cosmological "constant" can be assumed to

$$\Lambda = \widetilde{\Lambda} = \frac{8}{45}\pi^3 \frac{(kT)^4}{E_{Pl}^2 (\hbar c)^2}\frac{1}{N(T)} \,. \tag{4.53}$$

Now, for the massive universe, we compare the radiation or particle energy density with the vacuum energy density (4.52). For this goal, we apply the data of the X and Y gauge bosons [$t = t_{\mathrm{X,Y}} = 2.67 \cdot 10^{-40}\,\mathrm{s}$ (see Eq. (3.35)) and $T = T_{\mathrm{X,Y}} = 3.104 \cdot 10^{29}\,\mathrm{K}$ (see Eq. (3.36)) as well as $N(T) = 160.75$ (see Table V)]. For the solution (3.52), the particle energy density (3.50) is identical with the radiation energy density (see Eq. (2.22)), so that we have

$$\rho_{\mathrm{r}} c^2 = \frac{1}{4} \rho_{0\mathrm{C}} c^2 \frac{R_0^2}{(ct)^2} = 3.519 \cdot 10^{117}\,\mathrm{eV\,cm^{-3}}. \tag{4.54}$$

The vacuum energy density (4.52) yields

$$\rho_{\mathrm{vac}}(T) c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3} \frac{1}{N(T)} = 9.09 \cdot 10^{112}\,\mathrm{eV\,cm^{-3}}, \tag{4.55}$$

i.e. it can be neglected in comparison with the energy density (4.54) at the normal description of the massive universe. However, it has influence on the massive universe. For this goal, using Eqs. (4.48) and (4.51), we can introduce the two conditions

$$\rho_{\mathrm{vac}}(T) c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3} \frac{1}{N(T)} \geq \rho_{\mathrm{vac}} c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \tag{4.56}$$

and

$$\tilde{\Lambda} = \frac{8}{45} \pi^3 \frac{(kT)^4}{E_{\mathrm{Pl}}^2 (\hbar c)^2} \frac{1}{N(T)} \geq \Lambda = \frac{3\Omega_\Lambda}{R_0^2}. \tag{4.57}$$

Using $N(T) = 3.362644$ (see, e.g., Refs [4, 6]), by the two conditions (4.56) and (4.57), we get the lower limiting temperature

$$T = 51.41\,\mathrm{K}. \tag{4.58}$$

This temperature agrees excellently with the value $T = (1 + z_\Lambda) T_0 = 51.45\,\mathrm{K}$, where for $1 + z_\Lambda = 18.90$ the influence of the vacuum energy density (4.48) begins (see Eq. (7.14)). Thus, we assume that the vacuum energy density (4.55) influences the present, exponential expansion, which therefore is considered.

Thus, we treat now this present, accelerated expansion. According to the dark mass density (4.48), the present, accelerated (acc) expansion begins at

$$1 + z_{\text{acc}} = \frac{R_0}{R} = (2\Omega_\Lambda/\Omega_{\text{m}})^{1/3} = 1.632 \tag{4.59}$$

because of

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) = 0 \tag{4.60}$$

with $\rho = \rho_\Lambda + \rho_{\text{m}}$ from Eq. (2.26) and $P = -\rho_{\text{vac}}c^2 = -\Omega_\Lambda \rho_{0\text{C}} c^2$ ($w = -1$) [see Eqs. (3.77) to (3.80)].

According to Eq. (7.11), the present, accelerated expansion begun after the big bang at the time

$$t(z_{\text{acc}}) = 2.43 \cdot 10^{17}\ \text{s} = 7.70\ \text{Gyr}\,. \tag{4.61}$$

By Eq. (7.12), this time yields the corresponding scale factor (see Eq. (4.45))

$$R = R(z_{\text{acc}}) = R_0/(1 + z_{\text{acc}}) = 8.43 \cdot 10^{25}\,\text{m}\,. \tag{4.62}$$

According to Eqs. (4.50) and (4.62), the beginning of the present, accelerated expansion takes place before the time

$$t_{\text{acc}} = t_0 - t = 2.72 \cdot 10^{17}\ \text{s} = 8.62\ \text{Gyr}\,, \tag{4.63}$$

seen backward from the present age $t_0 = 4.358 \cdot 10^{17}\,\text{s}$ of the universe (see Table I). In other words, the time (4.63) is necessary to reach the present scale factor $R_0 = 1.375 \cdot 10^{26}\,\text{m}$ (see Eqs. (3.41) and (4.47) [z=0]) from the scale factor (4.62).

Between Eqs. (4.61) and (4.63), the discrepancy can be solved by Eq. (4.67) via a corresponding cosmological "constant" $\tilde{\Lambda}$ (see, e.g., Eq. (4.57)). For this goal, we apply, for example, Eq. (4.50), where the transformations $\Omega_\Lambda^{1/2} H_0 \to H$, $\rho_{\text{vac}} \to \rho_{\text{vac}}(T)$ or $\Lambda \to \tilde{\Lambda}$ are possible because the temperature has a constant value by the conditions (4.57) and (4.59) as well as (4.66).

Then, at conservation of the constant result (4.59), for example, in Eq. (4.50), we get the new condition

$$t_0 - \widetilde{t} = \frac{\ln(1+z_{\rm acc})}{(c^2\widetilde{\Lambda}/3)^{1/2}} = \frac{\ln(1+z_{\rm acc})}{(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)\,E_{\rm Pl}^2\,\hbar^2c^4)^{1/2}} \le$$

$$\le t_0 - t = \frac{\ln(1+z_{\rm acc})}{\Omega_\Lambda^{1/2} H_0}, \qquad (4.64)$$

so that we find the connection

$$(t_0 - \widetilde{t})\,(c^2\widetilde{\Lambda}/3)^{1/2} = (t_0 - \widetilde{t})\,(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)\,E_{\rm Pl}^2\,\hbar^2c^4)^{1/2} =$$

$$= (t_0 - t)\,\Omega_\Lambda^{1/2} H_0\,, \qquad (4.65)$$

i.e. the duration $t_0 - \widetilde{t}$ is given by

$$(t_0 - \widetilde{t}) = (t_0 - t)\,\frac{\Omega_\Lambda^{1/2} H_0}{(c^2\widetilde{\Lambda}/3)^{1/2}} =$$

$$= (t_0 - t)\,\frac{\Omega_\Lambda^{1/2} H_0}{(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)\,E_{\rm Pl}^2\,\hbar^2c^4)^{1/2}}\,. \qquad (4.66)$$

Consequently, because of the condition (4.57), the enlarged, present, accelerated expansion

$$R(z_{\rm acc}) = R_0\, e^{(c^2\widetilde{\Lambda}/3)^{1/2}(\widetilde{t}-t_0)} \qquad (4.67)$$

is defined by a smaller duration $t_0 - \widetilde{t}$ than $t_0 - t$ (see Eqs. (4.50) and (4.67)) at conservation of the redshift condition (4.62), i.e. the influence of the general vacuum energy density $\rho_{\rm vac}(T)\,c^2$ or the cosmological "constant" $\widetilde{\Lambda}$ on the massive universe is not negligible.

Therefore, we can solve the discrepancy between Eqs. (4.61) and (4.63) if in Eq. (4.66) we select $\widetilde{t} = t(z_{\rm acc}) = 2.43\cdot 10^{17}\,{\rm s}$, so that for $t_0 - t = 2.72\cdot 10^{17}\,{\rm s}$ Eq. (4.66) provides $(c^2\widetilde{\Lambda}/3)^{1/2} = 2.55\cdot 10^{-18}\,{\rm s}^{-1}$ with $\widetilde{\Lambda} = 2.16\cdot 10^{-52}\,{\rm m}^{-2}$ or $\rho_{\rm vac}(T)\,c^2 = 6.50\cdot 10^3\,{\rm eV\,cm}^{-3}$ at $T = 61\,{\rm K}$.

Thus, by this temperature $T = 61\,{\rm K}$, the influence of the dark energy begins a little earlier at the present, accelerated expansion. By this fact, we have explained this discrepancy for Eqs. (4.61) and (4.63).

Using the data of Tables III, IV and VI, all results, derived in this Sec. 4.5, are also valid for the anti-universe.

4.5 Some conclusions from the vacuum energy density or the cosmological "constant"

The four-dimensional quantum gravity, which is explicitly defined completely by Eqs. (4.34) and (4.37) as well as (4.39) to (4.43), gives uniquely the general vacuum energy density (4.37) or the cosmological "constant" (4.39) in the space range $R_{BB} \leq R \leq R_{Pl}$ of the massless universe, i.e. we have found a simple quantum-theoretical, four-dimensional description for the gravitation.

The expressions (4.41) and (4.43) mean clearly that the massless universe is exclusively characterized by the general vacuum energy density or the radiation (photon (particle)) energy density, respectively. They are identically described by Eq. (4.37).

Generally, the vacuum energy density, which is represented by the cosmological "constant", decreases by the expansion (see Secs. 3.6 as well as 4.1 to 4.4), since at this the distance R increases.

The vacuum energy density is again connected with the dark energy, which is repulsive, i.e. the dark energy determines the expansion dynamics of the massive universe, so that the dark energy is responsible for the accelerated expansion of the massive universe [see Eqs. (3.77) to (3.80) as well as (4.59) to (4.67)].

More generally, because by the expansion the vacuum energy density decreases, the dark energy must disappear in the future, i.e. the accelerated expansion of the massive universe must end by a negative acceleration (see Eqs. (7.26) and (7.27)).

We have uniquely determined the general vacuum energy density or the cosmological "constant" for the massless ($R_{BB} \leq R \leq R_{Pl}$) [see Sec. 4.3] and the massive ($R \geq R_{Pl}$) [see Sec. 4.4] universe, i.e. we have found a simple solution for the four-dimensional quantum gravity.

At $R = R_{\mathrm{Pl}}$, the excellent agreement of the vacuum energy densities (see Sec. 3.6), derived by the new inflation model (see Sec. 3.5) and the new quantum-statistical photon energy density (see Sec. 4.1), confirms the correctness of the estimation of the rest energies of the light neutrinos (see Eqs. (3.8) to (3.11)) and the SUSY GUT particles [X and Y gauge bosons (see Eq. (3.34)) as well as magnetic monopoles (see Eq. (3.44))].

In the massive universe the radiation energy density and the particle energy density are identical. They dominate clearly the quantum vacuum in form of the vacuum energy density (see Eqs. (4.54) and (4.55)), so that it can be neglected at the normal description of the massive universe. At the present, accelerated expansion, its small influence is demonstrated in Sec. 4.4.

Therefore, the energy $E_{\mathrm{Pl}} \geq kT \geq E_0(\mathrm{P})$ of the particles (see, e.g. Eq. (3.44)), which is a result of the new thermal equilibrium with the photons (see the paragraph after Eq. (3.36) in Sec. 3.2), arises from the expansion of the universes and anti-universes in opposite directions. This temperature of the photons determines the redshift condition $1 + z = T/T_0$, which defines the expansion dynamics of the universe via its mass density (see, e.g. Eq. (2.18) as well as Eqs. (2.3) and (2.4)). This new result, which solves simply the separation of matter and antimatter, does not agree with the old interpretation, in which for the universe the particles and antiparticles are produced coincidently by photons with the thermal energy $kT = E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}})$, so that this old interpretation has a problem at the separation of matter and antimatter (see also Sec. 3.2 after Eq. (3.36)).

Using again the data of Tables III, IV and VI, all results, considered in this Sec. 4.5, are also valid for the anti-universe.

5 The exact curvature of universe and anti-universe

By our new inflation model (see Sec. 3.5), which is valid for the universe and the anti-universe, the dimension of the early universe and anti-universe enlarges by the enormous factor $1+z_{\mathrm{M}}=2.916\cdot 10^{30}$ (see Sec. 3.5) or $\mathrm{e}^{H\Delta t}=2.916\cdot 10^{30}$ (see the beginning of Sec. 3.6), so that any deviation from the flat geometry is quickly levelled out, i.e. the late universe and anti-universe (see Sec. 3.5) must be Euclidian ($\Omega_{\mathrm{tot}}=1$ and $\mathrm{k}=0$).

This conclusion is supported by the relation (2.10), which for the new scale factor $R_0=c/H_0$ (see Eq. (3.41)) because of $R=R_0/(1+z)$ (see Eq. (3.59)) at $z=0$ (present day universe) yields

$$\mathrm{k}\frac{c^2}{R_0^2\,H_0^2}=\mathrm{k}=(\Omega_{\mathrm{tot}}-1)\,. \tag{5.1}$$

Then, via (5.1), the Planck 2013 results [10], which were used at the determination of the experimental limit $\Omega_{\mathrm{tot}}\approx 1$ for the curvature in Table I, permit uniquely to exclude the values $\Omega_{\mathrm{tot}}=2$ ($\mathrm{k}=+1$) and $\Omega_{\mathrm{tot}}=0$ ($\mathrm{k}=-1$), so that we must assume as exact value $\Omega_{\mathrm{tot}}=1$ ($\mathrm{k}=0$) {see Refs. [1-3]}.

For the present-day universe, the curvature Ω_{tot} (see above) was determined to $\Omega_{\mathrm{tot}}=1$, so that according to Eq. (2.10) we have (see also Refs. [2, 3]):

$$\Omega_{\mathrm{tot}}=\Omega_{\Lambda}+\Omega_{\mathrm{m}}+\Omega_{\mathrm{r}}=1 \tag{5.2}$$

with

$$\Omega_{\mathrm{r}}=\tfrac{1}{2}N(T)\,\Omega_{\gamma}\,, \tag{5.3}$$

where we have the present value $N(T)=3.362644$ [6] because the 3 types of massless or nearly massless neutrinos and antineutrinos are indirectly connected with the photons [6], i.e. we obtain thus

$$1=\Omega_{\Lambda}+\Omega_{\mathrm{m}}+1.681322\,\Omega_{\gamma}=\Omega_{\mathrm{tot}}\,. \tag{5.4}$$

The exact value $\Omega_{\mathrm{tot}}=1$ solves the flatness problem (see Sec. 2 to the end) because this condition $\Omega_{\mathrm{tot}}=1$ (see Refs. [1, 2]) means automatically also

$$\Omega_{\mathrm{tot}}(z)=1\,. \tag{5.5}$$

Namely, according to Ref. [4] for the redshift evolution $1+z$ the total density parameter $\Omega_{\mathrm{tot}}(z)$ of the universe is defined by

$$\Omega_{\text{tot}}(z) - 1 = (\Omega_{\text{tot}} - 1)\,\frac{(1+z)^2}{E^2(z)} \tag{5.6}$$

with

$$E(z) = \left[\Omega_\Lambda + \Omega_k (1+z)^2 + \Omega_m (1+z)^3 + \Omega_r (1+z)^4\right]^{1/2}, \tag{5.7}$$

$$\Omega_k = 1 - \Omega_{\text{tot}} = 0, \tag{5.8}$$

$$\Omega_\Lambda(z) = \frac{\Omega_\Lambda}{E^2(z)}, \tag{5.9}$$

$$\Omega_m(z) = \frac{\Omega_m}{E^2(z)}\,(1+z)^3 \tag{5.10}$$

and

$$\Omega_r(z) = \frac{\Omega_r}{E^2(z)}\,(1+z)^4, \tag{5.11}$$

so that for $\Omega_{\text{tot}} = 1$ indeed Eq. (5.6) yields Eq. (5.5) because of

$$\Omega_{\text{tot}}(z) = \Omega_\Lambda(z) + \Omega_m(z) + \Omega_r(z) = 1. \tag{5.12}$$

Consequently, the Euclidian geometry of the FRW universe (see Eq. (5.2)) is completely determined by the present values of the cosmological parameters throughout its history [4] including its future (see Sec. 7.3), so that because of the constant $k = 0$ the Friedmann equation

$$\frac{\dot{R}}{R}(z) = H_0\, E(z) = H_0 \left[\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_r (1+z)^4\right]^{1/2} \tag{5.13}$$

is simply solvable (see, e.g., Secs. 2, 3.5 and 7). Because of the data of Tables IV and VI, Eq. (5.4) is also valid for the anti-universe. Therefore, all results, derived in this Sec. 5, are also valid for the anti-universe.

