HADRONIC JOURNAL 43, 79 - 119 (2020)
DOI 10.29083/HJ.43.01.2020

BINDING ENERGY OF HELIUM $^{4}_{2}$He, CARBON $^{12}_{6}$C, DEUTERIUM $^{2}_{1}$H, AND TRITIUM $^{3}_{1}$H IN VIEW OF THE SHELL-NODAL ATOMIC MODEL AND DYNAMIC MODEL OF ELEMENTARY PARTICLES

George P. Shpenkov
Bielsko-Biala, Poland
g.shpenkov@gmail.com

Received February 11, 2020

Abstract

Derivation of binding energy of nucleons in deuterium, tritium, helium $^{4}_{2}He$, and carbon $^{12}_{6}C$ atoms on the basis of shell-nodal atomic model and dynamic model of elementary particles is presented in this paper. It is shown that the internal structure of nucleons at the level of constituent g-particles recalls the shell-nodal structure of silicon of the nucleon level. Calculated binding energies well agree with the binding energies estimated from mass-energy equation as the mass defect.

PACS numbers: 03.40.Kf, 03.65.GE, 11.90.+t, 12.10.-g, 12.90.+b, 13.75.Cs, 14.20.Dh, 14.40.-n, 14.60.-z, 21.10.Dr, 21.60.-n, 32.10.-f

Key words: wave equation, elementary particles, associated mass, exchange charge, binding energy, atomic structure, proton, neutron, deuterium, tritium, helium, carbon, nuclear forces, nuclear structure

1. Introduction

Shell-Nodal Atomic Model (SNM) allows explaining the structure of matter at atomic and subatomic levels in a more logical and simple way. Different sides of the model were already published [1 - 4] or are in printing being accepted for publication [5, 6]. The model is based on particular solutions of ordinary wave equation in spherical polar coordinates and on the Dynamic Model of Elementary Particles (DM) [7].

As follows from the SNM, atoms recalls molecules if one judges on their internal structure. Their discrete points-nodes are the places of location of nucleons, at least by two per node. Nucleon nodes are located on spherical atomic shells radius of which is defined by roots of Bessel functions [8]. The main features of SNM are presented here, in introduction, very concisely. In spite of this, the reader will be able to understand in outline origination of the intra-atomic shell-nodal structure and the derivation of binding energy of atoms on the basis of the SNM without turning to references.

The wave function $\hat{\Psi}$,

$$\hat{\Psi} = \hat{R}_l(kr)\hat{Y}_{l,m}(\theta,\varphi)\hat{T}(\omega t) = \Psi_p + i\Psi_k, \qquad (1.1)$$

of the wave equation

$$\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0, \quad \text{or} \qquad \Delta\hat{\Psi} + k^2\hat{\Psi} = 0, \qquad (1.2)$$

contains spherical harmonics

$$\hat{Y}_{l,m}(\theta,\varphi) = \Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi) = C_{l.m}P_{l.m}(\cos\theta)C_m \exp[\pm i(m\varphi + \alpha)], \qquad (1.3)$$

where $C_{l,m}$ and C_m are coefficients depending on the normalizing conditions, $P_{l,m}(\cos\theta)$ are Legendre adjoined functions, $\Phi_m(\varphi)$ are azimuthal functions, α is an initial phase of the azimuthal state,

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad (1.4)$$

is the wave number.

The difference between solutions of ordinary wave equation (1.2) and the Schrödinger's wave equation [9] is defined by the difference in radial

equations (because of the different wave numbers k), which yields the different radial solutions $\hat{R}_l(kr)$. Schrödinger's radial equation and its solutions are analysed in detail in [10, 11].

At integer values of the wave number m, the particular solution of the wave equation (1.2) has the standard form. If we present the number m in the form $m = \frac{1}{2}2s$, where $s \in N$, we arrive at

$$\hat{\psi} = A_l \hat{R}_l(\rho)\Theta_{l,s}(\theta)e^{\pm is\varphi} = A_l\sqrt{\pi/2\rho}H^{\pm}_{l+1/2}(\rho)\Theta_{l,s}(\theta)e^{\pm is\varphi} \qquad (1.5)$$

or

$$\hat{\psi} = A_l\sqrt{\pi/2\rho}(J_{l+1/2}(\rho) \pm iY_{l+1/2}(\rho))\Theta_{l,s}(\theta)e^{\pm is\varphi}, \qquad (1.6)$$

where A_l is the constant factor; $\rho = kr$; $H^{\pm}_{l+1/2}(\rho)$, $J_{l+1/2}(\rho)$ and $Y_{l+1/2}(\rho)$ (or $N_{l+1/2}(\rho)$) are the Hankel, Bessel and Neumann functions, correspondingly.

The form of the function (1.6) uniquely shows that it describes the kinematic structure of *standing spherical waves* in wave physical space. Thus the solution (1.6) yields the spatial geometry of disposition of specific points (nodes and antinodes) in which the wave $\hat{\Psi}$ function takes the zero and extremal values. With this, polar-azimuthal functions, potential and kinetic, define the *angular spatial coordinates*, respectively, of nodes and antinodes of the standing spherical waves.

Two terms in (1.6) are the potential and kinetic *spatial* constituents of the $\hat{\Psi}$-function; they have the following form

$$\hat{\psi}_p = Ac_l(\rho)/\rho = A\sqrt{\pi/2\rho}J_{l+1/2}(\rho)\Theta_{l,m}(\theta)e^{\pm im\varphi}, \qquad (1.7)$$

$$\hat{\psi}_k = \pm As_l(\rho)/\rho = \pm A\sqrt{\pi/2\rho}Y_{l+1/2}(\rho)\Theta_{l,m}(\theta)e^{\pm im\varphi}. \qquad (1.8)$$

The half-integer solutions of (1.2), at $l = m = (1/2)s$, have the form

$$\hat{\psi} = A\hat{R}_s(\rho)\Theta_s(\theta)e^{\pm i\frac{s}{2}\varphi}, \qquad (1.9)$$

where

$$\hat{R}_s(\rho) = \sqrt{\pi/2\rho} H^{\pm}_{\frac{s}{2}+\frac{1}{2}}(\rho), \tag{1.10}$$

$$\Theta_s(\theta) e^{\pm i\frac{s}{2}\varphi} = C_s \sin^{\frac{s}{2}}\theta(\cos\frac{s}{2}\varphi \pm i\sin\frac{s}{2}\varphi). \tag{1.11}$$

All spatial components are determined with the accuracy of a constant factor A, imposed by boundary conditions, which have no influence on the peculiarity of distribution of the *nodes* on radial spheres. The superposition of even and odd solutions defines the *even-odd solutions*. Odd solutions describe the nodes, lying in the equatorial plane of atomic space. In this plane there are also solutions in the form of *rings* in space (graphically shown further) separated by radial unstable shells.