6 The particle-defined, (present-day) cosmological parameter values for the massive universe and anti-universe

In Refs. [1-3], we have clearly and uniquely shown that for the supersymmetric grand unification in the early universe the light neutrinos, which are also responsible indirectly for the final state of the massive universe by a negative acceleration (see also Sec. 7.1), the astronomical unit changing (see also Sec. 7.2), the future of the massive universe via the astronomical unit changing (see also Sec. 7.3) and the redshift values of the reionization in the late, massive universe (see also Sec. 7.4), characterize the rest energy of the X, Y gauge bosons (see Sec. 3.2).

The magnetic monopoles determine the Planck length by their relativistic maximum energy (see also Sec. 7.1), the redshift evolutions (see Eqs. (3.58) and (3.59)), the new inflation model (see Sec. 3.5 and the beginning of Sec. 3.6) and the vacuum energy density (see, e.g., Secs. 3.6, 4.1, 4.3 and 4.4).

The present values of the cosmological parameters H_0, Ω_r, Ω_m and Ω_Λ define completely the Euclidian geometry (see Sec. 5) by the redshift conditions (see Sec. 3.5) for the massive universe throughout its history including its future (see Sec. 7.3), so that the Friedmann equation is uniquely solvable (see, e.g., Secs. 2, 3.5 and 7).

We can expect thus an unique connection between the present values of the cosmological parameters as well as the light neutrino densities of the universe [indirectly of the neutrino rest energies (see Eq. (3.25))] via the extremely heavy X, Y gauge bosons and magnetic monopoles.

Therefore, we assume that this connection can be described via the increase of the light neutrino density parameters by a factor of the products of the different ratios of the relativistic energy to the rest energy of the three particles of the supersymmetric grand unification [1, 2].

Thus, using Eqs. (3.72) to (3.75), we can form the four ratios

$$r_1 = \frac{E(\mathrm{M})}{E_0(\mathrm{M})} = \frac{E_\mathrm{Pl}}{E_0(\mathrm{M})}, \qquad (6.1)$$

$$r_2 = \frac{E(\mathrm{X})}{E_0(\mathrm{X})} = \frac{\tfrac{1}{2}E_0(\mathrm{M})}{E_0(\mathrm{X})}, \qquad (6.2)$$

$$r_3 = \frac{E(\mathrm{Y})}{E_0(\mathrm{Y})} = \frac{\tfrac{1}{2}E_0(\mathrm{M})}{E_0(\mathrm{Y})}, \qquad (6.3)$$

$$r_4 = \frac{E(\mathrm{X}) + E(\mathrm{Y})}{E_0(\mathrm{Y})} = \frac{E_0(\mathrm{M})}{E_0(\mathrm{Y})}. \qquad (6.4)$$

Then, using Eq. (3.20), the light neutrino density parameters (see Eqs. (3.27) to (3.29)), multiplied by the corresponding ratios (6.1) to (6.4), yield the following expressions for the particle-defined, present-day, cosmological parameter values of the massive universe [1, 2], whereat we distinguish the ionizable matter

$$\Omega_{\mathrm{b}} = \overline{\Omega}_{\mathrm{b}} + \Omega_\nu = \Omega_\nu(\nu_\mu) \cdot r_1 \cdot r_2 + \Omega_\nu = \Omega_\nu(\nu_\mu) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} + \Omega_\nu =$$

$$= (894^{+35}_{-36}) \Omega_\gamma = (0.02211^{+0.00089}_{-0.00091}) h^{-2} \qquad (6.5)$$

with

$$\overline{\Omega}_{\mathrm{b}} = \Omega_\nu(\nu_\mu) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} = (868^{+34}_{-35}) \Omega_\gamma = (0.02146^{+0.00086}_{-0.00089}) h^{-2}, \qquad (6.5\mathrm{a})$$

the dark matter

$$\Omega_{\mathrm{dm}} = \overline{\Omega}_{\mathrm{dm}} + \Omega_\nu = \Omega_\nu(\nu_\tau) \cdot r_1 \cdot r_2 + \Omega_\nu = \Omega_\nu(\nu_\tau) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} + \Omega_\nu =$$

$$= (4862^{+234}_{-206}) \Omega_\gamma = (0.1202^{+0.0059}_{-0.0052}) h^{-2} \qquad (6.6)$$

with

$$\overline{\Omega}_{\mathrm{dm}} = \Omega_\nu(\nu_\tau) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} = (4836^{+233}_{-205}) \Omega_\gamma = (0.1196^{+0.0059}_{-0.0051}) h^{-2}, \qquad (6.6\mathrm{a})$$

the pressureless (cold) matter

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}} = (5756^{+234}_{-206}) \Omega_\gamma = (0.1423^{+0.0059}_{-0.0052}) h^{-2} \qquad (6.7)$$

and the dark energy

$$\Omega_\Lambda = \Omega_\nu(\nu_\mu) \cdot r_1 \cdot r_3 + \Omega_\nu \cdot r_1 \cdot r_4 =$$

$$= \Omega_\nu(\nu_\mu) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{Y})} + \Omega_\nu \frac{E_{\mathrm{Pl}}}{E_0(\mathrm{Y})} =$$

$$= \left(12589^{+559}_{-505}\right) \Omega_\gamma = \left(0.311^{+0.015}_{-0.013}\right) h^{-2}. \qquad (6.8)$$

By Eq. (6.8), in contrast to Eqs. (6.5) and (6.6), the dark energy is possible because the X, Y gauge bosons are different particles in spite of their same masses.

Then, using Eq. (5.4), we have

$$1 = \Omega_\Lambda + \Omega_\mathrm{m} + 1.681322\,\Omega_\gamma = \left(18347^{+560}_{-506}\right)\Omega_\gamma\,, \tag{6.9}$$

so that in accordance with Table I and Refs. [1, 2] we get

$$\Omega_\gamma = (5.45^{+0.15}_{-0.17})\cdot 10^{-5}\,. \tag{6.10}$$

Consequently, by Eqs. (3.20) and (6.10), we find the reduced Hubble constant h [1, 2] to

$$h = 0.6736^{+0.0105}_{-0.0096}\,. \tag{6.11}$$

Now, in accordance with Table I, we can estimate the present values of the Hubble constant H_0 and the critical density $\rho_{0\mathrm{C}}$ to

$$\begin{aligned} H_0 &= h \times (9.777752\ \mathrm{Gyr})^{-1} = \\ &= (2.183^{+0.034}_{-0.031})\cdot 10^{-18}\ \mathrm{s}^{-1} = 67.36^{+1.05}_{-0.96}\ \mathrm{km\ s^{-1}\,Mpc^{-1}} \end{aligned} \tag{6.12}$$

and

$$\begin{aligned} \rho_{0\mathrm{C}} &= 1.05375(13)\cdot 10^{-5}\,h^2 (\mathrm{GeV}/c^2)\ \mathrm{cm}^{-3} = \\ &= (4.78^{+0.15}_{-0.14})\cdot 10^{3} (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}\,. \end{aligned} \tag{6.13}$$

By Eq. (7.11), for $z = 0$, we can uniquely determine the age of the present universe to

$$t_0 = \frac{2}{3 H_0 \Omega_\Lambda^{1/2}} \ln \frac{\sqrt{\Omega_\Lambda} + \sqrt{\Omega_\Lambda + \Omega_\mathrm{m}}}{\sqrt{\Omega_\mathrm{m}}} = 13.82^{+0.06}_{-0.07}\ \mathrm{Gyr}\,. \tag{6.14}$$

We summarize these estimated, (present-day) cosmological parameter values [1, 2] in Table VI, using Eqs. (6.5) to (6.14) without Eqs. (6.5 a) and (6.6 a).

Generally, using the data of Tables III and IV, all results, derived in this Sec. 6, are also valid for the anti-universe.

Because of $\Omega_\mathrm{tot} = 1$, we have obtained also the reduced Hubble constant, which to this day was only determined by observations [1, 2]. This fact confirms indirectly the correctness of the estimated rest energies of the light neutrinos and the SUSY GUT particles (X and Y gauge bosons as well as the magnetic monopoles).

Table VI. The estimated, (present-day) values of the cosmological parameters for universe and anti-universe (compare with Tables I and II). [a] Universe: "+"sign, anti-universe: "−"sign.

Symbol, equation	Value for universe and anti-universe
$\lvert \pm h \rvert$ [a]	$0.6736^{+0.0105}_{-0.0096}$
$\lvert \pm H_0 \rvert$ [a]	$100\,h\ \mathrm{km\ s^{-1} Mpc^{-1}} = 67.36^{+1.05}_{-0.96}\ \mathrm{km\ s^{-1} Mpc^{-1}} =$ $= h \times (9.777752\,\mathrm{Gyr})^{-1} = (2.183^{+0.034}_{-0.031}) \cdot 10^{-18}\ \mathrm{s^{-1}}$
$\lvert \pm t_0 \rvert$ [a]	$13.82^{+0.06}_{-0.07}$ Gyr
$\rho_{0C} = 3H_0^2 / 8\pi G_N$	$(4.78^{+0.15}_{-0.14}) \cdot 10^3 (\mathrm{eV}/c^2)\ \mathrm{cm^{-3}}$
$\Omega_b = \rho_b / \rho_{0C}$	$0.02211^{+0.00089}_{-0.00091}\ h^{-2} = 0.0487 \pm 0.0020$
$\Omega_{dm} = \rho_{dm} / \rho_{0C}$	$0.1202^{+0.0059}_{-0.0051}\ h^{-2} = 0.265^{+0.013}_{-0.012}$
$\Omega_\Lambda = \rho_\Lambda / \rho_{0C}$	$0.311^{+0.014}_{-0.013}\ h^{-2} = 0.686^{+0.020}_{-0.021}$
$\Omega_m = \rho_m / \rho_{0C}$	$0.1423^{+0.0059}_{-0.0052}\ h^{-2} = 0.314^{+0.013}_{-0.012}$
$\Omega_\gamma = \rho_\gamma / \rho_{0C}$	$(2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2} = (5.45^{+0.15}_{-0.17}) \cdot 10^{-5}$
Ω_{tot}	1
$\sum m_\nu c^2$	$(5.97^{+0.14}_{-0.13}) \cdot 10^{-2}\,\mathrm{eV}$
Ω_ν	$(6.35^{+0.16}_{-0.14}) \cdot 10^{-4} h^{-2}; \quad \Rightarrow \quad (1.402^{+0.040}_{-0.044}) \cdot 10^{-3}$

Table VI (continued)

Symbol, equation	**Value for universe and anti-universe**
$z_{\text{eq}} = \dfrac{2\Omega_{\text{m}}}{N(T)\Omega_{\gamma}} - 1$	3423^{+107}_{-94} (see Eq. (7.5))
z_{dec}	1090.7 (see Eq. (9.35))
$z_{\text{reion}}(\nu_{\tau})$	$11.60^{+0.31}_{-0.26}$ (see Eq. (7.45))

Between the calculated (see Table VI) and the observed (see Tables I and II) present values of the cosmological parameters, we have obtained an excellent agreement. This agreement between these parameters speaks indirectly for the correctness of the already obtained results at the massless and massive universes in this work as the new inflation model, the derivation of the four-dimensional quantum gravity via the vacuum energy density or the corresponding cosmological "constant", the matter-antimatter problem, the curvature, the horizon problem, the monopole problem, the flatness problem, the initial condition problem, the accelerated expansion as well as the influence of the general vacuum energy density or its cosmological "constant on the massive universes (see Secs. 3 to 5).

Besides, for the massive universes, this agreement between the calculated and observed parameters (see above) confirms also the results of the next chapters as the mean negative acceleration, the end of the accelerated expansion by this negative acceleration, the astronomical unit changing, the future and the age of the final state of the (massive) universes, the rest energy and number density of the heavy and the sterile neutrinos, the description of the present dark matter and dark energy by sterile neutrinos, the direct description of the (present) cosmological parameter values via the rest energies of the heavy and the sterile neutrinos, the transformation of the various types of all neutrinos into one another and the transition from the final state to the big bang, which permits the estimation of the parameters of the big bang and the lifetime of the sterile neutrinos (see Secs. 7 to 11).

Because of the excellent agreement between estimated and observed parameters, we use in the next chapters the data of Table I.

7 New results for universe and anti-universe

7.1 The final state of the universe as well as the end of the present, accelerated expansion by a mean, negative acceleration

The modern relativistic cosmology is based on the General Relativity ($R_{\mathrm{Pl}} \leq R \leq \infty$) and the quantum theory ($0 \leq R \leq \infty$). However, the different ranges of validity for the General Relativity and the quantum theory lead to an incompatibility between these theories, so that the Friedmann-Lemaitre equations (see Eqs. (2.3) and (2.4)), which are based on the (classical) General Relativity, have a limit of their applicability on massive particles [1, 3]. Then, for massive particles of the energy E, the uncertainty relation $R = \hbar c/2E$, multiplied by their Schwarzschild radius $R = 2G_N E/c^4$ (see Eq. (4.26)), which describes these massive particles (matter), yields this limit [1, 3] as the so-called Planck lenght

$$R_{\mathrm{Pl}} = c\,t_{\mathrm{Pl}} = \sqrt{\frac{\hbar G_N}{c^3}} = (1.616200 \pm 0.000096) \cdot 10^{-35}\,\mathrm{m}. \tag{7.1}$$

Then, derived Planck units are the Planck time, the Planck energy and the Planck temperature:

$$t_{\mathrm{Pl}} = \frac{R_{\mathrm{Pl}}}{c} = \sqrt{\frac{\hbar G_N}{c^5}} = (5.39106 \pm 0.00032) \cdot 10^{-44}\,\mathrm{s}, \tag{7.2}$$

$$E_{\mathrm{Pl}} = \frac{\hbar c}{R_{\mathrm{Pl}}} = \frac{\hbar}{t_{\mathrm{Pl}}} = \sqrt{\frac{\hbar c^5}{G_N}} = (1.220932 \pm 0.000073) \cdot 10^{28}\,\mathrm{eV}, \tag{7.3}$$

$$T_{\mathrm{Pl}} = \frac{E_{\mathrm{Pl}}}{k} = \sqrt{\frac{\hbar c^5}{G_N k^2}} = (1.416833 \pm 0.000086) \cdot 10^{32}\,\mathrm{K}. \tag{7.4}$$

The above-mentioned incompatibility between the General Relativity and the quantum theory can only be solved by the quantum gravity, which must reconcile the gravitation with the quantum theory (see Sec. 4.4).

For the estimation of the Planck energy (see Eq. (7.3)), we can also as usually [1, 3, 9] consider two particles with the mass $m = E/c^2$, which have the distance of the reduced Compton wavelength $\lambda_{\mathrm{C}} = \hbar c/E$. Then, the gravitation yields the potential energy $V = G_N m^2/\lambda_{\mathrm{C}} = G_N E^3/c^5\hbar$. An upper limit for the correctness of this classical approach gives the assumption that this potential cannot exceed the limiting energy $E = E_{\mathrm{Pl}} = \sqrt{\hbar c^5/G_N}$ (Planck energy (see Eq. (7.3))) because according to the indeterminacy principle the quantum fluctuations of the distance cause an energy uncertainty in order of magnitude of this Planck energy, so that it is not possible to describe these particles by a well-defined rest energy. Thus, because of $E_0(\mathrm{M}) = 6.849 \cdot 10^{26}\,\mathrm{eV}$, the rest energy of the magnetic monopoles, which is well-defined because of $E_0(\mathrm{M}) \ll E_{\mathrm{Pl}}$, is again connected with the scale factor of the Planck length $R_{\mathrm{Pl}} = ct_{\mathrm{Pl}} = \sqrt{\hbar G_N/c^3} = \lambda_{\mathrm{C}}$ via their relativistic energy $E_{\mathrm{Pl}} = \hbar c/R_{\mathrm{Pl}} = \hbar/t_{\mathrm{Pl}}$ (see Eq. (3.72)) or their redshift condition (see Eq. (3.48)).

In contrast to the large mass of the monopoles ($E_0(\mathrm{M}) = 6.849 \cdot 10^{26}\,\mathrm{eV}$), which is connected with the Planck length via their redshift condition (see above), we can assume that the small mass of the electron neutrinos ($E_0(\nu_{\mathrm{e}}) = 1.589 \cdot 10^{-3}\,\mathrm{eV}$) defines an upper limiting value of the scale factor for the Friedmann-Lemaitre equations as the final state of the universe via their blueshift condition [1, 3].

Thus, we assume that this final state can be estimated as follows. Because of $\Omega_{\mathrm{m}} = \{\Omega_{\mathrm{dm}} + \Omega_\nu(\nu_\tau) + \Omega_\nu(\nu_\mu) + \Omega_\nu(\nu_{\mathrm{e}})\} + \Omega_{\mathrm{b}}$ and $N(T) = 3.362644$, for this goal, we use the redshift condition of the matter-radiation equality (see Tables I, II and VI):

$$1+z_{\text{eq}}=\frac{2\Omega_{\text{m}}}{N(T)\,\Omega_{\gamma}}=$$
$$=\frac{2\left(\left\{\Omega_{\text{dm}}+\Omega_{\nu}(\nu_{\tau})+\Omega_{\nu}(\nu_{\mu})+\Omega_{\nu}(\nu_{\text{e}})\right\}+\Omega_{\text{b}}\right)}{N(T)\,\Omega_{\gamma}}, \tag{7.5}$$

so that by Eq. (3.26) we can also assume

$$1+z(\nu_{\text{e}})=\frac{2\left\{\Omega_{\text{m}}-\left(\left\{\Omega_{\text{dm}}+\Omega_{\nu}(\nu_{\tau})+\Omega_{\nu}(\nu_{\mu})\right\}+\Omega_{\text{b}}\right)\right\}}{N(T)\,\Omega_{\gamma}}=$$
$$=\frac{2\Omega_{\nu}(\nu_{\text{e}})}{N(T)\,\Omega_{\gamma}}=0.406^{+0.020}_{-0.025}\,. \tag{7.6}$$

Then, the neutrinos must be considered together with the photons (see Eq. (2.12)). However, the matter-dominated universe expands for the redshift condition $1+z_{\text{eq}}>>1+z\geq 1$, i.e., the scale factor R increases up to R_0, i.e. $R\leq R_0$. At $1+z\leq 1$, we have a blueshift, so that R increases from R_0, i.e. $R\geq R_0$.

As above-mentioned, we can assume that the blueshift $1+z(\nu_{\text{e}})=0.406$ determines a final state of the universe, where the particles do not move more practically. Therefore, a mean, negative acceleration, which effectively must finish with the present, exponential expansion, must exist. After the end of this present, accelerated expansion, we observe a stop of the particles which have moved still further as a result of a slow, linear expansion by the astronomical unit changing up to the final state of the universe. This astronomical unit changing is calculated in Sec 7.2.

Now, we determine the values of the final state and the mean, negative acceleration as well as the end of the present, accelerated expansion as follows.

At Eq. (2.25), for $z_{\text{eq}}>>z>>1$, we can neglect Ω_{Λ}, so that the matter-dominated universe is described by the Friedmann equation of the form

$$\left(\frac{\dot{R}}{R}\right)^2=\Omega_{\text{m}}H_0^2\left(\frac{R_0}{R}\right)^3. \tag{7.7}$$

Eq. (7.7) has the solutions

$$R = (3/2)^{2/3}\,\Omega_{\mathrm{m}}^{1/3} R_0 (H_0 t)^{2/3} \tag{7.8}$$

and

$$t = t(z) = \frac{2}{3H_0\left[\Omega_{\mathrm{m}}(1+z)^3\right]^{1/2}}\,. \tag{7.9}$$

However, for Eq. (2.26), at $z \ll z_{\mathrm{eq}}$, we cannot more neglect Ω_Λ, so that by Eq. (2.3) the matter-dominated universe is defined by

$$\left(\frac{\dot{R}}{R}\right)^2 = H_0^2(\Omega_{\mathrm{m}}\left(\frac{R_0}{R}\right)^3 + \Omega_\Lambda) \tag{7.10}$$

with the solutions

$$t = t(z) = \frac{2}{3H_0\Omega_\Lambda^{1/2}} \ln\frac{\sqrt{\Omega_\Lambda(1+z)^{-3}} + \sqrt{\Omega_\Lambda(1+z)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}} \tag{7.11}$$

and

$$\frac{R}{R_0} = \frac{1}{1+z} = \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3}\left[\sinh(3/2\,\Omega_\Lambda^{1/2} H_0 t)\right]^{2/3} =$$

$$= \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3}\left[\frac{e^{3/2\,\Omega_\Lambda^{1/2} H_0 t} - e^{-3/2\,\Omega_\Lambda^{1/2} H_0 t}}{2}\right]^{2/3}. \tag{7.12}$$

By Eqs. (7.9) and (7.11), we can form the equation

$$\left(\frac{\Omega_\Lambda}{\Omega_{\mathrm{m}}}\right)^{1/2}\frac{1}{(1+z_\Lambda)^{3/2}} =$$

$$= \ln\frac{\sqrt{\Omega_\Lambda(1+z_\Lambda)^{-3}} + \sqrt{\Omega_\Lambda(1+z_\Lambda)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}}\,. \tag{7.13}$$

Using the corresponding density parameters in Table I, Eq. (7.13) yields the redshift condition

$$1+z_\Lambda = 18.90^{+0.48}_{-0.49}, \tag{7.14}$$

so that in Eqs. (2.25) or (2.26) the influence of the dark energy (Ω_Λ) begins for $z_\Lambda = 18.90$ according to Eqs. (7.9) or (7.11) at

$$t = t(z_\Lambda) = 6.63\cdot 10^{15}\ \mathrm{s} = 0.210\ \mathrm{Gyr}. \tag{7.15}$$

Then, taking Eqs. (3.59) as well as (7.6), according to Refs. [1, 3], for the scale factor of the final state of the universe (see above), we get

$$R = R(\nu_e) = \frac{R_0}{1+z(\nu_e)} = (3.39^{+0.27}_{-0.23})\cdot 10^{26}\ \mathrm{m}. \tag{7.16}$$

By Eq. (7.14), for the beginning influence of the dark energy, we find its minimum scale factor [1, 3]:

$$R = R_\Lambda = \frac{R_0}{1+z_\Lambda} = (7.28\pm 0.32)\cdot 10^{24}\ \mathrm{m}. \tag{7.17}$$

Because of the expressions (7.16) and (7.17), which define the range of influence of the dark energy (Ω_Λ), for the mean, negative acceleration (MNA), we must assume the average of their redshift conditions [1, 3]:

$$1+z_{\mathrm{MNA}} = \frac{R_0}{\sqrt{R_\Lambda R(\nu_e)}} = \left([1+z_\Lambda][1+z(\nu_e)]\right)^{1/2} = 2.77^{+0.10}_{-0.12}, \tag{7.18}$$

so that we obtain the corresponding scale factor [1, 3]:

$$R = R_{\mathrm{MNA}} = \frac{R_0}{1+z_{\mathrm{MNA}}} = (4.96^{+0.31}_{-0.27})\cdot 10^{25}\ \mathrm{m}. \tag{7.19}$$

Consequently, by Eqs. (2.4) and (2.26), via Eqs. (7.18) and (7.19), for the matter-dominated universe, [where because of $1+z_{\mathrm{MNA}} = 2.77$ we can still assume $P=0$, i.e. $\Omega_\Lambda \ll \Omega_m(1+z_{\mathrm{MNA}})^3$ in Eq. (2.26)], the mean negative acceleration [1, 3] is effectively defined by

$$\ddot{R}_{\mathrm{MNA}} = a = -\frac{1}{2}H_0^2\left[\Omega_m(R_0/R_{\mathrm{MNA}})^3 + \Omega_\Lambda\right]R_{\mathrm{MNA}} =$$
$$= -\frac{1}{2}H_0 c\left[\Omega_m(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right] =$$
$$= (-8.71^{+0.76}_{-0.87})\cdot 10^{-8}\ \mathrm{cm\ s^{-2}}. \tag{7.20}$$

According to Eq. (7.11), this negative acceleration (7.20) exists from the time

$$t = t_{\mathrm{MNA}} = t(z_{\mathrm{MNA}}) = 1.162 \cdot 10^{17}\ \mathrm{s} = 3.68\ \mathrm{Gyr}. \tag{7.21}$$

The upper assumption $P = 0$ is reasonable because for $P \neq 0$ according to Eqs. (4.59) and (4.62), the present, accelerated (acc) expansion begins at

$$1 + z_{\mathrm{acc}} = \frac{R_0}{R} = 1.632 \tag{7.22}$$

and

$$R = R_0/(1 + z_{\mathrm{acc}}) = 8.43 \cdot 10^{25}\,\mathrm{m}, \tag{7.23}$$

i.e. according to Eq. (7.11) the present, accelerated expansion begun at the time

$$t = t(z_{\mathrm{acc}}) = 2.43 \cdot 10^{17}\ \mathrm{s} = 7.70\ \mathrm{Gyr} \tag{7.24}$$

after the big bang.

The value (7.20) agrees excellently with the observed [18] Pioneer (P) anomaly

$$\ddot{R}_{\mathrm{P}} = a_{\mathrm{P}} = (-8.74 \pm 1.33) \cdot 10^{-8}\ \mathrm{cm\,s^{-2}}. \tag{7.25}$$

This excellent agreement is an evidence for the correctness of $E_0(\nu_{\mathrm{e}})$ and $\Omega_\nu(\nu_{\mathrm{e}})$ as well as of the assumption of a final state of the universe. In Sec 7.3, we will return to this final state. Consequently, Eq. (7.20) confirms the significance of the electron neutrino for the universe.

With that, we have solved the mystery of the Pioneer anomaly [1-3, 18, 19]. Therefore, the thermal recoil pressure computer simulation [20] is doubtful because it explains the Pioneer anomaly as an unreal physical effect.

By Eq. (2.4), for $\Lambda = 0$, according to $\rho = \rho_{\mathrm{vac}}\,c^2 = \Omega_\Lambda \rho_{0\mathrm{C}}\,c^2$ and $P = -\rho_{\mathrm{vac}}\,c^2 = -\Omega_\Lambda \rho_{0\mathrm{C}}\,c^2$ (see Eqs. (3.77) to (3.80)), because of $w = -1$, the acceleration of the present, accelerated expansion is determined by

$$\ddot{R} = \Omega_\Lambda H_0^2 R. \tag{7.26}$$

Consequently, for the end of this present, accelerated expansion, the effective (eff) equilibrium condition $\ddot{R} + \ddot{R}_{\mathrm{MNA}} = 0$ yields the effective scale factor

$$R_{\mathrm{eff}} = \frac{\ddot{R}_{\mathrm{MNA}}}{\Omega_\Lambda H_0^2} = 2.67 \cdot 10^{26}\ \mathrm{m}. \tag{7.27}$$

We will return to this problem (7.27) in Sec. 7.3.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 7.1, are also valid for the anti-universe.

Therefore, since the universe and the anti-universe are completely symmetric, the measured flyby anomaly, which in the works [1-3] is attributed to the mean negative acceleration, must be observed also by the antihuman on the anti-earth in the anti-universe. This fact is a strong argument for the existence of the Pioneer anomaly as a real physical effect (see above).

7.2 The astronomical unit changing

An astronomical unit changing was observed by two independent researcher groups, analysing radio echoes from the planets [21, 22]. According to Krasinsky and Brumberg [21], the astronomical unit changing was found to

$$\frac{d}{dt}\mathrm{AU} = (15 \pm 4)\ \mathrm{cm\,yr}^{-1}, \tag{7.28}$$

where $\mathrm{AU} = 1.49597870700(3) \cdot 10^{11}\ \mathrm{m}$ is the astronomical unit [8].

Because of Eqs. (2.26) and (7.19), we have

$$\rho = \rho_{\mathrm{MNA}} = \frac{3H_0^2}{8\pi G_N}\left[\Omega_{\mathrm{m}}(R_0/R_{\mathrm{MNA}})^3 + \Omega_\Lambda\right], \tag{7.29}$$

so that because of $P = 0$ (see Eq. (7.20)) as well as $\mathrm{k} = 0$ (see Sec. 5) and $\Lambda = 0$ the Friedmann-Lemaitre Equations (see Eqs. (2.3) and (2.4)) yield

$$\frac{1}{2}\left(\frac{\dot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}}\right)^2 + \frac{\ddot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}} = 0\,. \tag{7.30}$$

Therefore, we use the equation

$$\frac{1}{2}\left(\frac{\dot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}}\right)^2 = \frac{4\pi G_N}{3}\rho_{\mathrm{MNA}} \tag{7.31}$$

for the introduction of the velocity

$$v = \frac{1}{2}\frac{\dot{R}^2_{\mathrm{MNA}}}{H_0\,R_{\mathrm{MNA}}} = \frac{1}{2}H_0\left[\Omega_{\mathrm{m}}(R_0/R_{\mathrm{MNA}})^3 + \Omega_\Lambda\right]R_{\mathrm{MNA}}\,. \tag{7.32}$$

Thus, via Eq. (7.32), the mean astronomical unit changing [1-3] can be defined by

$$\frac{d}{dt}\mathrm{AU} = \frac{1}{2}\frac{\dot{R}^2_{\mathrm{MNA}}}{H_0 R_{\mathrm{MNA}}}\mathrm{corr(AU)} =$$

$$= \frac{1}{2}H_0(\Omega_{\mathrm{m}}\left(\frac{R_0}{R_{\mathrm{MNA}}}\right)^3 + \Omega_\Lambda)\,R_{\mathrm{MNA}}\,\mathrm{corr(AU)}\,, \tag{7.33}$$

where the correction constant corr(AU) is necessary as a proportionality factor, since the radio echoes, which are observed on the surface of the earth, are reflected on the surface of the planets. Consequently, the radio signals run twice the distance $\Delta\mathrm{AU} = \mathrm{AU} - (R_{\mathrm{earth}} + R_{\mathrm{sun}})$, so that this corrective constant must be assumed to

$$\mathrm{corr(AU)} = 2\frac{\mathrm{AU} - \Delta\mathrm{AU}}{R_0} =$$

$$= 2\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} = 2\times(5.105\pm0.093)\cdot10^{-18}\,, \tag{7.34}$$

where $R_{\mathrm{earth}} = 6.378137\cdot10^6\,\mathrm{m}$ and $R_{\mathrm{sun}} = (6.9551\pm0.0004)\cdot10^8\,\mathrm{m}$ are the equatorial radii of the earth and the sun [8]. Plausibly, the denominator of this corr(AU) must be defined by the scale factor R_0 for the present-day universe ($z = 0$). Thus, by Eqs. (7.19) as well as (7.33) and (7.34), we find the mean astronomical unit changing [1-3] to

$$\frac{d}{dt}\mathrm{AU} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} =$$

$$= (2.521^{+0.18}_{-0.21})\cdot10^{18}\,\mathrm{cm\,yr^{-1}}\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} = (12.9^{+1.2}_{-1.3})\,\mathrm{cm\,yr^{-1}}\,. \tag{7.35}$$

The theoretical value (7.35) agrees excellently with the result (7.28) of the analysis of all available radiometric measurements in the time span 1961-2003 [21].

This result is in accordance with a later analysis of the solar-system anomalies in Ref. [23], which for the astronomical unit changing permits the limiting conditions $0 \le {}^{d}\!/_{dt}\,\mathrm{AU} \le 20\,\mathrm{cm\,yr^{-1}}$. The very small velocity (7.35) is presently very difficult to observe because it is covered up by the present, accelerated expansion.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 7.2, are also valid for the anti-universe.