$\hat{\Psi}$-Function represents any parameter of the *wave* field such as, for example, potential-kinetic displacement, potential-kinetic speed, physical potential-kinetic probability, *etc.*

The radial component $\hat{R}_l(kr)$ of the wave function $\hat{\Psi}$ (1.1) describes the radial field of displacements of the wave parameter, which the $\hat{\Psi}$-function represents in the wave equation (the density of potential-kinetic phase probability, in the work [12]), the polar component $\Theta_{l,m}(\theta)$ describes the polar displacements, and $\hat{\Phi}_m(\varphi)$ describes the azimuth displacements.

The potential solutions define the coordinates of rest, whereas the conjugate kinetic solutions define the coordinates of maxima of motion. Thus, the potential solutions give us the spatial coordinates of equilibrium domains (nodes of standing spherical waves) in the wave atomic space.
Thus, we should distinguish two solutions, potential and kinetic, do not mixing them [13]. Rest and motion (nodes and antinodes) are the two *qualitatively different* states. Kinetic harmonics are the same, in form, as potential harmonics, but they are displaced in space in the radial direction and turned in the azimuthal direction, around the z-axis, with respect to potential harmonics (just like $\cos m\varphi$ with respect to $\sin m\varphi$ in (1.3)) so that the kinetic extrema are between the corresponding potential extrema (as alternated nodes and antinodes in standing waves).

The form of the radial equation and its solutions $\hat{R}_l(kr)$ depend on the concrete problem, which imposes the definite requirements on the wave number k. However, for any model of an object of study, the radial solutions define the characteristic spheres of extrema and zeros of the radial function.

For a variety of problems, it is sufficient to know at most that such characteristic spheres exist.

Evidently, the polar-azimuth equation is common (universal) for all spherical objects if they are described by the general wave equation (1.2).

Radial solutions $\hat{R}_l(kr)$ entered in (1.5) are defined by roots of Bessel functions [8]. They give the equilibrium spherical shells of standing spherical waves in the wave field of potential and kinetic displacements.

Polar-azimuthal functions of (1.5) define polar-azimuthal coordinates of *nodes and antinodes of standing spherical waves*, located on these shells.

Polar components $\Theta_{l,m}(\theta)$ of the $\hat{\Psi}$-function (1.1) define characteristic parallels of extrema (principal and collateral) and zeros on radial spheres (shells). Azimuthal components $\hat{\Phi}_m(\varphi)$ define characteristic meridians of extrema and zeros. Potential and kinetic polar-azimuthal functions $\hat{Y}_{l,m}(\theta,\varphi)$ select together the distinctive coordinates of extrema and zeros on the radial shells.

The geometry of characteristic states on radial shells is expressed by extrema and zeros of the polar-azimuthal components of the $\hat{\Psi}$-function. The *potential* solutions of the $\hat{\Psi}$-function (for $l = 0, 1, 2, 3, 4, 5$) are depicted graphically in Fig. 1.1 for a constant value of the radial coordinate r. In this figure, with an example for $l = 5$ and $m = \pm 2$, it is also showed how the presented discrete (nodal) structure of the three dimensional wave space is obtained from the aforementioned solutions for the different wave numbers l and m.

The graphs of the solutions indicate that there are *principle* (designated in Fig. 1.1 by shaded points) and *collateral* (designated by the smaller unnumbered hollow points) extrema, which determine, correspondingly, stable and metastable states of probabilistic events.

Principal *potential* polar-azimuthal nodes are numbered in Fig. 1.1 by ordinal numbers. The principal polar-azimuth extrema (potential and kinetic, $m \neq 0$) mainly define the geometry of radial shells of atomic space, whereas collateral extrema ($m \neq 0$) play the secondary role. Both principal and collateral extrema are points-nodes of the steady-state discrete geometry of the wave field of matter-space-time of atoms.

As it turned out, the shell-nodal structure, presented in Fig. 1.1, is reminiscent of *spherical resonant cavities* [14] (described by Bessel functions as well) having internal oscillating electric and magnetic mode fields. And

what is more, as appears from the comprehensive analysis, all types of *elementary crystal lattices represent*, in essence, elementary *nodal structure of standing waves* in a limited three-dimensional wave physical space [1, 6].

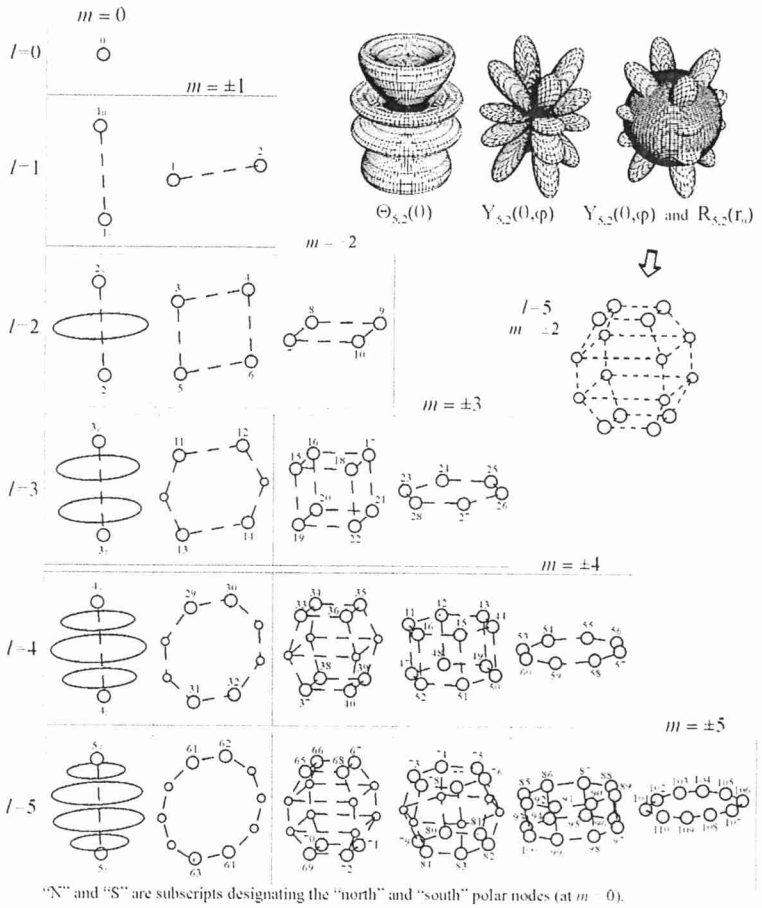


Fig. 1.1. Spatial solutions $\psi_{l,m}(\rho,\theta,\varphi)_p = C_\psi R_l(\rho)\Theta_{l,m}(\theta)Cos m\varphi$ (for $r = const$) of the wave equation (1.2) for *spherical standing waves* presented in the form indicating the space distribution of *potential* extrema-nodes (discrete elements of the shell-nodal structure of atoms); numbers 1, 2, 3, …, 110 are the ordinal numbers of the principal potential polar-azimuth nodes coinciding with atomic numbers of the elements Z [1].