7.3 The future of the universe by a slow linear expansion

Using Eqs. (2.27) and (7.27), the effective equilibrium takes place at the time

$$t_{\mathrm{eff}} = t_0 + \frac{1}{\Omega_\Lambda^{1/2} H_0} \ln \frac{R_{\mathrm{eff}}}{R_0} = 8.034 \cdot 10^{17}\ \mathrm{s} = 25.46\ \mathrm{Gyr}\,. \tag{7.36}$$

Now, for Eq. (7.27), we estimate the influence of the astronomical unit changing (auc) of Eq. (7.35) on the exponential (exp) scale factor R_{exp}, i.e. we must consider the relation

$$R_{\mathrm{eff}} = R_{\mathrm{exp}} + \Delta R_{\mathrm{auc}} = 2.67 \cdot 10^{26}\ \mathrm{m}\,. \tag{7.37}$$

For this goal, we assume as relative changing

$$\frac{\Delta R_{\mathrm{auc}}}{R_{\mathrm{MNA}}} = \frac{R_{\mathrm{eff}} - R_{\mathrm{MNA}}}{R_{\mathrm{MNA}}} = \frac{\Delta t_{\mathrm{auc}}}{\mathrm{AU}} \frac{d}{dt}\mathrm{AU} = \frac{t_{\mathrm{eff}} - t_{\mathrm{MNA}}}{\mathrm{AU}} \frac{d}{dt}\mathrm{AU} \tag{7.38}$$

with

$$\frac{1}{\mathrm{AU}} \frac{d}{dt}\mathrm{AU} = 2.733 \cdot 10^{-20}\ \mathrm{s}^{-1}. \tag{7.39}$$

Then, using Eqs. (7.19) and (7.21) as well as (7.36) and (7.39), we get

$$\Delta R_{\mathrm{auc}} = R_{\mathrm{MNA}} \frac{t_{\mathrm{eff}} - t_{\mathrm{MNA}}}{\mathrm{AU}} \frac{d}{dt}\mathrm{AU} = 9.32 \cdot 10^{23}\ \mathrm{m}\,. \tag{7.40}$$

In first approximation, the result (7.40) can be neglected in the relation (7.37).

At $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see Eqs. (7.27) and (7.37)), we observe the end of the present, accelerated expansion, i.e. in future, the present exponential expansion, which is determined by the dark energy, gives way to a slow linear expansion of the universe by the astronomical unit changing. Therefore, we can estimate the age of the universe for its final (f) state because we know its scale factor $R_{\text{f}} = R(\nu_{\text{e}}) = 3.39 \cdot 10^{26}\,\text{m}$ (see Eq. (7.16)), i.e. similar to Eq. (7.38) we have here the total relative changing

$$\frac{\Delta R_{\text{auc}}}{R_{\text{MNA}}} = \frac{R_{\text{f}} - R_{\text{MNA}}}{R_{\text{MNA}}} = \frac{\Delta t_{\text{auc}}}{\text{AU}} \frac{d}{dt}\text{AU} = \frac{t_{\text{f}} - t_{\text{MNA}}}{\text{AU}} \frac{d}{dt}\text{AU}\,. \qquad (7.41)$$

Consequently, via the results (7.19) and (7.21), for the age of the final state of the universe, Eq. (7.41) yields

$$t_{\text{f}} = t_{\text{MNA}} + \frac{R_{\text{f}} - R_{\text{MNA}}}{R_{\text{MNA}}} \left(\frac{1}{\text{AU}} \frac{d}{dt}\text{AU} \right)^{-1} =$$

$$= 2.136 \cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}\,. \qquad (7.42)$$

To the result (7.42), where we must still take into account the influence of the general vacuum energy density or the corresponding cosmological "constant" of the massive universe, we will return in Eqs. (11.33) to (11.44).

However, for the future, in a time span $\Delta\tau \approx 200\,\text{yr}$, the changing of the astronomical unit (see Eq. (7.35)) is well measurable [1-3] because we have at this directly an AU changing of about $26\,\text{m}$ and the error of the present AU determination is only $3\,\text{m}$ (see AU after Eq. (7.28)).

If in the future this conclusion can be confirmed, the Pioneer anomaly and the astronomical unit changing are uniquely proved, so that they can be counted to the most important astronomical discoveries within living memory.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 7.3, are also valid for the anti-universe.

7.4 The Redshift values of the reionization

Much information about the intergalactic medium (IGM), which permeates the space between the galaxies, has been obtained through its absorption of light from distant quasars, whereby one of the most important tests of the presence of a diffuse component of the intergalactic neutral (mainly hydrogen (HI)) gas is the Gunn-Peterson effect [1-4].

This effect uses the fact that in the intergalactic neutral gas the density of these HI atoms, defined by $n_{\rm HI}(z) \approx 2.42 \cdot 10^{-11} \tau(z) h E(z)\ \mathrm{cm}^{-3}$ for $\Omega_{\rm k} = 0$ (see Sec. 5), can be estimated according to Ref. [4] by the measurement of its optical depth $\tau(z)$ from flux decrement in quasar spectra at the wavelength $\lambda_{\rm Ly\alpha} = 121.6\,\mathrm{nm}$ in $\mathrm{Ly}\alpha$ absorption (of the neutral hydrogen), which has a very large cross-section for

$$T_{\rm IGM} << T_{\rm Ly\alpha} = \frac{2\pi\hbar c}{k\,\lambda_{\rm Ly\alpha}} = 1.183 \cdot 10^5\ \mathrm{K}. \tag{7.43}$$

The observations show that at a redshift of order $z \approx 5\text{-}6$ the neutral hydrogen, left over from the time of recombination, becomes reionized by this ultraviolet light ($\lambda_{\rm Ly\alpha} = 121.6\,\mathrm{nm}$) from the massive stars (quasars) [4, 6].

For example, the condition (7.43) can be fulfilled by the neutrinos [1-3] if they determine $T_{\rm IGM}$.

Thus, via Eqs. (3.27) to (3.29), analogously to Eqs. (7.5) and (7.6), we assume that shortly prior to and after the value $z_{\rm MNA} = 1.77$ (see Eq. (7.18)) the redshifts $z_{\rm reion}$ of the neutrinos

$$z_{\rm reion}(\textstyle\sum_i \nu_i) = \frac{2\sum_i \Omega_\nu(\nu_i)}{N(T)\,\Omega_\gamma} - 1 = 14.27^{+0.37}_{-0.33}, \tag{7.44}$$

$$z_{\rm reion}(\nu_\tau) = \frac{2\,\Omega_\nu(\nu_\tau)}{N(T)\,\Omega_\gamma} - 1 = 11.60^{+0.31}_{-0.26} \tag{7.45}$$

and

$$z_{\rm reion}(\nu_\mu) = \frac{2\,\Omega_\nu(\nu_\mu)}{N(T)\,\Omega_\gamma} - 1 = 1.263^{+0.036}_{-0.042} \tag{7.46}$$

determine the beginning, the half and the end of the reionization [1-3], respectively.

This assumption is supported by the neutrino temperatures

$$T(\sum_i \nu_i) = \{1 + z_{\text{reion}}(\sum_i \nu_i)\} T_0 = 41.62\ \text{K}, \quad (7.47)$$

$$T(\nu_\tau) = \{1 + z_{\text{reion}}(\nu_\tau)\} T_0 = 34.34\ \text{K} \quad (7.48)$$

and

$$T(\nu_\mu) = \{1 + z_{\text{reion}}(\nu_\mu)\} T_0 = 6.168\ \text{K}, \quad (7.49)$$

since they fulfil the condition (7.43). Then, the neutrinos are indirectly connected with the photons via $N(T) = 3.362644$ (see Eq. (2.12)). Then, the mean redshift condition

$$\overline{1+\bar{z}_{\text{reion}}} = = \left([1 + z_{\text{reion}}(\sum_i \nu_i)] \times [1 + z_{\text{reion}}(\nu_\tau)] \times [1 + z_{\text{reion}}(\nu_\mu)]\right)^{1/3} = 7.58, \quad (7.50)$$

which yields $\bar{z} = 6.58$, must agree approximately with the above-mentioned redshift $z \approx 5 - 6$.

Therefore, the assumptions (7.44) to (7.46) show the significance of these neutrinos for the universe. The value (7.45) agrees excellently with the redshifts of the half reionization in the Tables I and II, which were found on completely other way. This fact is a strong argument for the correctness of the assumptions (7.44) and (7.46) as the beginning and the end of the reionization, respectively. Using the data of Tables III, IV and VI, all results, derived in this Sec. 7.4, are also valid for the anti-universe.

8 Rest energy and number density of heavy (anti)neutrinos

Using Eqs. (3.9) to (3.11) and the connections (6.5) to (6.8) (except Eqs. (6.5a), 6.6a) and (6.7)), we can estimate the rest energy and the number density for three heavy neutrinos [1, 2], denoted therefore as a heavy baryon (ionizable matter) neutrino $\tilde{\nu}_{\text{b}}$, a heavy dark matter neutrino $\tilde{\nu}_{\text{dm}}$ as well as a heavy dark energy neutrino $\tilde{\nu}_\Lambda$ with the rest energies $E_0(\tilde{\nu}_{\text{b}})$, $E_0(\tilde{\nu}_{\text{dm}})$ and

$E_0(\widetilde{\nu}_\Lambda)$ as well as the number densities $n(\widetilde{\nu}_b)$, $n(\widetilde{\nu}_{dm})$ and $n(\widetilde{\nu}_\Lambda)$, respectively. Thus, we obtain their rest energy and number density as follows:

$$E_0(\widetilde{\nu}_b) = E_0(\nu_\mu)\frac{E_{Pl}}{2E_0(X)} + \sum_i E_0(\nu_i) = \\ = 2.08. \pm 0.16 \text{ eV} \quad (8.1)$$

and

$$n(\widetilde{\nu}_b) = \frac{\Omega_b \rho_{0C} c^2}{E_0(\widetilde{\nu}_b)} = 112 \text{ cm}^{-3}, \quad (8.2)$$

$$E_0(\widetilde{\nu}_{dm}) = E_0(\nu_\tau)\frac{E_{Pl}}{2E_0(X)} + \sum_i E_0(\nu_i) = 11.31^{+0.94}_{-0.87} \text{ eV} \quad (8.3)$$

and

$$n(\widetilde{\nu}_{dm}) = \frac{\Omega_{dm} \rho_{0C} c^2}{E_0(\widetilde{\nu}_{dm})} = 112 \text{ cm}^{-3} \quad (8.4)$$

as well as

$$E_0(\widetilde{\nu}_\Lambda) = \sum_i E_0(\nu_i)\frac{E_{Pl}}{E_0(Y)} + E_0(\nu_\mu)\frac{E_{Pl}}{2E_0(Y)} = \\ = 29.3^{+2.4}_{-2.3} \text{ eV} \quad (8.5)$$

and

$$n(\widetilde{\nu}_\Lambda) = \frac{\Omega_\Lambda \rho_{0C} c^2}{E_0(\widetilde{\nu}_\Lambda)} = 112 \text{ cm}^{-3}. \quad (8.6)$$

Because the light neutrino density parameters (see Eq. (3.25)) were derived by aid of the present number density of the light neutrinos, the number density of the heavy neutrinos (see (8.2), (8.4) and (8.6)) must be identical with the number density of the light neutrinos [1, 2], which is defined by Eq. (3.24).

The excellent agreement between the number densities of the heavy and light neutrinos confirms again the correctness of the estimated present values of the cosmological parameters and the rest energy of the heavy neutrinos [1, 2]. Consequently, we must observe a relationship between the Eqs. (8.2), (8.4) and (8.6) as well as (3.24). Thus, all these neutrinos must provide a contribution to the ionizable matter (see Eq. (6.5)). Eq. (8.2) dominates the number density of the particles of the ionizable matter because the number density $n_{\mathrm{b}} = 2.482(32)\cdot 10^{-7}\,\mathrm{cm}^{-3}$ of the baryons [5] is much smaller than the number density (3.24) of the light neutrinos. We will return to this point in Sec. 9.3.

The particle densities Ω_{b}, Ω_{dm} and Ω_{Λ} of the universe were identified as the corresponding estimated present cosmological parameter values of the three heavy neutrinos (see Sec. 6). Thus, because of $\Omega_{\mathrm{b}} < \Omega_{\mathrm{dm}} < \Omega_{\Lambda}$, for these heavy neutrinos, we must observe again the hierarchy $E_0(\widetilde{\nu}_{\mathrm{b}}) < E_0(\widetilde{\nu}_{\mathrm{dm}}) < E_0(\widetilde{\nu}_{\Lambda})$ (see Eqs. (8.1), (8.3) and (8.5)), so that the heavy neutrinos must be independent particles. These heavy neutrinos could be identical with the long-sought right handed neutrinos [24].

In Sec. 9.3, via the results of Secs. 9.1 and 9.2, we show that these heavy neutrinos are correspondingly connected with their excited energy states in form of the far heavier unstable, sterile neutrinos (see Eq. (9.29)), denoted as the sterile baryon neutrino $\hat{\nu}_{\mathrm{b}}$, the sterile dark matter neutrino $\hat{\nu}_{\mathrm{dm}}$ and the sterile dark energy neutrino $\hat{\nu}_{\Lambda}$ [1, 2] with the rest energies $E_0(\hat{\nu}_{\mathrm{b}})$, $E_0(\hat{\nu}_{\mathrm{dm}})$ and $E_0(\hat{\nu}_{\Lambda})$ as well as the number densities $n(\hat{\nu}_{\mathrm{b}})$, $n(\hat{\nu}_{\mathrm{dm}})$ and $n(\hat{\nu}_{\Lambda})$, respectively. Consequently, a hierarchy $E_0(\hat{\nu}_{\mathrm{b}}) < E_0(\hat{\nu}_{\mathrm{dm}}) < E_0(\hat{\nu}_{\Lambda})$ can be expected again. By the data of Tables III, IV and VI, all results, derived in this Sec. 8, are also valid for the anti-universe.

9 Sterile neutrinos in universe and anti-universe

9.1 Sterile dark matter neutrinos as dark matter particles

In work [25], a weak unidentified emission line $E = (3.56 \pm 0.03)\,\mathrm{keV}$ was detected in a stacked $XMM-Newton$ spectrum of 73 galaxy clusters (spanning a redshift range $0.01-0.35$). This line is characterized by the photon flux

$$\Phi = \Phi(\hat{\nu}_{\mathrm{dm}}) = (4.0^{+0.8}_{-0.8}[^{+1.8}_{-1.2}])\cdot 10^{-6}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} \qquad (9.1)$$

at 68% [90%] confidence levels. For this line, in Ref. [25], a dark matter interpretation was assumed via the photon decay of the sterile neutrinos (long-sought dark matter particle candidates). This assumption is reasonable because they decay via the gravitation in one photon with the energy of their half rest energy and possess a lifetime larger than the age of the universe [1, 2]. Therefore, we have assumed $\Phi = \Phi(\hat{\nu}_{\mathrm{dm}})$.

Then, via the 68% confidence level, we can form the photon (γ) density

$$n_\gamma = \frac{\Phi}{c} = \frac{\Phi(\hat{\nu}_{\mathrm{dm}})}{c} = (1.33 \pm 0.27)\cdot 10^{-16}\,\mathrm{cm}^{-3}. \qquad (9.2)$$

Using Eq. (8.4), we find

$$\frac{n_\gamma}{n(\tilde{\nu}_{\mathrm{dm}})} = \frac{\Phi(\hat{\nu}_{\mathrm{dm}})}{c\,n(\tilde{\nu}_{\mathrm{dm}})} = (1.19 \pm 0.24)\cdot 10^{-18}. \qquad (9.3)$$

By Eqs. (7.18) and (7.35), the astronomical unit changing is given via

$$\frac{d}{dt}\mathrm{AU} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} =$$

$$= (12.9^{+1.2}_{-1.3})\ \mathrm{cm\,yr}^{-1}. \qquad (9.4)$$

Taking the ratio (9.3) instead of the factor $(R_{\mathrm{earth}} + R_{\mathrm{sun}})/R_0$ in Eq. (9.4), we get a remaining (photon to heavy dark matter neutrino) astronomical part "unit" changing

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{n_\gamma/n(\tilde{\nu}_{\mathrm{dm}})} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{n_\gamma}{n(\tilde{\nu}_{\mathrm{dm}})} =$$

$$= (3.00^{+0.82}_{-0.85})\ \mathrm{cm\,yr}^{-1}, \qquad (9.5)$$

so that we can form the difference

$$\frac{d}{dt}\mathrm{AU} - \left(\frac{d}{dt}\mathrm{AU}\right)_{n_\gamma/n(\tilde{\nu}_{\mathrm{dm}})} = (9.9^{+1.8}_{-1.9})\ \mathrm{cm\,yr}^{-1}. \qquad (9.6)$$

If our considerations are correct, this difference must be equivalent to the dark matter astronomical part "unit" changing [1, 2]:

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_{\mathrm{dm}}} = c\left[\Omega_{\mathrm{dm}}(1+z_{\mathrm{MNA}})^2\right]\frac{R_{\mathrm{earth}}+R_{\mathrm{sun}}}{R_0} =$$

$$= (9.8^{+1.3}_{-1.4})\ \mathrm{cm\,yr^{-1}}, \qquad (9.7)$$

since the expression (9.5) is practically equivalent to a remaining ((baryon-neutrino)-dark energy) astronomical part "unit" changing

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{(\overline{\Omega}_{\mathrm{b}}+\Omega_\nu)+\Omega_\Lambda} =$$

$$= c\left[(\overline{\Omega}_{\mathrm{b}}+\Omega_\nu)(1+z_{\mathrm{MNA}})^2+\Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}}+R_{\mathrm{sun}}}{R_0} =$$

$$= (3.05^{+0.35}_{-0.37})\ \mathrm{cm\,yr^{-1}} \quad \text{(see Eq. (9.5))}. \qquad (9.8)$$

Consequently, because of Eqs. (9.3) to (9.7), the theoretical photon flux $\Phi(\hat{\nu}_{\mathrm{dm}})$ for the sterile dark matter neutrinos can be determined via

$$\frac{d}{dt}\mathrm{AU}-\left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_{\mathrm{dm}}} = 3.1\ \mathrm{cm\,yr^{-1}} =$$

$$= c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2+\Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{\Phi(\hat{\nu}_{\mathrm{dm}})}{c\,n(\tilde{\nu}_{\mathrm{dm}})} \qquad (9.9)$$

to

$$\Phi(\hat{\nu}_{\mathrm{dm}}) \cong 4.1\cdot 10^{-6}\,\mathrm{cm^{-2}\,s^{-1}}. \qquad (9.10)$$

Indeed, Eqs. (9.1) and (9.10) agree excellently, so that the existence of the sterile dark matter neutrinos with the energy $2E = E_0(\hat{\nu}_{\mathrm{dm}})$ is confirmed by their photon decay in consequence of the emission line $E = (3.56 \pm 0.03)\,\mathrm{keV}$, observed in Ref. [25]. Thus, because of Eqs. (9.1) and (9.10), we provide for the first time the evidence that in Ref. [25] the discovery of this emission line is the first direct observation of the present dark matter in form of the sterile dark matter neutrinos because they are connected with the heavy dark matter neutrinos (see also Eqs. (9.12)).

Therefore, the number density of these sterile dark matter neutrinos $\hat{\nu}_{\mathrm{dm}}$ with the rest energy $E_0(\hat{\nu}_{\mathrm{dm}}) = 7120\,\mathrm{eV}$ (see Ref. [1, 2]) can be estimated

$$n(\hat{\nu}_{\mathrm{dm}}) = \frac{\Omega_{\mathrm{dm}}\rho_{0\mathrm{C}}c^2}{E_0(\hat{\nu}_{\mathrm{dm}})} = 0.178 \pm 0.011\,\mathrm{cm}^{-3} \quad (E_0(\hat{\nu}_{\mathrm{dm}}) = 2E)\,, \qquad (9.11)$$

so that the connection between the sterile dark matter neutrinos and the heavy dark matter neutrinos (see above) is defined via

$$n(\hat{\nu}_{\mathrm{dm}}) = n(\widetilde{\nu}_{\mathrm{dm}})\frac{E_0(\widetilde{\nu}_{\mathrm{dm}})}{E_0(\hat{\nu}_{\mathrm{dm}})} \quad \text{or} \quad E_0(\hat{\nu}_{\mathrm{dm}}) = E_0(\widetilde{\nu}_{\mathrm{dm}})\frac{n(\widetilde{\nu}_{\mathrm{dm}})}{n(\hat{\nu}_{\mathrm{dm}})}\,. \qquad (9.12)$$

Using the data of Tables III, IV and VI, all results, derived in this Sec. 9.1, are also valid for the anti-universe.

9.2 Sterile dark energy neutrinos as dark energy particles

Because all heavy neutrinos possess the same number density (see Eqs. (8.2), (8.4) and (8.6)), we can assume that also the three sterile neutrino types (see also Eq. (9.29)) have the same smaller number density [1, 2] analogous to Eq. (9.11). Assuming also for the sterile dark energy neutrinos their number density to $n(\hat{\nu}_\Lambda) = n(\hat{\nu}_{\mathrm{dm}}) = 0.178\,\mathrm{cm}^{-3}$ (see Eq. (9.11)), we can similar via Eq. (9.12) estimate their rest energy $2E = 2E(\hat{\nu}_\Lambda) = E_0(\hat{\nu}_\Lambda)$ to

$$\begin{aligned} &E_0(\hat{\nu}_\Lambda) = 2E(\hat{\nu}_\Lambda) = \\ &= \frac{n(\widetilde{\nu}_\Lambda)}{n(\hat{\nu}_\Lambda)}E_0(\widetilde{\nu}_\Lambda) \cong 18436\,\mathrm{eV} \quad \text{(see Eqs. (8.5) and (8.6))}\,, \end{aligned} \qquad (9.13)$$

so that because of their photon decay we can observe the emission line

$$E = E(\hat{\nu}_\Lambda) = \frac{n(\widetilde{\nu}_\Lambda)}{2n(\hat{\nu}_\Lambda)}E_0(\widetilde{\nu}_\Lambda) \cong 9.22\,\mathrm{keV}\,. \qquad (9.14)$$

Similar to Sec. 9.1, its photon flux $\Phi(\hat{\nu}_\Lambda)$ can be estimated via the remaining (photon to heavy dark energy neutrino) astronomical part "unit" changing [1, 2]:

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{n_\gamma/n(\tilde{\nu}_\Lambda)} = c\left[\Omega_\mathrm{m}(1+z_\mathrm{MNA})^2 + \Omega_\Lambda/(1+z_\mathrm{MNA})\right]\frac{\Phi(\hat{\nu}_\Lambda)}{c\,n(\tilde{\nu}_\Lambda)} =$$

$$= \frac{d}{dt}\mathrm{AU} - \left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_\mathrm{dm}} - \left(\frac{d}{dt}\mathrm{AU}\right)_{(\overline{\Omega}_\mathrm{b}+\Omega_\nu)} =$$

$$= c\left[\Omega_\Lambda/(1+z_\mathrm{MNA})\right]\frac{R_\mathrm{earth}+R_\mathrm{sun}}{R_0} = 1.194\ \mathrm{cm\,yr^{-1}} \qquad (9.15)$$

to

$$\Phi = \Phi(\hat{\nu}_\Lambda) \cong 1.59\cdot 10^{-6}\ \mathrm{cm^{-2}\,s^{-1}}. \qquad (9.16)$$

In Eq. (9.15), similar to the expressions (9.7) or (9.8), we have used the term

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{(\overline{\Omega}_\mathrm{b}+\Omega_\nu)} = c\left[(\overline{\Omega}_\mathrm{b}+\Omega_\nu)(1+z_\mathrm{MNA})^2\right]\frac{R_\mathrm{earth}+R_\mathrm{sun}}{R_0}. \qquad (9.17)$$

Consequently, if our considerations are correct, the emission line (9.14) must also be observed in a stacked *XMM – Newton* spectrum of many galaxy clusters.

This line must be characterized by the photon flux (9.16). In this case, we can assume the observation of present dark energy by sterile dark energy neutrinos.

Then, for the mystery of the dark matter (see Sec. 9.1) and the dark energy (see this chapter) [1, 2], we can show that a hypothesis of their "joint" origin is confirmed by the sterile neutrinos in their two different states denoted as sterile dark matter neutrino (see Sec. 9.1) as well as sterile dark energy neutrino (see Eqs. (9.13) and (9.16)), respectively. Indeed, the same values of the different ratios

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_\mathrm{dm})} = \frac{E(\hat{\nu}_\Lambda)}{E(\hat{\nu}_\mathrm{dm})} = \frac{E_0(\tilde{\nu}_\Lambda)}{E_0(\tilde{\nu}_\mathrm{dm})} = \frac{\Omega_\Lambda}{\Omega_\mathrm{dm}} = 2.59 \pm 0.17 \qquad (9.18)$$

support this explanation [1, 2]. Assuming for the photon energy fluxes of the dark energy and the dark matter the relation $E(\hat{\nu}_\Lambda)\Phi(\hat{\nu}_\Lambda) = E(\hat{\nu}_\mathrm{dm})\Phi(\hat{\nu}_\mathrm{dm})$, we can use the inverse flux values of Eqs. (9.10) and (9.16) for the ratio

$$\frac{1/\Phi(\hat{\nu}_\Lambda)}{1/\Phi(\hat{\nu}_{\mathrm{dm}})} = 2.58 \rightarrow \frac{\Omega_\Lambda}{\Omega_{\mathrm{dm}}} = 2.59 \pm 0.17\,, \tag{9.19}$$

so that Eq. (9.19) provides also a strong argument for this explanation [1, 2] because of Eq. (9.18). Since Eqs. (9.18) and (9.19) yield also the energies (9.13) or (9.14), the result (9.19) confirms also our assumption for the number densities of the sterile neutrinos (see above).

Using the data of Tables III, IV and VI, all results, derived in this Sec. 9.2, are also valid for the anti-universe.

9.3 Sterile baryon neutrinos

Similar as in Sec. 9.2, we can explain the significance of the sterile baryon (ionizable matter) neutrinos [1, 2].

Assuming also their number density to $n(\hat{\nu}_{\mathrm{b}}) = n(\hat{\nu}_{\mathrm{dm}}) = 0.178\ \mathrm{cm}^{-3}$ (see Eq. (9.11)), we can (similar to Eqs. (9.12) or (9.13)) estimate their energy $2E = 2E(\hat{\nu}_{\mathrm{b}}) = E_0(\hat{\nu}_{\mathrm{b}})$ to

$$\begin{aligned} &E_0(\hat{\nu}_{\mathrm{b}}) = 2E(\hat{\nu}_{\mathrm{b}}) = \\ &= \frac{n(\tilde{\nu}_{\mathrm{b}})}{n(\hat{\nu}_{\mathrm{b}})} E_0(\tilde{\nu}_{\mathrm{b}}) \cong 1309\ \mathrm{eV} \quad \text{(see Eq. (8.1) and (8.2))}\,, \end{aligned} \tag{9.20}$$

so that because of their photon decay we must observe the emission line

$$E = E(\hat{\nu}_{\mathrm{b}}) = \frac{n(\tilde{\nu}_{\mathrm{b}})}{2n(\hat{\nu}_{\mathrm{b}})} E_0(\tilde{\nu}_{\mathrm{b}}) \cong 654\ \mathrm{eV}. \tag{9.21}$$

Similar to Secs. 9.1 or 9.2, its photon flux $\Phi(\hat{\nu}_{\mathrm{b}})$ can be estimated via the remaining (photon to heavy baryon neutrino) astronomical part "unit" changing (see Refs. [1, 2]):

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{n_\gamma/n(\tilde{\nu}_\mathrm{b})} = c\left[\Omega_\mathrm{m}(1+z_\mathrm{MNA})^2 + \Omega_\Lambda/(1+z_\mathrm{MNA})\right]\frac{\Phi(\hat{\nu}_\mathrm{b})}{c\,n(\tilde{\nu}_\mathrm{b})} =$$

$$= \frac{d}{dt}\mathrm{AU} - \left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_\mathrm{dm}} - \left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_\Lambda} =$$

$$= c\left[\Omega_\mathrm{b}\,(1+z_\mathrm{MNA})^2\right]\frac{R_\mathrm{earth}+R_\mathrm{sun}}{R_0} = 1.805\ \mathrm{cm\,yr^{-1}} \qquad (9.22)$$

to

$$\Phi = \Phi(\hat{\nu}_\mathrm{b}) \cong 2.41\cdot 10^{-6}\ \mathrm{cm^{-2}\,s^{-1}}. \qquad (9.23)$$

In Eq. (9.22), similar to Eqs. (9.7) and (9.17), we have used the term

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_\Lambda} =$$

$$= c\left[\Omega_\Lambda/(1+z_\mathrm{MNA})\right]\frac{R_\mathrm{earth}+R_\mathrm{sun}}{R_0}. \qquad (9.24)$$

Consequently, if our considerations are correct, the emission line (9.21) must be observed in a stacked *XMM – Newton* spectrum of many galaxy clusters. This line must be characterized by the photon flux (9.23). In this case, we can assume that the observation of these sterile baryon neutrinos confirms the contribution of all independent matter neutrino types (see Eq. (9.29)) to the present, ionizable matter.

Now, for the ionizable matter, we estimate this contribution of the matter neutrinos characterized by the matter quantities Ω_ν, Ω_b, Ω_dm and Ω_Λ. For this goal, using the number density of these neutrino types in Secs. 3.1 and 8, we calculate their neutrino energy density ε_ν [1, 2] to

$$\varepsilon_\nu = \Omega_\nu \rho_{0\mathrm{C}} c^2 = n_0(\nu_i)\sum_i E_0(\nu_i) = n(\tilde{\nu}_\mathrm{b})\sum_i E_0(\nu_i) =$$

$$= n(\tilde{\nu}_\mathrm{dm})\sum_i E_0(\nu_i) = n(\tilde{\nu}_\Lambda)\sum_i E_0(\nu_i) = 6.69\ \mathrm{eV\,cm^{-3}}. \qquad (9.25)$$

Using from Eq. (9.25) the four different unique terms with the joint factor $\sum_i E_0(\nu_i) = 5.97\cdot 10^{-2}\mathrm{eV}$ (see Eq. (3.11)), for this contribution of all these neutrinos, we find for their total matter neutrino energy density [1, 2]:

$$4\varepsilon_\nu \approx 26.8\ \mathrm{eV\,cm^{-3}}. \qquad (9.26)$$

By Eq. (8.2), the ionizable matter is defined by the total baryon energy density [1, 2, 6]:

$$\varepsilon_\mathrm{b} = \Omega_\mathrm{b}\rho_{0\mathrm{C}}c^2 = n(\widetilde{\nu}_\mathrm{b})E_0(\widetilde{\nu}_\mathrm{b}) = n_\mathrm{b}E_0(\mathrm{p}) \approx 232.9\ \mathrm{eV\,cm^{-3}}, \qquad (9.27)$$

where $n_\mathrm{b} = 2.482\cdot10^{-7}\mathrm{cm^{-3}}$ [5] and $E_0(\mathrm{p}) = 9.38272\cdot10^{8}\mathrm{eV}$ [5, 8] are the baryon number density and the proton rest energy, respectively.

Then, using the difference $\varepsilon_\mathrm{b} - 4\varepsilon_\nu \approx 206.1\mathrm{eVcm^{-3}}$, we show that the photon flux of Eq. (9.23) can be estimated by the semi-empirical expression [1, 2]:

$$\Phi = \Phi(\hat{\nu}_\mathrm{b}) \approx \frac{E_0(\widetilde{\nu}_\mathrm{b})}{E_0(\mathrm{p})}\,\frac{\tfrac{1}{6}(\varepsilon_\mathrm{b} - 4\varepsilon_\nu)}{E_0(\mathrm{p})}\,c \approx$$
$$\approx 2.43\cdot10^{-6}\ \mathrm{cm^{-2}\,s^{-1}}, \qquad (9.28)$$

where here the factor $\tfrac{1}{6}$ takes into account that we have 6 independent matter neutrinos [3 light neutrinos and 3 heavy neutrinos {3 sterile neutrinos (see Eq. (9.29))}]. Thus, the expression $\varepsilon_\mathrm{tot} = \tfrac{1}{6}(\varepsilon_\mathrm{b} - 4\varepsilon_\nu) \approx 34.35\mathrm{eVcm^{-3}}$ yields the total energy density per sterile neutrino $\{n(\widetilde{\nu}_j) \propto n(\hat{\nu}_j)$ [see Secs. 9.1 and 9.2]} for the photon flux (9.23), where we have considered the influence of all independent matter neutrinos (see Eqs. (9.25) and (9.26)). We have so shown that we obtain a smaller photon flux than the expected photon flux according to the ratio $\Omega_\mathrm{b}/\Omega_\mathrm{dm}$ (see, for example, Eq. (9.19)). The excellent agreement of Eqs. (9.23) and (9.28) supports our explanation of the physical meaning of the sterile baryon neutrinos, which are responsible for the photon flux (9.23) at the ionizable matter.