Thus, the spatial shell-nodal structure presented in Fig. 1.1 uniquely determines the structure of matter at the atomic and molecular levels, in particular, the intra-atomic structure and the structure of crystals.

The quasi-similarity of the geometry of external shells, for the same quantum number m and different quantum numbers l, clearly seen from Fig. 1.1, reveals the nature of Mendeleev' Periodic Law [5, 15]. A great body of other important consequences, originated from the above solutions, relates to the new data concerning the atomic structure, periodicity, symmetries, the nature and structure of isotopes [16], *etc.* These and other relevant data one can find in the reference works of the author.

2. The shell-nodal structure of helium ${}^{4}_{2}He$, and carbon ${}^{12}_{6}C$

The completely realized polar-azimuthal n-th potential shell (with potential nodes) is defined, in accordance with (1.2), by the function

$$\Psi_{l,m}(\rho_{l,n},\theta,\varphi)_p = C_\Psi R_l(\rho_{l,n})\Theta_{l,m}(\theta)\cos(m\varphi+\alpha), \qquad (2.1)$$

where $\rho_{l,n}$ is the relative radius of the n-th external radial shell. The geometry of angular disposition of nodes is determined by polar-azimuthal functions $\Theta_{l,m}(\theta)\cos(m\varphi+\alpha)$ of (2.1). The latter and their sections are presented in Fig. 2.1 for hydrogen, helium, and carbon atoms. In the case of carbon atom, two configurations of functions, $\Theta_{1,1}(\theta)\cos\varphi$ and $\Theta_{1,1}(\theta)\cos(\varphi+\pi/2)$, different by initial azimuthal phases ($\alpha = 0$ and $\alpha = \pi/2$), are shown in Fig. 2.1. The octahedral structure ($\alpha = \pi/2$) is realized in the diamond structure of carbon.

The nodal structure of ${}^{4}_{2}He$ and ${}^{12}_{6}C$, originated from (2.1), is conditionally shown in Fig. 2.2. The carbon's nodal structure is depicted with the plane disposition of its six principal polar-azimuth nodes ($\alpha = 0$). All principal polar-azimuthal nodes in the stable isotopes, which are presented in this figure, are filled with paired nucleons - say H-atoms (to which we refer the hydrogen atoms, protons and neutrons). Polar nodes are situated at the polar azis z, forming something like "spinal" of atoms.

The nodal structure of carbon isotope ${}^{12}_{6}C$ and its polar-azimuth functions are shown also in Fig. 2.3. The carbon atom has the central empty node ($m = 0$, $l = 0$) and four spherical shells: two shells ($m = 0$, $l = 1, 2$) with four empty

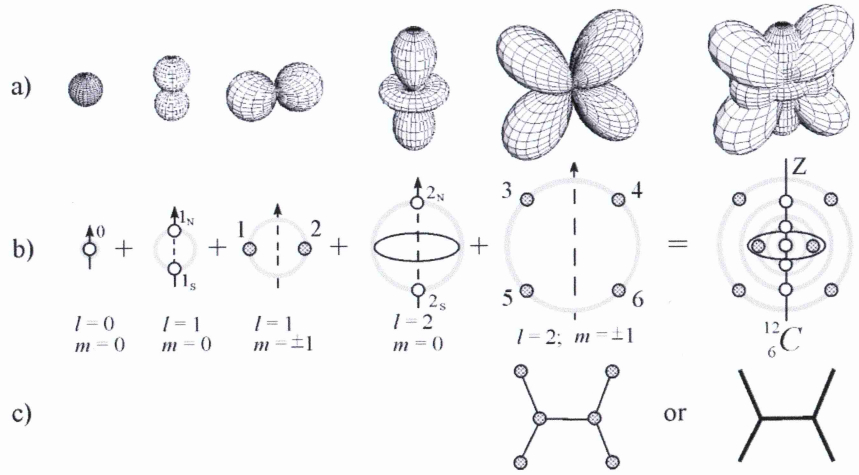


Fig. 2.3. Plots of potential polar-azimuth functions $\Theta_{l,m}(\theta)\mathrm{Cos}m\varphi$ ($l=0,1,2$; $m=0,\pm1$) (a), their extremal points on radial extremal shells $R_l(\rho)$ (b), and the symbolic designation of carbon $_6$C (c).

Six potential polar-azimuth nodes (at $m=\pm1$), completed every by two H-atoms, lie in one plane: two potential nodes are in the inner shell (l = 1) and the four ones are in the outer shell (l = 2). Six empty kinetic nodes (not shown here) lie in a perpendicular plane with respect to the plane of disposition of potential nodes, on kinetic radial shells.

Hydrogen is mainly in coherent states in Nature, in particular, in the form of coupled atoms – hydrogen molecules H_2. Paired H-atoms, filling polar-azimuth nodes, apparently provide for the stable state of atomic shells. The *condition of coupling*, observed in nature, is inherent not only for H-atoms in nodes of individual atoms, but probably also for individual atoms themselves in solids, liquids, and molecules built on their basis.

The distance r between nodes is defined by roots of Bessel functions $z_{n,m}=kr$, as follows from solutions (1.6) of the wave equation (1.2).

The derivation of binding energy of nucleons, located in nodes of atoms, rests on the Law of Universal Exchange, which originates from the Dynamic Model of Elementary Particles (DM) [7]. Therefore, we will recall now the main features of the DM and show the formula of exchange used for calculations of binding energies here.

potential-kinetic polar nodes (situated along the z-axis) and one *ring*, two shells ($m=\pm1; l=1,2$) with six completed *potential polar-azimuth nodes* and six empty *kinetic polar-azimuth nodes* (the lasts are shown neither in Fig. 1.1 nor in other figures).