Now, we can determine the nature of the heavy and the sterile neutrinos. Because of Eqs. (9.12) and (9.13) as well as (9.20), the ratio of the rest energies of the heavy and the sterile neutrinos is constant, i.e. for $j = \mathrm{b}, \mathrm{dm}, \Lambda$ we observe $E_0(\widetilde{\nu}_j)/E_0(\hat{\nu}_j) = n(\hat{\nu}_j)/n(\widetilde{\nu}_j) = 1.589\cdot10^{-3}$ [1, 2].

Therefore, the heavy neutrino types are again coupled with the sterile neutrinos by the same velocities

$$v_j = (1 - [E_0(\tilde{\nu}_j)/E_0(\hat{\nu}_j)]^2)^{1/2} c = 0.99999874c \,. \qquad (9.29)$$

Thus, the stable, heavy neutrinos have a ground state as independent particles with own, excited energy states in form of the instable sterile neutrinos [1, 2].

Finally, according to the ratio $\Omega_{\rm dm}/\Omega_{\rm b}$, the expected (heavy baryon neutrino) flux, which is determined by the 3 light neutrinos for Ω_ν (see Eq. (6.5)), can be estimated by

$$\tilde{\Phi} = \tilde{\Phi}(\tilde{\nu}_{\rm b}) = \tilde{\Phi}(\Omega_{\rm dm}/\Omega_{\rm b}) \approx \frac{E_0(\tilde{\nu}_{\rm b})}{E_0(\rm p)} \frac{(\varepsilon_{\rm b} + 12\varepsilon_\nu)}{E_0(\rm p)} c \approx$$
$$\approx 2.22 \cdot 10^{-5}\,{\rm cm}^{-2}\,{\rm s}^{-1}, \qquad (9.30)$$

where the quantity $12\varepsilon_\nu$ is defined via the expression (9.26), multiplied by the number 3 of the light (matter) neutrinos, i.e. $\varepsilon_{\rm b} + 12\varepsilon_\nu \approx 313.3\,{\rm eV\,cm}^{-3}$ represents the total energy density of the heavy baryon neutrinos. Indeed, because of (9.10) and (9.19), according to $\Omega_{\rm dm}/\Omega_{\rm b} \approx 5.44$, the result (9.30) agrees with

$$\tilde{\Phi} = \tilde{\Phi}(\tilde{\nu}_{\rm b}) \approx \frac{\Omega_{\rm dm}}{\Omega_{\rm b}} \Phi(\hat{\nu}_{\rm dm}) \approx 2.23 \cdot 10^{-5}\,{\rm cm}^{-2}\,{\rm s}^{-1}. \qquad (9.31)$$

Using the data of Tables III, IV and VI, all results, derived in this Sec. 9.3, are also valid for the anti-universe.

9.4 A fourth type at the heavy and the sterile neutrinos

The origin of the CMB radiation which comes to us from the last scattering surface at the redshift $z \cong 1091$ of the photon decoupling [1-5, 8], is so connected with this photon decoupling [4]. Except this redshift, we have estimated all important cosmological parameters (see also Tables I and II as well as VI). Therefore, using Eqs. (6.2) and (6.4), we introduce the new ratio [1, 2]:

$$r_5 = \left(r_2 \frac{1}{r_4} \right)^{1/2} = \left(\frac{E(\mathrm{X})}{E_0(\mathrm{X})} \frac{E_0(\mathrm{Y})}{E(\mathrm{X}) + E(\mathrm{Y})} \right)^{1/2} = \frac{1}{\sqrt{2}} . \qquad (9.32)$$

Then, by the neutrino rest energy $E_0(\nu_e)$ according to Eq. (3.8) as well as the ratios of Eqs. (6.1) and (6.2), we can assume via (9.32) a fourth heavy photon neutrino, denoted as the heavy "CMB" neutrino [1, 2] with the rest energy

$$E_0(\tilde{\nu}_{\mathrm{CMB}}) = E_0(\nu_e)\, r_1 r_2 r_5 = E_0(\nu_e) \frac{E_{\mathrm{Pl}}}{2\sqrt{2}\, E_0(\mathrm{X})} = 0.2564\ \mathrm{eV}, \qquad (9.33)$$

since the neutrino rest energies $E_0(\nu_\mu)$ and $E_0(\nu_\tau)$ were already explicitly used in Eqs. (8.1) and (8.3) as well as (8.5). Because of the new thermal equilibrium, this heavy "CMB" photon neutrino could be responsible for the decoupling (dec) of the photons [1, 2], since the redshift condition

$$1 + z_{\mathrm{dec}} = E_0(\tilde{\nu}_{\mathrm{CMB}}) / kT_0 = 1091.7 \qquad (9.34)$$

yields the redshift of the photon decoupling to

$$z_{\mathrm{dec}} = 1090.7 \qquad (9.35)$$

and this value (9.35) agrees well with the corresponding values $z_{\mathrm{dec}} = 1091 \pm 1$ (see Ref. [8]), $z_{\mathrm{dec}} = 1090.2 \pm 0.7$ (see Ref. [5]) and $z_{\mathrm{dec}} = 1090.43 \pm 0.54$ (see Ref. [10]). By the new, present, heavy "CMB" photon neutrino density parameter [1, 2]:

$$\Omega_{\tilde{\nu}}(\tilde{\nu}_{\mathrm{CMB}}) = \Omega_\nu(\nu_e)\, r_1 r_2 r_5 = 6.019 \cdot 10^{-3} \quad (\text{see Eq. (3.26)}), \qquad (9.36)$$

the new rest energy (9.33) determines the corresponding number density [1, 2] (see also Sec. 8) to

$$n(\tilde{\nu}_{\mathrm{CMB}}) = \frac{\Omega_{\tilde{\nu}}(\tilde{\nu}_{\mathrm{CMB}})\, \rho_{0C} c^2}{E_0(\tilde{\nu}_{\mathrm{CMB}})} = 112\ \mathrm{cm}^{-3} . \qquad (9.37)$$

Using again the rest energy (9.33), a fourth sterile neutrino [2], denoted as the sterile "CMB" photon neutrino with the rest energy $E_0(\hat{\nu}_{\mathrm{CMB}})$ [analogous to Eq. (9.12)] and the number density $n(\hat{\nu}_{\mathrm{CMB}}) = n(\hat{\nu}_{\mathrm{dm}}) = 0.178\,\mathrm{cm}^{-3}$ (see Eq. (9.11)), is then defined by

$$E_0(\hat{\nu}_{\mathrm{CMB}}) = E_0(\tilde{\nu}_{\mathrm{CMB}})\,\frac{n(\tilde{\nu}_{\mathrm{CMB}})}{n(\hat{\nu}_{\mathrm{CMB}})} \cong 161.33\,\mathrm{eV}. \qquad (9.38)$$

Consequently, the total "CMB" speculation can be confirmed finally by the observation of the X-ray line with the energy $E = \frac{1}{2}E_0(\hat{\nu}_{\mathrm{CMB}}) = 80.7\,\mathrm{eV}$ (see Sec. 9.1).

Now, similar to the expressions (9.17) or (9.24), we use the term

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{(\overline{\Omega}_{\mathrm{b}}+\Omega_\nu(\nu_\tau)+\Omega_\nu(\nu_\mu))+\Omega_\Lambda} =$$

$$= c\left[(\overline{\Omega}_{\mathrm{b}} + \Omega_\nu(\nu_\tau) + \Omega_\nu(\nu_\mu))\,(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\times$$

$$\times\,\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0}\,. \qquad (9.39)$$

Then, for the line $E = \frac{1}{2}E_0(\hat{\nu}_{\mathrm{CMB}}) = 80.7\,\mathrm{eV}$, the photon flux $\Phi(\hat{\nu}_{\mathrm{CMB}})$ [1, 2] can be estimated via

$$\left(\frac{d}{dt}\mathrm{AU}\right)_{n_\gamma/n(\tilde{\nu}_{\mathrm{CMB}})} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{\Phi(\hat{\nu}_{\mathrm{CMB}})}{c\,n(\tilde{\nu}_{\mathrm{CMB}})} =$$

$$= \frac{d}{dt}\mathrm{AU} - \left(\frac{d}{dt}\mathrm{AU}\right)_{\Omega_{\mathrm{dm}}} - \left(\frac{d}{dt}\mathrm{AU}\right)_{(\overline{\Omega}_{\mathrm{b}}+\Omega_\nu(\nu_\tau)+\Omega_\nu(\nu_\mu))+\Omega_\Lambda} =$$

$$= c\left[\Omega_\nu(\nu_{\mathrm{e}})\,(1+z_{\mathrm{MNA}})^2\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} \qquad (9.40)$$

to

$$\Phi(\hat{\nu}_{\mathrm{CMB}}) = \frac{\Omega_\nu(\nu_{\mathrm{e}})\,(1+z_{\mathrm{MNA}})^2\left(\left[R_{\mathrm{earth}} + R_{\mathrm{sun}}\right]/R_0\right)}{\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})}\,c\,n(\tilde{\nu}_{\mathrm{CMB}}) \cong$$

$$\cong 1.84\cdot 10^{-9}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} \quad \text{(see, e.g., Eqs. (9.22) and (9.23))}. \qquad (9.41)$$

In Eq. (9.39), we must additionally consider the neutrino density parameters $\Omega_\nu(\nu_\tau)$ and $\Omega_\nu(\nu_\mu)$ because they were already explicitly used for Eqs. (6.5) and (6.6) as well as (6.8). At this new "CMB" photon neutrino, for $j = \mathrm{CMB}$, Eq. (9.29) is also valid.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 9.4, are also valid for the anti-universe.

9.5 A semi-empirical explanation for the sterile neutrino calculations

In Secs. 9.1 to 9.4, at the calculation of the sterile neutrinos, we observe a characteristical constant, which for $j = \mathrm{b, dm}, \Lambda, \mathrm{CMB}$ is defined by

$$\frac{E_0(\hat{\nu}_j)}{E_0(\tilde{\nu}_j)} = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)} \cong 629.2\,, \tag{9.42}$$

Therefore, we assume that this constant must be attributable to the light and the heavy neutrinos.

If we use $E_0(\tilde{\nu}_\Lambda) = 29.3\,\mathrm{eV}$ (see Eq. (8.5)) and $E_0(\nu_\tau) = 0.0493\,\mathrm{eV}$ (see Eq. (3.9)) as well as $E_0(\tilde{\nu}_\mathrm{b}) = 2.08\,\mathrm{eV}$ (see Eq. (8.1)) and $\sum_i E_0(\nu_i) = 0.0597\,\mathrm{eV}$ (see Eq. (3.11)), we find semi-empirically the connection

$$\frac{E_0(\hat{\nu}_j)}{E_0(\tilde{\nu}_j)} = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)} = \frac{E_0(\tilde{\nu}_\Lambda)}{E_0(\nu_\tau)} + \frac{E_0(\tilde{\nu}_\mathrm{b})}{\sum_i E_0(\nu_i)} = 629.2\,, \tag{9.43}$$

where we have again $j = \mathrm{b, dm}, \Lambda, \mathrm{CMB}$.

This connection is useful because the number densities of the sterile neutrinos (see Secs. 9.1 to 9.4) are generally defined by

$$n(\hat{\nu}_j) = \frac{E_0(\tilde{\nu}_j)}{E_0(\hat{\nu}_j)} n(\tilde{\nu}_j) = 0.178\ \mathrm{cm}^{-3}\,, \tag{9.44}$$

since the number densities $n(\tilde{\nu}_j) = 112\,\mathrm{cm}^{-3}$ of the heavy neutrinos are known (see Eqs. 8.2), (8.4), (8.6) and (9.37)). Analogously, the rest energies of the sterile neutrinos (see Secs. 9.1 to 9.4) are generally given by

$$E_0(\hat{\nu}_j) = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)} E_0(\tilde{\nu}_j), \tag{9.45}$$

since the rest energies $E_0(\tilde{\nu}_j)$ of the heavy neutrinos are also known (see Eqs. (8.1), (8.3), (8.5) and (9.33)).

Consequently, our interpretations in Secs. 9.1 to 9.4 are independent on the observation of the X-ray line $E = (3.56 \pm 0.03)\,\mathrm{keV}$.

Therefore, our description of the dark matter and dark energy particles is unique if we apply the sterile dark matter (see Sec. 9.1) and the sterile dark energy neutrinos (see Sec. 9.2), respectively.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 9.5, are also valid for the anti-universe.

10 Special properties of neutrinos in universe and anti-universe

10.1 Definition of present-day, cosmological parameters by the rest energy of the heavy and the sterile (anti)neutrinos

Using $j = \mathrm{b}, \mathrm{dm}, \Lambda, \mathrm{CMB}$, between the rest energies of the heavy (see Eqs. (8.1), (8.3), (8.5) and (9.33)) and the sterile neutrinos (see, for example, Eq. (9.45)), we have generally the connection

$$E_0(\tilde{\nu}_j) = \frac{n(\hat{\nu}_j)}{n(\tilde{\nu}_j)} E_0(\hat{\nu}_j), \tag{10.1}$$

where the constant $n(\hat{\nu}_j)/n(\tilde{\nu}_j) \equiv 1.589 \cdot 10^{-3}$ (see Eq. (9.42)). Consequently, via the results of Secs. 6 as well as 8 and 9, using the expression (3.25) with $\Omega_\gamma = 5.46 \cdot 10^{-5}$, we obtain

$$\Omega_\mathrm{b} = 2.3472 \cdot 10^{-2} \frac{E_0(\tilde{\nu}_\mathrm{b})}{\mathrm{eV}} = 3.7297 \cdot 10^{-5} \frac{E_0(\hat{\nu}_\mathrm{b})}{\mathrm{eV}}, \qquad (10.2)$$

$$\Omega_\mathrm{dm} = 2.3472 \cdot 10^{-2} \frac{E_0(\tilde{\nu}_\mathrm{dm})}{\mathrm{eV}} = 3.7297 \cdot 10^{-5} \frac{E_0(\hat{\nu}_\mathrm{dm})}{\mathrm{eV}}, \qquad (10.3)$$

$$\Omega_\Lambda = 2.3472 \cdot 10^{-2} \frac{E_0(\tilde{\nu}_\Lambda)}{\mathrm{eV}} = 3.7297 \cdot 10^{-5} \frac{E_0(\hat{\nu}_\Lambda)}{\mathrm{eV}} \qquad (10.4)$$

and

$$\Omega_\mathrm{CMB} = 2.3472 \cdot 10^{-2} \frac{E_0(\tilde{\nu}_\mathrm{CMB})}{\mathrm{eV}} = 3.7297 \cdot 10^{-5} \frac{E_0(\hat{\nu}_\mathrm{CMB})}{\mathrm{eV}}. \qquad (10.5)$$

Consequently, by the rest energies of the heavy and the sterile neutrinos, we find the corresponding, present, cosmological parameters, i.e. these particle-defined cosmological parameters (see Sec. 6) are a function of the rest energy of the heavy and the sterile neutrinos (see Secs. 8 and 9). Therefore, they represent the present, cosmological parameters of the heavy and the sterile neutrinos.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 10.1 for the universe, are also valid for the anti-universe.

10.2 Transformations of the various types of all neutrinos into one another

At the light neutrinos, if we assume the result (3.11) as a new neutrino (see also the expressions (8.1) and (8.3)), i.e. we have also four light neutrino types. These light and the four heavy neutrinos possess the same number density (see Eqs. (3.22), (3.24), (8.2), (8.4), (8.6) and (9.37)). This fact is our argument for the transformations of various types of the light and heavy neutrinos into one another. Therefore, for example, we must observe the transformations at the light neutrinos (see Eq. (3.22) for $i = \mu, \tau$)

$$\frac{\Omega_\nu(\nu_\mu)}{E_0(\nu_\mu)} = \frac{\Omega_\nu(\nu_\tau)}{E_0(\nu_\tau)} = 0.02347\,\mathrm{eV}^{-1} \tag{10.6}$$

and the heavy neutrinos (see Eqs. (8.1) to (8.4) as well as (10.2) and (10.3))

$$\frac{\Omega_\mathrm{b}}{E_0(\tilde{\nu}_\mathrm{b})} = \frac{\Omega_\mathrm{dm}}{E_0(\tilde{\nu}_\mathrm{dm})} = 0.02347\,\mathrm{eV}^{-1} \tag{10.7}$$

as well as correspondingly between the light and the heavy neutrinos

$$\frac{\Omega_\nu(\nu_\mu)}{E_0(\nu_\mu)} = \frac{\Omega_\mathrm{dm}}{E_0(\tilde{\nu}_\mathrm{dm})} = 0.02347\,\mathrm{eV}^{-1}\,. \tag{10.8}$$

Using Eqs. (9.43) or (10.1), the number density of the heavy neutrinos is also given by

$$n(\tilde{\nu}_j) = \frac{E_0(\hat{\nu}_j)}{E_0(\tilde{\nu}_j)} n(\hat{\nu}_j)\,, \tag{10.9}$$

where the constant $E_0(\hat{\nu}_j)/E_0(\tilde{\nu}_j) = n(\tilde{\nu}_j)/n(\hat{\nu}_j) \equiv 629.2$. Consequently, for $j = \mathrm{dm}$, similar to the expressions (10.6) to (10.8), we can form the transformations between the light and the sterile neutrinos

$$\frac{\Omega_\nu(\nu_\mu)}{E_0(\nu_\mu)} = \frac{E_0(\hat{\nu}_\mathrm{dm})}{E_0(\tilde{\nu}_\mathrm{dm})}\frac{n(\hat{\nu}_\mathrm{dm})}{\rho_{0\mathrm{C}}c^2} = 0.02347\,\mathrm{eV}^{-1} \tag{10.10}$$

as well as heavy and sterile neutrinos

$$\frac{\Omega_\mathrm{b}}{E_0(\tilde{\nu}_\mathrm{b})} = \frac{E_0(\hat{\nu}_\mathrm{dm})}{E_0(\tilde{\nu}_\mathrm{dm})}\frac{n(\hat{\nu}_\mathrm{dm})}{\rho_{0\mathrm{C}}c^2} = 0.02347\,\mathrm{eV}^{-1}\,. \tag{10.11}$$

Thus, we have shown uniquely the existence of transformations of various types of the light, the heavy and sterile neutrinos into one another. The existence of the sterile neutrinos is clear because the X-ray line

$E = (3.56 \pm 0.03)\,\text{keV}$ was observed by several, independent researcher groups (as long-sought dark matter particle candidate) [25-30]. Therefore, by the observation of this X-ray line, our calculated results (see Sec. 9 and Eqs. (10.1) to (10.11)), which are independent on the experiment by Eq. (9.43), must be seen as the basis of a direct evidence not only of the dark matter and the dark energy but also of all neutrino types.

Using the data of Tables III, IV and VI, all results, derived in this Sec. 10.1 for the universe, are also valid for the anti-universe.

11 The parameter of the big bang and the lifetime of the sterile neutrinos

At the time $t_{\text{eff}} = 8.034 \cdot 10^{17}\,\text{s} = 25.46\,\text{Gyr}$ (see Eq. (7.36)) or the scale factor $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see Eq. (7.27)), we have the end of the present, accelerated expansion, so that here the final value of the dark (d) energy E_{d} is found via the vacuum energy density $\rho_{\text{vac}} c^2 = \Omega_\Lambda \rho_{0\text{C}} c^2 = 3.27 \cdot 10^3\,\text{eV}\,\text{cm}^{-3}$ (see Eq. (4.48)) to

$$E_{\text{d}} = \Omega_\Lambda \rho_{0\text{C}} c^2 \, 4/3 \, \pi \, d_{\text{eff}}^3 \,, \tag{11.1}$$

where the distance d_{eff} is defined by

$$d_{\text{eff}} = R_{\text{eff}} \, r_{\text{eff}} \tag{11.2}$$

with r_{eff} as the dimensionless coordinate distance (see Eqs. (3.59 a) and (3.59 b)).

We assume that this dark energy E_{d} results from the decay of all sterile neutrinos (see Sec. 9). Thus, the half, dark energy $1/2\, E_{\text{d}}$ must be formed by massless photons, which arise from the decay of all sterile neutrinos ($\hat{\nu}$) by the gravitation in these photons with the total, thermal energy

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\gamma}\langle kT_{\gamma}\rangle = \sum_{\gamma}\langle \tfrac{1}{2}E_0(\hat{\nu})\rangle\,, \tag{11.3}$$

so that the other, half, dark energy $\tfrac{1}{2}E_{\mathrm{d}}$ is determined by the massive, sterile neutrino relic with the total, thermal energy

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\frac{1}{2}\hat{\nu}}\langle kT_{\frac{1}{2}\hat{\nu}}\rangle = \sum_{\frac{1}{2}\hat{\nu}}\langle \tfrac{1}{2}E_0(\hat{\nu})\rangle\,. \tag{11.4}$$

Therefore, their gravitational potential energy $V_{\mathrm{gr}}(d_{\mathrm{eff}})$ can be defined by

$$V_{\mathrm{gr}}(d_{\mathrm{eff}}) = \frac{G_N}{c^4}\frac{(\tfrac{1}{2}E_{\mathrm{d}})^2}{d_{\mathrm{eff}}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(\tfrac{1}{2}E_{\mathrm{d}})^2}{d_{\mathrm{eff}}}\,. \tag{11.5}$$

Using $V_{\mathrm{gr}}(d_{\mathrm{eff}}) = \tfrac{1}{2}E_{\mathrm{d}}$, we obtain

$$d_{\mathrm{eff}} = \frac{G_N}{c^4}\tfrac{1}{2}E_{\mathrm{d}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\tfrac{1}{2}E_{\mathrm{d}}\,. \tag{11.6}$$

By Eqs. (11.1) and (11.6), we find the condition

$$d_{\mathrm{eff}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\tfrac{1}{2}\Omega_{\Lambda}\rho_{0\mathrm{C}}c^2\,\tfrac{4}{3}\pi\, d_{\mathrm{ef}}^3 \tag{11.7}$$

with the solution

$$d_{\mathrm{eff}} = \left(\frac{E_{\mathrm{Pl}}^2}{\hbar c\,\tfrac{1}{2}\Omega_{\Lambda}\rho_{0\mathrm{C}}c^2\,\tfrac{4}{3}\pi}\right)^{\frac{1}{2}} = 3.321\cdot 10^{26}\,\mathrm{m}\,, \tag{11.8}$$

so that Eq. (11.2) gives

$$r_{\mathrm{eff}} = \frac{d_{\mathrm{eff}}}{R_{\mathrm{eff}}} = 1.2438\,. \tag{11.9}$$

Using Eqs. (11.1) and (11.8), we get

$$E_{\mathrm{d}} = \Omega_{\Lambda}\rho_{0\mathrm{C}}c^2\,\tfrac{4}{3}\pi\, d_{\mathrm{eff}}^3 = 5.017\cdot 10^{89}\,\mathrm{eV}\,. \tag{11.10}$$

By the result (11.10), the assumptions (11.3) and (11.4) are trivially defined by

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\gamma}\langle kT_{\gamma}\rangle = \sum_{\gamma}\langle \tfrac{1}{2}E_0(\hat{\nu})\rangle = 2.509\cdot 10^{89}\,\mathrm{eV} \tag{11.11}$$

and

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\frac{1}{2}\hat{\nu}} \left\langle kT_{\frac{1}{2}\hat{\nu}} \right\rangle = \sum_{\frac{1}{2}\hat{\nu}} \left\langle \tfrac{1}{2}E_0(\hat{\nu}) \right\rangle = 2.509 \cdot 10^{89}\,\mathrm{eV}\,. \tag{11.12}$$

Then, by Eq. (11.11), the thermal photon number density $n_\gamma(R_{\mathrm{eff}})$ is found to

$$n_\gamma(R_{\mathrm{eff}}) = 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\tfrac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3 = 1.353 \cdot 10^{282}\,\mathrm{cm}^{-3}\,. \tag{11.13}$$

Because of the decay conditions of the sterile neutrinos (see Sec. 9.1), for the massive, sterile neutrino relic, its number density $n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{eff}})$ must be equivalent to $n_\gamma(R_{\mathrm{eff}})$, i.e. we have

$$n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{eff}}) = n_\gamma(R_{\mathrm{eff}}) = 1.353 \cdot 10^{282}\,\mathrm{cm}^{-3}\,. \tag{11.14}$$

At $R_{\mathrm{eff}} = 2.67 \cdot 10^{26}\,\mathrm{m}$ (see above), in the massive universe, because of $n_\gamma(R_{\mathrm{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{eff}})$, we have a stable equilibrium between these massless photons and this massive, sterile neutrino relic, since still no complete decay of the sterile neutrinos has taken place.

Taking the result (11.8), the lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos can be assumed by

$$\tau_{\hat{\nu}} = \frac{d_{\mathrm{eff}}}{c} = 1.108 \cdot 10^{18}\,\mathrm{s} = 35.11\,\mathrm{Gyr}. \tag{11.15}$$

Consequently, for $t_{\mathrm{eff}} = 8.034 \cdot 10^{17}\,\mathrm{s} = 25.46\,\mathrm{Gyr}$ (see above), if we use the denotation $n(\hat{\nu})$ for the total number density of the sterile neutrinos, by the universal decay law

$$n_{\hat{\nu}}(t_{\mathrm{eff}}) = n(\hat{\nu})\,\mathrm{e}^{-t_{\mathrm{eff}}/\tau_{\hat{\nu}}}\,, \tag{11.16}$$

we find

$$n_{\hat{\nu}}(t_{\mathrm{eff}}) \approx \tfrac{1}{2}n(\hat{\nu}). \tag{11.17}$$

Therefore, we assume

$$n_{\hat{\nu}}(t_{\mathrm{eff}}) = n_\gamma(R_{\mathrm{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{eff}}) = 1.353 \cdot 10^{282}\,\mathrm{cm}^{-3}\,, \tag{11.18}$$

so that for the total number density of the sterile neutrinos we get

$$n(\hat{\nu}) = 2n_\gamma(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) =$$

$$= 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\text{d}}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\text{cm}^{-3}. \qquad (11.19)$$

Then, for the final state of the massive universe, at $R_{\text{f}} = R(\nu_{\text{e}}) = 3.39\cdot 10^{26}\,\text{m}$ (see Eq. 7.16)) or $t_{\text{f}} = 2.136\cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}$ (see Eq. (7.42)), we can assume a complete decay of all sterile neutrinos with $n(\hat{\nu}) = 2.706\cdot 10^{282}\,\text{cm}^{-3}$, so that we get now correspondingly an unstable equilibrium between the massless photons with

$$n_\gamma(R_{\text{f}}) = 2n_\gamma(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\text{d}}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\text{cm}^{-3} \qquad (11.20)$$

and the massive, sterile neutrino relic with

$$n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\text{d}}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\text{cm}^{-3}. \qquad (11.21)$$

This unstable equilibrium leads to a transition from the final state of the massive universe to the big bang of the massless universe. Therefore, for example, by Eqs. (4.43) and (11.20), because of particle conservation, we have the condition

$$n_\gamma(R_{\text{BB}}) = \frac{2}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{1}{R_{\text{BB}}^3} =$$

$$= n_\gamma(R_{\text{f}}) = 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\text{d}}}{\hbar c}\right)^3 \qquad (11.22)$$

with the solution

$$R_{\text{BB}} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{\frac{1}{3}}\frac{\hbar c}{\frac{1}{2}E_{\text{d}}} = 2.069\cdot 10^{-98}\,\text{m}. \qquad (11.23)$$

Then, the remaining parameters of the big bang are defined as follows. By Eq. (4.40), the big bang has taken place at the time

$$t_{\mathrm{BB}} = \frac{R_{\mathrm{BB}}}{c} = \left(\frac{\pi^4 \Omega_\gamma}{292.2273}\right)^{1/3} \frac{\hbar}{\tfrac{1}{2} E_{\mathrm{d}}} = 6.901 \cdot 10^{-107}\ \mathrm{s}\,. \qquad (11.24)$$

According to Eqs. (4.37) and (4.39), for the big bang, the vacuum energy density or the cosmological "constant" are given by

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c R_{\mathrm{BB}}^2} =$$

$$= \frac{2}{45} \frac{\Omega_\gamma^{1/3}}{\pi^{2/3}} (292.2273)^{2/3} \frac{E_{\mathrm{Pl}}^2}{(\hbar c)^3} (\tfrac{1}{2} E_{\mathrm{d}})^2 = 4.227 \cdot 10^{247}\ \mathrm{eV\,cm^{-3}} \qquad (11.25)$$

or

$$\Lambda = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R_{\mathrm{BB}}^2} =$$

$$= \frac{16}{45}(\pi\Omega_\gamma)^{1/3} (292.2273)^{2/3} \left(\frac{\tfrac{1}{2} E_{\mathrm{d}}}{\hbar c}\right)^2 = 1.406 \cdot 10^{192}\ \mathrm{m}^{-2}, \qquad (11.26)$$

respectively. Using Eqs. (4.34), for the big bang, the thermal energy or the temperature are found to

$$kT_{\mathrm{BB}} = \frac{E_{\mathrm{Pl}}^2}{\hbar c} R_{\mathrm{BB}} = \left(\frac{\pi^4 \Omega_\gamma}{292.2273}\right)^{1/3} \frac{E_{\mathrm{Pl}}^2}{\tfrac{1}{2} E_{\mathrm{d}}} = 1.583 \cdot 10^{-35}\ \mathrm{eV} \qquad (11.27)$$

or

$$T_{\mathrm{BB}} = \frac{E_{\mathrm{Pl}}^2}{\hbar c k} R_{\mathrm{BB}} = \left(\frac{\pi^4 \Omega_\gamma}{292.2273}\right)^{1/3} \frac{E_{\mathrm{Pl}}^2}{\tfrac{1}{2} E_{\mathrm{d}}\, k} = 1.837 \cdot 10^{-31}\ \mathrm{K}\,. \qquad (11.28)$$

With that, we have solved the problem "big bang". The transition from the final state of the massive universe ($R_{\mathrm{f}} \geq R \geq R_{\mathrm{Pl}}$) to the big bang of the massless universe ($R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) leads to a cyclic evolution of the total [massless ($R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) and massive ($R_{\mathrm{f}} \geq R \geq R_{\mathrm{Pl}}$)] universe.

Finally, we estimate still the lifetime of the sterile neutrinos. For this goal, we must still determine the initial, sterile neutrino number density. Therefore, we use the fact that the present day universe contains a relic neutrino (ν) background with the temperature $T_{\nu,0} = (4/11)^{1/3} T_0 = 1.945\,\mathrm{K}$, where ν characterizes the light neutrinos, which were assumed as massless or nearly massless.

For the formation of the particle-defined cosmological parameters (see Sec. 6), which were identified as the cosmological parameters of the heavy neutrinos (see Sec. 8), the necessary energy for the light neutrinos was taken from the early universe, so that the light and the heavy neutrinos have the same number density, i.e. $n(\nu) = n(\tilde{\nu}) = 112\,\mathrm{cm}^{-3}$ (see Sec. 8). Consequently, the light (ν) and the heavy ($\tilde{\nu}$) neutrino background must possess the same temperature $T_{\nu,0} = T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ according to the ideal gas law because of a constant pressure.