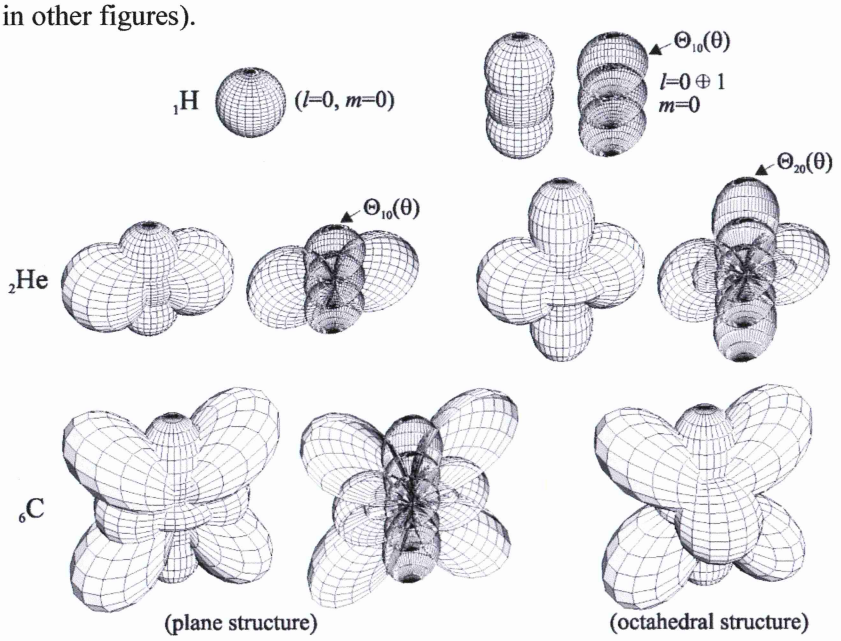
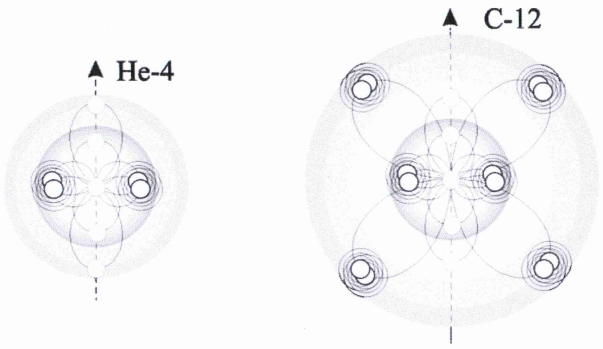


Fig. 2.1. The structure of potential polar-azimuthal functions (integer solutions) for hydrogen $_1H$, helium $_2He$, and carbon $_6C$.



Fi.2. 2. The nodal structure of helium ^4_2He and carbon $^{12}_6C$.

3. Dynamic model of elementary particles; the law of central exchange

We regard elementary particles as dynamic spherical formations of a complicated internal structure being in dynamic equilibrium with environment through the wave process of the definite frequency ω_e characteristic for the atomic and subatomic levels [7]. Longitudinal oscillations of their spherical wave shells in the radial direction provide an interaction of the particles with other objects and the ambient field of matter-space-time. The logical triad: *matter-space-time* expresses an indissoluble bond of matter, space, and time (incessant wave motion).

The wave shell, or in another words, the characteristic sphere of an elementary particle restricts the main part of the particle from its field part merging gradually with the ambient field of matter-space-time. The main part (core) is the *basis* of the particle, whereas the field part represents its *superstructure*. Such a model interprets an elementary particle as a particular pulsing physical point (like a micropulsar) of an arbitrary level of matter-space-time, restricted by the characteristic wave sphere and being in rest in the field-space.

The existence and interactions of the particles are, in essence, a continuous process of wave *exchange* of matter-space-time. The wider (and, hence, truer) notion *exchange* is thus more correct because it reflects behavior of elementary particles in their dynamic equilibrium with the ambient field, at rest and motion, and interactions with other objects (and particles themselves). In other words, the notion *exchange* is more appropriate from the point of view of the physics of the complex behavior of elementary particles viewed as dynamic micro-objects belonging to one of the interrelated levels of the many-level Universe.

The ratio of mass dm and volume dV of elementary particles defines their *absolute-relative density* ε:

$$\varepsilon = dm / dV = \varepsilon_0 \varepsilon_r, \qquad (3.1)$$

where $\varepsilon_0 = 1\, g \cdot cm^{-3}$ is the *absolute unit density*, and ε_r is the *relative density*.

Masses of all dynamic formations (micro-particles) in the Universe, according to the DM, have *associated field character* with respect to the deeper level of the field of matter-space-time; therefore, their own (proper, rest)

masses do not exist. Associated mass, or briefly the mass of the particle is defined from the formula

$$m = \frac{4\pi r^3 \varepsilon_0 \varepsilon_r}{1 + k_e^2 r^2}, \tag{3.2}$$

where r is the radius of the wave shell;

$$k_e = 2\pi / \lambda = \omega_e / c \tag{3.3}$$

is the wave number corresponding to the *fundamental frequencies* ω_e of the field of exchange, which is characteristic for the atomic and subatomic levels of the Universe.

The *speed of wave exchange* is presented in the form

$$\hat{\upsilon} = \upsilon(kr) e^{i\omega t}, \tag{3.4}$$

The volumetric rate of mass exchange of the particles with environment called the *exchange charge*, or merely the charge, is defined as

$$\hat{Q} = d\hat{m} / dt = S\hat{\upsilon}\varepsilon, \tag{3.5}$$

where S is the area of a closed surface separating the space of an elementary particles from the surrounding field of matter-space-time, υ is the speed of wave exchange (interaction) at the separating surface. Strictly speaking, the exchange charge is the measure of the rate of exchange of matter-space-time, or briefly the *power of mass exchange*. In this wider sense, the area of exchange S does not necessary concern the closed surface. The symbol "^" expresses the contradictory (or complex) *potential-kinetic* character of physical space-fields [17, 18].

Derivations carried out in [1, 7] have shown that the charge of exchange $\hat{Q}$ has the active-reactive character. It is defined as

$$\hat{Q} = \frac{4\pi r^3 \varepsilon_0 \varepsilon_r}{1 + k_e^2 r^2} (k_e r \omega_e + i\omega_e) = Q_a + iQ_r, \tag{3.6}$$

where

$$Q_a = \frac{4\pi r^3 \varepsilon_0 \varepsilon_r}{1 + k_e^2 r^2} kr\omega \qquad \text{and} \qquad Q_r = \frac{4\pi r^3 \varepsilon_0 \varepsilon_r}{1 + k_e^2 r^2} \omega \tag{3.7}$$

are *active* and *reactive charges*, correspondingly.