At the formation of sterile neutrinos, the necessary energy is taken from the relic neutrino ($\tilde{\nu}$) background, whereat the number density of the sterile neutrinos ($\hat{\nu}$) decreases to $n(\hat{\nu}) = 0.178\,\mathrm{cm}^{-3}$ (see Sec. 9). Therefore, at constant pressure, for the heavy neutrino relic with $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$, we must here assume the new volume $V(\hat{\nu}) \propto 1/n(\hat{\nu})$, so that we get a cooling of this relic neutrino background from $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ to $T_{\hat{\nu},0}$ in the old volume $V(\tilde{\nu}) \propto 1/n(\tilde{\nu})$, so that the ideal gas law yields

$$T_{\hat{\nu},0} = \frac{n(\hat{\nu})}{n(\tilde{\nu})} T_{\tilde{\nu},0} = 3.09 \cdot 10^{-3}\,\mathrm{K}\,, \qquad (11.29)$$

i.e. the corresponding thermal photon energy is given by

$$kT = k(11/4)^{1/3} T_{\hat{\nu},0} = 3.73 \cdot 10^{-7}\,\mathrm{eV}\,. \qquad (11.30)$$

Then, using Eqs. (4.34) and (11.30), for the quantum gravity of the massless universe ($R \le R_{\mathrm{Pl}}$), we obtain the distance

$$R = R_{\hat{\nu}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} kT_{\hat{\nu},0} = 4.938 \cdot 10^{-68}\,\mathrm{cm}\,, \qquad (11.31)$$

so that Eqs. (4.41) and (4.43) yield the initial, thermal number density of the massive relics or the massless photons from the decay of the sterile neutrinos to

$$n_{\hat{\nu}}(R_{\hat{\nu}}) = n_\gamma(R_{\hat{\nu}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R^3} = 1.989 \cdot 10^{197}\ \mathrm{cm}^{-3}. \tag{11.32}$$

Therefore, because of the universal decay law, by $n_\gamma(R_{\hat{\nu}}) = 1.989 \cdot 10^{197}\,\mathrm{cm}^{-3}$ (see Eq. (11.32)) and $n_\gamma(R_\mathrm{f}) = 2.706 \cdot 10^{282}\,\mathrm{cm}^{-3}$ (see Eq. (11.20)), we can assume

$$n_\gamma(R_{\hat{\nu}}) = n_\gamma(R_\mathrm{f})\,\mathrm{e}^{-t_\mathrm{f}/\tau_{\hat{\nu}}}, \tag{11.33}$$

where t_f is the age of the final state of the massive universe, whereas $\tau_{\hat{\nu}}$ is the lifetime of the sterile neutrinos.

Now, we have two possibilities. Firstly, in the variant 1, we can calculate $\tau_{\hat{\nu}} = \tau_{\hat{\nu}1}$, using the age $t_\mathrm{f} = t_{\mathrm{f}1} = 2.136 \cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}$ (see Eq. (7.42)) for the final state of the massive universe. Secondly, in the variant 2, we can estimate the age $t_\mathrm{f} = t_{\mathrm{f}2}$ for the final state of the massive universe, taking the lifetime $\tau_{\hat{\nu}} = \tau_{\hat{\nu}2} = 1.108 \cdot 10^{18}\,\mathrm{s} = 35.11\,\mathrm{Gyr}$ (see Eq. (11.15)).

The variant 1 yields

$$\tau_{\hat{\nu}1} = t_{\mathrm{f}1}\left(\ln\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\}\right)^{-1} = 1.090 \cdot 10^{18}\,\mathrm{s} = 34.52\,\mathrm{Gyr}, \tag{11.34}$$

whereas the variant 2 provides

$$t_{\mathrm{f}2} = \tau_{\hat{\nu}2}\ln\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\} = 2.172 \cdot 10^{20}\,\mathrm{s} = 6883\,\mathrm{Gyr}. \tag{11.35}$$

Now, we show that the variant 2 is correct. For this goal, we introduce the distances

$$d_{\mathrm{f}1} = R_\mathrm{f} r_{\mathrm{f}1} = ct_{\mathrm{f}1} = 6.404 \cdot 10^{28}\,\mathrm{m} \tag{11.36}$$

and

$$d_{\mathrm{f}2} = R_\mathrm{f} r_{\mathrm{f}2} = ct_{\mathrm{f}2} = 6.512 \cdot 10^{28}\,\mathrm{m}, \tag{11.37}$$

where $r_{\mathrm{f}1}$ and $r_{\mathrm{f}2}$ are the dimensionless coordinate distances, i.e. because of the energy conservation from Eq. (11.10) we can form the energy densities

$$\rho_{\rm f1} c^2 = \Omega_\Lambda \rho_{0\rm C} c^2 \left(\frac{d_{\rm eff}}{d_{\rm f1}} \right)^3 = 4.560 \cdot 10^{-4}\ {\rm eV\,cm^{-3}} \tag{11.38}$$

and

$$\rho_{\rm f2} c^2 = \Omega_\Lambda \rho_{0\rm C} c^2 \left(\frac{d_{\rm eff}}{d_{\rm f2}} \right)^3 = 4.337 \cdot 10^{-4}\ {\rm eV\,cm^{-3}}, \tag{11.39}$$

so that we have the ratios

$$\frac{\rho_{\rm f1}}{\rho_{\rm f2}} = \left(\frac{d_{\rm f2}}{d_{\rm f1}} \right)^3 = \left(\frac{t_{\rm f2}}{t_{\rm f1}} \right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}} \right)^3 \cong 1.052\,. \tag{11.40}$$

The discrepancy between $t_{\rm f} = t_{\rm f1} = 2.136 \cdot 10^{20}\,{\rm s} = 6768\,{\rm Gyr}$ (see Eq. (7.42)) and $t_{\rm f2} = 2.172 \cdot 10^{20}\,{\rm s} = 6883\,{\rm Gyr}$ (see Eq. (11.35)) has the plausible reason that the derivation of $t_{\rm f} = t_{\rm f1}$ is alone based on the vacuum energy density $\rho_{\rm vac} c^2 = \Omega_\Lambda \rho_{0\rm C} c^2 = 3.27 \cdot 10^3\ {\rm eV\,cm^{-3}}$ (see Eq. (4.48)) or the cosmological "constant" $\Lambda = 3\Omega_\Lambda / R_0^2 = 1.087 \cdot 10^{-52}\,{\rm m^{-2}}$ (see Eq. (4.51)), i.e. the limiting conditions (4.56) and (4.57) yield here the temperature $T = T_{\rm f1} = 51.41\,{\rm K}$ (see Eq. (4.58)). However, this discrepancy can be explained uniquely by Eq. (4.66), using the assumptions for the times $\tilde{t} = \tau_{\hat{\nu}2} \ln 2 = 7.680 \cdot 10^{17}\,{\rm s}$ (see Eqs. (11.15) to (11.21)) and $t = t_{\rm eff} = 8.034 \cdot 10^{17}\,{\rm s}$ (see Eq. (7.36)), so that Eq . (4.66) provides

$$\left(\frac{c^2 \tilde{\Lambda}}{3} \right)^{1/2} = \frac{t_{\rm eff} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} \Omega_\Lambda^{1/2} H_0 = 1.997 \cdot 10^{-18}\,{\rm s^{-1}}\,. \tag{11.41}$$

By Eq. (11.41), we obtain

$$\tilde{\Lambda} = 1.331 \cdot 10^{-52}\,{\rm m^{-2}}, \tag{11.42}$$

so that the temperature $T = T_{\rm f2} = 54.07\,{\rm K}$ results from Eqs. (4.53) and (11.42) for $N(T) = 3.362644$ (see Eq. (2.12) or Refs. [4, 6]. For constant pressure $P = -\Omega_\Lambda \rho_{0C} c^2$, the ideal gas law yields following connection between the temperature T and the volume V for the state changing according to Eq. (11.40):

$$\frac{T_{f2}}{T_{f1}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052\,. \tag{11.43}$$

Thus, we get

$$t_{f2} = t_{f1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 6883\ \text{Gyr} \tag{11.44}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 35.11\ \text{Gyr}\,, \tag{11.45}$$

i.e. $t_{f2} = 6883\,\text{Gyr}$ (see Eq. (11.35)) and $\tau_{\hat{\nu}2} = 35.11\,\text{Gyr}$ (see Eq. (11.15)) are correct.

Because of Eqs. (11.34) and (11.35), the results (11.44) and (11.45) provide the evidence, that for $R \le R_{\text{Pl}}$ our assumptions, $\text{const} = 1/N(T)$ and $N(T) = 1/2\Omega_\gamma$ (see Eqs. (4.35) to (4.38)), are correct.

Consequently, we have reliably determined the final age of the massive universe and the lifetime of the sterile neutrinos.

Using the data of Tables III, IV and VI, the results, derived in this Sec. 11, are also valid for a cyclic evolution of the total (massless and massive) anti-universe.

12 Summary

We have shown that from the big bang ($R = R_{\text{BB}}$) by the momentary formation of universe and anti-universe, which expand in opposite directions, the virtual particle-antiparticle (photon-photon) pairs of the quantum vacuum are separated into two real radiation or particle fields ($R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) as the massless universe or anti-universe. These massless universes were described via the gravitation. On this way, at $R = R_{\mathrm{Pl}}$, if the real photons of the massless universe or anti-universe possess the thermal energy $kT_{\mathrm{Pl}} = E_{\mathrm{Pl}}$, the gravitation between these real photons yields the relativistic energy $E_{\mathrm{Pl}} = E(\mathrm{P})$ or $E_{\mathrm{Pl}} = E(\overline{\mathrm{P}})$ for all particles (P) and antiparticles ($\overline{\mathrm{P}}$) not only in the massless universes but also in the massive universes because of their joint boundary $R = R_{\mathrm{Pl}}$, i.e. the separation between matter and antimatter. These massive universes are described by the Friedmann-Lemaitre Equations and the new inflation model.

We have explained generally the long-sought quantum gravity as a result of the unique connection between vacuum energy density and cosmological "constant", i.e. we have found a simple solution for this four-dimensional quantum gravity, which influences not only the massless but also the massive universes because their space-time continuum is penetrated with the quantum vacuum.

Generally, this four-dimensional quantum gravity explains the particle formation, which permits the above-mentioned conclusion that at $R = R_{\mathrm{Pl}}$ all particles and antiparticles have the Planck energy as relativistic energy in the massless ($R \leq R_{\mathrm{Pl}}$) and massive ($R \geq R_{\mathrm{Pl}}$) universe and anti-universe.

The excellent usefulness of this four-dimensional quantum gravity is shown at its application for the transition from the final state of the massive universes to the big bang as the beginning of the massless universes as a consequence of the dark energy, which is completely converted into massless photons and the corresponding, massive relics by the decay of the sterile neutrinos via the gravitation. This result supports generally our hypothesis from a joint origin of the dark matter and the dark energy by sterile neutrinos (see below).

Using the known properties of particles and antiparticles, we have derived the necessary quantities for the description of the massless and massive universes, so that we can prove that the particle-defined and antiparticle-defined, present day, cosmological parameters, which determine completely the history and future of the massive universes and anti-universe, have identical values. Therefore, we can prove that all results for the universe are also valid for the anti-universe.

Using the results of the neutrino oscillation experiments together with cosmological data, we have presented an approximate solution for the neutrino problem (by the estimation of the neutrino masses and the neutrino densities of the universe). Via this estimation and by the magnetic neutrino moment of the standard $SU(2)\times U(1)$ electroweak theory, we have derived the masses of the X and Y gauge bosons. We have also obtained the mass of the magnetic monopoles via the big bang model, so that in accordance with the particle physics we have exactly determined the coupling constant of the supersymmetric grand unified theories which characterize the early universe.

Thus, using the two scale factors of absorption at the redshift condition before and after the inflation as different functions of the Hubble constant, we have obtained an exact and unique description of this inflation and its transition to the standard model as well as the vacuum energy density and the curvature of the universe, so that we have confirmed the presented, qualitative solutions for the pressing problems of the standard cosmology as the horizon problem, the monopole problem and the initial condition problem as well as the flatness problem. At this, we have shown that the massive universe and anti-universe are Euclidian via the derivation of $\Omega_{tot} = 1$ $(k = 0)$ and $\Omega_{tot}(z) = 1$ $(k = 0)$.

However, the old inflation scenario, which solves qualitatively the most pressing problems of the cosmology, cannot be explained in the framework of the known particle physics. This difficulty does not exist for our new inflation model, which describes not only qualitatively, but also quantitatively these most pressing problems within the cosmology.

Via the matter-dominated universe, we have shown that the influence of the dark energy, which determines the present-day accelerated expansion of the universe, ends in equilibrium with a negative acceleration, whereat we have explained the Pioneer anomaly and the flyby anomaly via this mean negative acceleration as well as the astronomical unit changing. After the end of the present, accelerated expansion, i.e. in future, this present exponential expansion gives way to a slow linear expansion of the universe by the astronomical unit changing. Therefore, the age of the universe for its final state, which is determined by the redshift condition of the electron neutrino, can be estimated to $6883\,\mathrm{Gyr}$ via the transition from the final state of the massive universes to the big bang as the beginning of the massless universes. Via the neutrinos, we have obtained the redshift values for the beginning, the half and the end of the reioinization.

The evolution of the universe is based on its early epoch which is defined by SUSY GUTs. Therefore, we have estimated the present values of the cosmological parameters by aid of the light neutrino density parameters as well as by the factor of the products of the different ratios of the relativistic energy to the rest energy of the three extremely heavy particles of the supersymmetric grand unification. We have obtained an excellent agreement between the observed and estimated present values of the cosmological parameters Ω_b, Ω_{dm} and Ω_Λ, so that because of $\Omega_{tot} = 1$ we can estimate also the reduced Hubble constant, which to this day was only determined by observations.

These facts confirm also the estimated astrophysical solutions (see above) for the neutrino rest energies, the neutrino density parameters, the rest energies of the X and Y gauge bosons as well as the magnetic monopoles, the inflation, the matter-antimatter problem, the curvature of the universe, the horizon problem, the monopole problem, flatness problem, the initial condition problem, the reionization, the Pioneer anomaly, the astronomical unit changing, the final state and the future of the universe.

Because these particle-defined cosmological parameter values Ω_b, Ω_{dm} and Ω_Λ are proportional the light neutrino density parameters, which are again proportional to the rest energy of light neutrinos, we can so estimate the rest energy of three heavy neutrinos denoted therefore as the heavy baryon (ionizable matter) neutrino, the heavy dark matter neutrino and the heavy dark energy neutrino. Thus, the particle densities Ω_b, Ω_{dm} and Ω_Λ of the universe were identified as the corresponding, heavy neutrino densities of the universe. By the aid of the electron neutrino, we have also shown the existence possibility of a fourth heavy "CMB" neutrino. All these heavy neutrinos have the same number density as the light neutrinos.

The smallest, heavy neutrino, i.e. the heavy "CMB" neutrino, describes brilliantly the redshift of the photon decoupling, whereas the three other, heavy neutrinos, i.e. the heavy baryon neutrino, the heavy dark matter neutrino and the heavy dark energy neutrino, define excellently the present, cosmological parameter values. These special properties of the heavy neutrinos confirm indirectly the correctness of the rest energies of the light neutrinos as well as the X, Y gauge bosons and the magnetic monopoles.

We have shown that these stable heavy (right handed) neutrinos are correspondingly coupled with the far heavier, unstable, sterile neutrinos (with the same however smaller number density), so that they were denoted so as the sterile baryon (ionizable matter) neutrino, the sterile dark matter neutrino, the sterile dark energy neutrino and the sterile "CMB" neutrino. Our results show that the stable, heavy neutrinos are independent particles with own excited energy states in form of the instable sterile neutrinos.

We have provided for the first time the evidence that the sterile dark matter neutrinos are responsible for the present dark matter. We have so confirmed the hypothesis for a "joint" origin of the present dark matter and dark energy by sterile neutrinos in form of the sterile dark matter neutrinos and the sterile dark energy neutrinos.

We have shown uniquely the existence of transformations of various types of the light, the heavy and sterile neutrinos into one another via their number density.

By the transition from the final state of the universe to the big bang, we have clearly shown that the evolution of the total (massless and massive) universe is cyclic. With that, in contrast to the present theory, which postulates an infinite, Euclidian universe, we have proved that the Euclidian universe is finite.

We have estimated the data for their big bang, which takes place at the distance $R = R_{\mathrm{BB}} = 2.069 \cdot 10^{-98}$ m and the time $t = t_{\mathrm{BB}} = 6.901 \cdot 10^{-107}$ s.

Its vacuum energy density or cosmological "constant" are defined by $\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = 4.227 \cdot 10^{247}$ eV cm^{-3} or $\Lambda = 1.406 \cdot 10^{192}$ m^{-2}. The lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos was found to $\tau_{\hat{\nu}} = 35.11$ Gyr.

Using the data of Tables III, IV and VI, we have proved that all results, derived in this work for the universe, are also valid for the anti-universe.

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