The active component Q_a defines the dispersion during exchange, which in a steady-state process of exchange is compensated by the inflow of motion and matter from the deeper levels of space.

We see that the reactive component of charge Q_r, called in contemporary physics the "*electric*" charge (further for brevity, the *charge of exchange* Q) is connected with the associated mass m (3.2) by the relation

$$Q = m\omega_e . \qquad (3.8)$$

From (3.8) it follows that the *dimensionality* of the exchange charge is $g \cdot s^{-1}$. Thus, the DM reveals the physical meaning of the electric charge. The exchange ("electric") charge is the measure of the *rate of exchange* of matter-space-time, or briefly the power of mass exchange; its alternative value changes with the fundamental frequency ω_e.

The derivation carried out first in [19] (details one can find in [7], see also [20]) leads to the following formula of correspondence between exchange charge Q and Coulomb charge q_C:

$$Q = q_C \sqrt{4\pi\varepsilon_0} . \qquad (3.9)$$

We recall, here and further in the DM, $\varepsilon_0 = 1\, g \cdot cm^{-3}$ is the absolute unit density. Hence, the *exchange* (*reactive*) *charge* of an electron at the level of the fundamental frequency ω_e is

$$e = e_C \sqrt{4\pi\varepsilon_0} = 1.702691627 \cdot 10^{-9}\ g \cdot s^{-1}, \qquad (3.10)$$

where $e_C = 4.803204401 \cdot 10^{-10}\ CGSE_q$ is the Coulomb charge of an electron in CGSE system of units. The dimensionality of the unit $1\mathrm{CGSE}_q$ is actually expressed by fractional powers of absolute units of matter and space of CGS systems, $g^{1/2} \cdot cm^{3/2} \cdot s^{-1}$, that has no physical meaning. An introduction of the SI unit of the electric charge, 1 Coulomb, did not change the situation, because the dimensionality of the charge is actually, as before, expressed by fractional powers of the units of matter and space, $kg^{1/2} \cdot m^{3/2} \cdot s^{-1}$ [21].

The exchange charge of the value (3.10) (electron charge) represents an *elementary quantum of the rate of mass exchange*. On the basis of (3.8) and (3.10), knowing the mass of the electron m_e, we found the *fundamental frequency of the wave field of exchange* at the *subatomic level*

$$\omega_e = e/m_e = 1.869162505 \cdot 10^{18}\ s^{-1}. \qquad (3.11)$$

It is the frequency of the field called in modern physics "*electrostatic*". Note in this connection that there are no "static" fields in the Universe where all is in incessant wave motion.

The radius of wave shells of particles r is derived from the formula of mass (3.2), where $\varepsilon_r = 1$, $k_e = 1/\lambda\!\!\!^{-}_e$, with

$$\lambda\!\!\!^{-}_e = c/\omega_e = 1.603886538 \cdot 10^{-8}\ cm \qquad (3.12)$$

Thus, the notion *rest mass* of elementary particles is not valid of principle for the DM. Accordingly, one could conclude that the rest mass of elementary particles does not exist. The *associated nature of mass*, as the field mass of the central wave exchange, naturally originates from this model. Moreover, in the DM, particles are boundless in size. Characteristic spheres (wave shells) of the particles restrict only their inner and external spaces. According to the definition, geometrical space (spherical volume) of an elementary particle, restricted by its wave spherical shell, is the external world of the particle.

From the above it follows that the physical field-space of the Universe represents by itself an infinite series of spaces embedded in each other [recalling a set of nesting dolls, or infinite functional series $f(x) = \sum_{k=1}^{\infty} u_k(x)$]. This series of spaces expresses the fundamental concept of natural philosophy concerning the infinite divisibility of matter. Every level of space is the basis level for the nearest above-situated level and, simultaneously, it is the level of superstructure for the nearest below-situated level. This means that above-situated field-spaces are formed on the basis of below-lying field-spaces. Accordingly, there is no meaning to the concept of "very last elementary particle" in the common classical sense of this phrase.

As follows from the DM [7], the law of central exchange has the form

$$F = \omega^2 \frac{m_1 m_2}{4\pi\varepsilon_0 r^2}, \qquad (3.13)$$

where ω is the fundamental frequency of exchange, m_1 and m_2 are associated masses defined by (3.2), $\varepsilon_0 = 1\,g \cdot cm^{-3}$ is the absolute unit density. This law lies at the foundation of nature. If $\omega = \omega_e$ (3.11), this law describes exchange (interactions) at the atomic and subatomic levels:

$$F_e = \omega_e^2 \frac{m_1 m_2}{4\pi\varepsilon_0 r^2}. \qquad (3.14)$$

A particular case of the law (3.13) is the law of universal gravitation, which we present in the form

$$F_g = \omega_g^2 \frac{m_1 m_2}{4\pi\varepsilon_0 r^2}, \qquad (3.15)$$

where ω_g is the fundamental frequency at the gravitational (mega) level of the Universe, *i.e.*, the fundamental gravitational frequency. Its magnitude, defined on the basis knowing the value of the gravitational constant $G = 6.6742 \cdot 10^{-8}\, cm^3 \cdot g^{-1} \cdot s^{-2}$, is equal to

$$\omega_g = \sqrt{4\pi\varepsilon_0 G} = 9.1581 \cdot 10^{-4}\, s^{-1}. \qquad (3.16)$$

The existence of the *gravitational frequency* ω_g and, hence, the corresponding *gravitational radius* of elementary particles $\lambda\!\!\!^{-}_g = c/\omega_g = 327.35\, Mkm$ shows the indissoluble bond of micro- and mega-objects of the Universe in the unit complex of the Infinitely Small and Infinitely Big, as the coexisting polar oppositions.

According to the DM, and the law of the universal exchange (3.14), the energy of exchange (interaction) of particles is defined, at atomic and subatomic levels, by the formula

$$E = \omega_e^2 \frac{m_1 m_2}{8\pi\varepsilon_0 r}, \qquad (3.17)$$

where $\omega_e m_1 = q_1$ and $\omega_e m_2 = q_1$ are exchange charges of interacting particles. We will use juct this formula at the derivation of internodal binding energies of nucleons in atoms.

4. The binding energy of helium ^4_2He

The binding energy of nucleons in atoms is attributed to three causes and consists of:

(1) the binding energy of paired nucleons in nodes, *i.e.*, in essence, it is the energy of deuterons;

(2) the binding energy of nucleon nodes with atomic shells to which these nodes belong; and
(3) the energy of internodal exchange (interaction) of nucleons.

The derivation of the **first** constituent of the binding energy on the basis of the DM and SNM with use of (3.17), we will consider latter at the derivation of the binding energy of deuterium. Here, in order to take into account the binding energy of deuterium, we will use the value obtained from the mass defect formula:

$$\Delta E = c^2 \Delta m \tag{4.1}$$

A deuteron is the nucleus of a deuterium atom, and consists of one proton and one neutron. The mass of the constituents is

$$m_p + m_n = 1.007276 + 1.008665 = 2.015941 amu\,. \tag{4.2}$$

The atomic mass of the deuteron D (^{2}H) is 2.013553 *amu*; hence, the mass difference is $\Delta m = 0.002388 amu$. Thus, according to (4.1), a deuteron's binding energy is

$$E_D = c^2 \Delta m = 2.224 MeV\,. \tag{4.3}$$

The **second** constituent of the binding energy is defined from the following conditions. In a *spherical atomic field*, radial amplitudes of oscillations of *H*-units in nodes of the *n*-th atomic shell are determined by the expression

$$\hat{A}_s = A\hat{e}_l(kr)/kr \tag{4.4}$$

originated from solutions of (1.2) for the radial function $\hat{R}_l(kr)$ [1]. Then, the energy of oscillations takes the form:

$$E_s = \frac{m_p \omega^2 A_s^2}{2} = \frac{1}{2} h\nu = \frac{m_p \omega^2}{2}\left(\frac{A}{kr}\right)^2 e_l^2(kr) = \frac{m_p c^2 A^2}{2r^2} e_l^2(kr), \tag{4.5}$$

where m_p is the mass of *H*-unit. Obviously, that

$$h = 2\pi m_p \upsilon_s A_s = 2\pi m_p \omega \left(\frac{A}{kr}\right)^2 e_l^2(kr) = \frac{2\pi m_p c A^2}{kr} e_l^2(kr), \tag{4.6}$$

where

$$e_l(kr) = |\hat{e}_l(kr)| = \sqrt{\frac{\pi kr}{2}\left(J^2_{l+½}(kr) + Y^2_{l+½}(kr)\right)}, \qquad (4.7)$$

$J_{l+½}(kr)$ and $Y_{l+½}(kr)$ are the Bessel functions.

At $n = 0$, in the wave zone ($kr = 1$), we have

$$h = 2\pi m_p c A^2 / r_0 . \qquad (4.8)$$

From this, we define the constant A:

$$A = \sqrt{\frac{hr_0}{2\pi m_p c}} . \qquad (4.9)$$

In the wave zone, $r_0 = ƛ_e$, then assuming that the radial action for the mass m_p is $h = 2\pi m_p \upsilon ƛ_e$, we arrive at

$$A = \sqrt{\frac{hr_0}{2\pi m_p c}} = ƛ_e \sqrt{\frac{\upsilon}{c}} . \qquad (4.10)$$

If one assumes further the speed υ to be equal to the Bohr speed, the constant A takes the value of

$$A = 1.370113189 \cdot 10^{-9}\, cm . \qquad (4.11)$$

Accepted suppositions lead to the following energy of the H-unit in a node, at $m_p = m_u$ (atomic mass unit):

$$E_s = \frac{m_u \omega^2 A_s^2}{2} = \frac{m_u \omega^2 A^2}{2(kr)^2} e_l^2(kr) = \frac{\pi\left(J^2_{l+½}(kr) + Y^2_{l+½}(kr)\right)}{2kr} \frac{m_u \omega^2 A^2}{2} . \qquad (4.12)$$

At the level of the fundamental frequency ω_e, we have

$$w_u = \frac{m_u \omega_e^2 A^2}{2} = 3398.72\ keV \qquad (4.13)$$

and

$$E_s = \frac{m_u \omega_e^2 A_s^2}{2} = \frac{w_u}{z_{l,s}^2} e_l^2(z_{l,s}), \qquad (4.14)$$

where $z_{l,s} = kr$ is the root of Bessel functions [8] .

The binding energy (4.14) is only an estimation of the bond of an atomic shell with the *n*-node, because it was obtained on the basis of a series of suppositions, which should be regarded as preliminary axioms. A transition from one *n*-shell into another is defined by the energy of transition:

$$\Delta E_s = w_u \left(\frac{e_p^2(z_{p,m})}{z_{p,m}^2} - \frac{e_q^2(z_{q,n})}{z_{q,n}^2} \right). \qquad (4.15)$$

The root $z_{l,s} = y_{0,1} = 0.89357697$ defines the equilibrium distance

$$r_{He} = y_{0,1} \lambda\!\!\!^{-}_e = 1.433196073 \cdot 10^{-8}\, cm \qquad (4.16)$$

between two polar-azimuthal nodes on the external atomic shell of helium ${}_2^4He$ (see Fig. 2.2). Hence, according to (4.14), the binding energy of a nucleon node with the atomic shell in helium is

$$E_{shell} = \frac{w_u}{y_{0,1}^2} e_0^2(y_{0,1}) = 3.92109\, MeV\,. \qquad (4.17)$$

The **third** constituent of the binding energy of helium, the energy of internodal exchange, is determined by the formula (3.17). The *quantum* of internodal nucleon exchange, $q = m\omega_e$, under formation of internodal bonds at the nucleon ("nuclear") level, is the *nucleon's exchange charge*. It means that the *exchange charge* of two such quanta maximum, by one per every node (proton's or neutron's exchange charge), can take part in the internodal exchange. Then, at $r = r_{He}$ (4.16), $\omega_e = 1.869162505 \cdot 10^{18}\, s^{-1}$, and the proton's exchange charge $q_p = m_p \omega_e$, where

$$m_1 = m_2 = m_p = 1.67262171 \cdot 10^{-24}\, g \qquad (4.18)$$

is the associated mass of a proton, we have the energy of exchange (per two pairs of nucleons)

$$E_{exch} = \omega_e^2 \frac{m_p^2}{8\pi\varepsilon_0 r_{He}} = 16.91883553\,MeV\,. \qquad (4.19)$$

Hence, the exchange binding energy per nucleon is

$$E_{exch/n} = E_{exch}/4 = 4.229708883\,MeV\,, \qquad (4.19a)$$

and per single internodal nucleon bond (pair),

$$E_{exch/b} = E_{exch}/2 = 8.459417765\,MeV\,. \qquad (4.19b)$$

For estimation, we take into account the double bond between nodes in helium-4 realized by the elementary quantum of internodal nucleon exchange. It means that two pairs of nucleons participate in the internodal bond, so that the whole value (4.19) must be taken in this case.

As a result, the binding energy of helium atom ${}_2^4He$ obtained as the sum of three constituents: (4.3), (4.17), and (4.19), is defined by the expression

$$E_{He,atom} = 2E_D + 2E_{shell} + E_{exch} \qquad (4.20)$$

and equal to

$$E_{He,atom} = 2\cdot 2.224 + 2\cdot 3.92109 + 16.91883553 = 29.209\,MeV. \qquad (4.21)$$

In the shell-nodal atomic model there is not such a notion as a nucleus. The value (4.20) was obtained for the helium atom as a whole independently of an existence of two electrons in the helium atom. The contribution of two electrons in the binding energy is insignificant. The energies of electron bond with proton (ionization energies) and of interelectron exchange (interaction) are very small in comparison with the energy of internucleon exchange. Actually, according to the formula of exchange (3.17), we have

$$E_{e-exch} = \omega_e^2 \frac{m_e^2}{8\pi\varepsilon_0 r_{He}} \approx 5.24\,eV\,, \qquad (4.22)$$

where $\omega_e m_e = e = 1.702691627\cdot 10^{-9}\ g\cdot s^{-1}$ is the exchange charge of an electron. The energy obtained naturally defines the difference between the two energies of ionization of the helium atom:

$$E_{e-bond} = E_{ion}^{(2)} - E_{ion}^{(1)} = (54.42 - 49.18)\,eV = 5.24\,eV\,. \qquad (4.23)$$

Thus, finally, at the subtraction of energy of two electrons $2E_e = 2m_e c^2$ from (4.21), the binding energy of helium ion ${}_2^4He^{-2}$ ("nucleus") is

$$E_{He,ion} = E_{He,atom} - 2E_e = 29.209 - 2 \cdot 0.510998902 \approx 28.187\,MeV\,. \quad (4.24)$$

If we substitute the neutron's mass $m_n = 1.67492728 \cdot 10^{-24}\,g$ in place of m_p in (4.18), we will arrive at

$$E_{He,ion} \approx 28.23\,MeV\,. \qquad (4.25)$$

Resulting magnitudes, (4.24) and (4.25), almost coincides with the binding energy

$$\Delta E_{He} = c^2 \Delta m \approx 28.3\,MeV \qquad (4.26)$$

obtained for the helium nucleus on the basis of the formula on the mass defect.

5. The binding energy of carbon ${}_6^{12}C$

Basing on solutions of Eq. (1.2), we must take into account only those shortest internodal bonds in the carbon atom ${}_6^{12}C$ which distinguish by the shortest distances between wave shells of internodal nucleons. Angular directions of such bonds are in more or less extent conditioned by the space geometry of polar-azimuthal functions (see Fig. 2.1, 2.2, and 2.3). Only along these directions shown in Fig. 5.1a the chemical bonds between nucleon nodes of different atoms are realized at formation of molecules and crystals [4].

Five internodal bonds responsible for the binding energy in the carbon atom have the same length $r_1 \approx 2.7 r_0$ (where r_0 is the Bohr radius), defined by the root of Bessel functions $y_{0,1}$ (as in the case of the helium atom):

$$y_{0,1} = 0.89357697, \qquad r_1 = y_{0,1} \lambda\!\!\!^{-}_e = 1.433196073 \cdot 10^{-8}\,cm$$

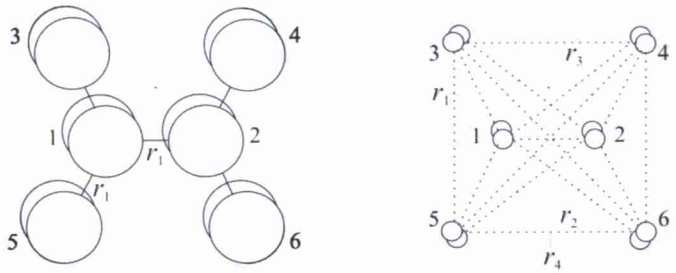


Fig. 5.1. The geometry of internodal nucleon ("nuclear") bonds in the carbon ${}^{12}_{6}C$ atom (a) and characteristic internodal distances (b) (between their centers) defined by the roots of Bessel functions.

All other characteristic internodal distances in the carbon atom, shown in Fig. 5.1b, are not arbitrary as well. They are defined, as r_1, by the roots of Bessel functions. This is justified by regularities of wave processes, described by the Bessel functions, that influences the strictly definite structure of the material spaces at all level:

$$y_{1/2,1} = 1.57079633 \qquad r_2 = y_{1/2,1}\lambda\!\!\!^{-}_e = 2.519379088 \cdot 10^{-8}\, cm$$

$$j_{0,1} = 2.40482556 \qquad r_3 = j_{0,1}\lambda\!\!\!^{-}_e = 3.857067342 \cdot 10^{-8}\, cm \quad (5.1)$$

$$j'_{1,1} = 1.84118378 \qquad r_4 = j'_{1,1}\lambda\!\!\!^{-}_e = 2.953049879 \cdot 10^{-8}\, cm$$

As in the case of the helium atom, three constituents of the binding energy must be taken into account. The first one, considering the coupling of two nucleons in a node in the form of deuteron, gives us the deuteron's binding energy E_D of 2.224 *MeV* (4.3) per node.

The second constituent of the binding energy takes into account the bond of a node with the atomic shell where this node is located. According to (4.14) and (4.15), for the 1st and 2nd nodes (Fig. 5.1) situated at the internal atomic shell (the shell of helium), we have $z_{l,m} = y_{0,1}$ and $E_{\text{int},shell} = 3.92109\, MeV$ (4.17).

Transitions of nucleons from the internal shell to the external shell, where four nodes are located, are defined by the formula of energy of transitions (4.15). For $z_{p,m} = y_{0,1}$ and $z_{q,n} = y_{1/2,1}$, we have $E_{trans} = 2.54363\,MeV$. The binding energy for every of four nodes of external shell is

$$E_{ext,shell} = \frac{w_u}{y_{1/2,1}^2} e_{1/2}^2(y_{1/2,1}) = 1.37745\,MeV. \qquad (5.2)$$

The third constituent of the binding energy of the carbon atom ${}^{12}_{6}C$, the energy of internodal exchange, is determined by the formula (3.17). According to the latter, an elementary binding energy, caused by exchange interaction between two nodes a distance r_1 apart, is

$$E_{exch} = \omega_e^2 \frac{m_p^2}{8\pi\varepsilon_0 r_1} = 16.91883553\,MeV \qquad (5.3)$$

(as in the case of the helium atom (4.19)).

The exchange energy (5.3) (of the quantum of nucleon exchange $q_p = m_p\omega_e$) of the 1st node (Fig. 5.1a) expends on three equal bonds with 2nd, 3rd, and 5th nodes; and the 2nd node, with 1st, 4th, and 6th nodes. Hence the binding energy per node (we mean 1st and 2nd nodes here) is

$$E_{exch,1} = (1/3)E_{exch} = 5.639611843\,MeV/node. \qquad (5.4)$$

Every node of the 3rd, 4th, 5th, and 6th nodes are connected only with one node (1st or 2nd). Hence, the binding energy per node (for nodes from 3rd to 6th) is

$$E_{exch,2} = (1/2)E_{exch} = 8.459417765\,MeV/node. \qquad (5.5)$$

Thus, we have the following internodal binding energies between the nodes of the numbers

(1-2):

$$E_{exch,1-2} = 2E_{exch,1} = 11.27922369\,MeV/bond; \qquad (5.6)$$

(3-1), (5-1), (4-2), (6-2):

$$E_{exch,3-1} = E_{exch,1} + E_{exch,2} = 14.09902961\,MeV/bond \qquad (5.7)$$

Thus, the total energy of internodal exchanges is

$$E_{\text{int},exch} = E_{exch,1-2} + 4E_{exch,3-1} = 67.67534212\,MeV\,. \qquad (5.8)$$

A resulting sum of all constituents of binding energy of the carbon atom ${}^{12}_{6}C$, calculated for q_p: (4.3), (4.17), (5.2), and (5.8), is

$$E_{C,atom} = 6E_D + 2E_{\text{int},shell} + 4E_{ext,shell} + E_{\text{int},exch} = 94.37132212\,MeV\,. \quad (5.9)$$

(Calculations for the exchange charge of a neutron $q_n = m_n\omega_e$ give 94.48781375 *MeV*).

At subtraction of the energy of four valent electrons, $4E_e = 2.022\,MeV$, from (5.9), we arrive at the energy of the carbon ion ${}^{12}_{6}C^{-4}$,

$$E_{C,ion} = E_{C,atom} - 4E_e = 92.34932212\,MeV\,. \qquad (5.10)$$

Thus, the binding energy of the carbon ion ${}^{12}_{6}C^{-4}$, obtained here on the basis of shell-nodal atomic model and the DM, is in well agreement with the binding energy of the carbon nucleus ${}^{12}_{6}C$ equal to $92.488\,MeV$, calculated from the formula on the mass difference, $\Delta E = c^2\Delta m$.

For the derivation described above, the used value of the first constituent of the binding energy of helium and carbon atoms, 2.224 *MeV*, originates from the well-known formula $E_D = c^2\Delta m$ (4.3). It is the binding energy of deuteron D (${}^{2}_{1}H$). We have the right to take this value assuming that according to shell-nodal atomic model the coupled protons and neutrons in nodes are in the form of deuteron.

We will show further that the binding energy of deuteron E_D is also derived on the new basis accepted in this work, just like the derivation of the binding energy of helium and carbon atoms. This aim is achieved on the basis of the supposition that solutions of the wave Eq. (1.2), resulted in the shell-nodal structure of atomic and interatomic (crystal or molecular) spaces (Fig. 1.1), are also valid for the subatomic (intra-nucleon) space. It means that basic constituents of atoms, protons and neutrons, have the same shell-nodal internal structure depicted graphycally in Fig. 1.1. In this connection, we will explain first of all our point of view on the nucleon structure and answer to the

question: what particles of the subatomic level are the main "building bricks" for nucleons?

Let us proceed to elucidate now this question as it is solved in the framework of the Dynamic Model of Elementary Particles.

6. The spectrum of associated masses of elementary particles

The spectrum of associated masses follows from the DM [1, 7]. In dependence on the character of exchange, we distinguish the masses in the longitudinal exchange (at motion-rest in the cylindrical field of matter-space-time), the masses in the transversal exchange (transversal oscillations of the wave beam), and the masses in the tangential exchange (at motion-rest in the cylindrical space-field).

We show here only the derivation of the spectrum of masses (taken from the author's work [1]) playing the role in the longitudinal exchange, because the latter leads to masses of the particles constituent of atoms as, for example, π-mesons, μ-mesons, γ-quanta, *etc*. This will help understanding the concept set forth first in [1] and developed here for the internal spatial structure of nucleons.

Motion-rest in the cylindrical field of matter-space-time can be presented, at a part of the axial line of length dz (Fig. 6.1), (in the simplest case) by the equation of exchange:

$$\rho_z dz \frac{\partial^2 \Psi}{\partial t^2} = -\frac{\partial F}{\partial z} dz , \qquad (6.1)$$

where ρ_z is the linear density of mass, Ψ is the axial displacement, and F is the power of exchange.

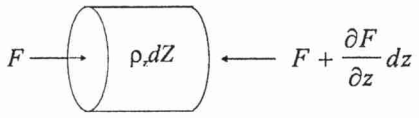


Fig. 6.1. A graph of power of the elementary longitudinal exchange.

Let w be the density of energy of basis and p be the density of energy of superstructure. In a linear approximation, the relative change of energy is $\frac{pS\partial z}{wS\partial z}$, where $wS\partial z$ is the energy of an elementary differential volume $S\partial z$, and $pS\partial z$ is its change.

Assuming that the relative change of energy of exchange is equal to the relative linear change of the elementary volume of space-field, $\frac{pS\partial z}{wS\partial z} = \frac{F\partial z}{wS\partial z} = -\frac{\partial \Psi}{\partial z}$, we obtain $F = -wS\frac{\partial \Psi}{\partial z}$. As a result, the equation of motion-rest takes the form

$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{wS}{\rho_z}\frac{\partial^2 \Psi}{\partial z^2} \qquad \text{or} \qquad \frac{\partial^2 \Psi}{\partial z^2} = \frac{\rho_z}{wS}\frac{\partial^2 \Psi}{\partial t^2}.$$

An element of a beam is $\partial z = c\partial t$; hence,

$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{wS}{\rho_z c^2}\frac{\partial^2 \Psi}{\partial t^2}$$

and

$$c = \sqrt{wS/\rho_z}.$$

If we consider the exchange with the density of energy E at the level of Young modulus, then

$$c = \sqrt{ES/\rho_z}. \qquad (6.2)$$

and

$$\omega = kc = k\sqrt{ES/\rho_z}, \qquad (6.2a)$$

where k is the wave number, which takes a series of discrete values.
Let us determine the characteristic spectrum of frequencies. For the hard-facing alloys, the Young modulus is approximately within $600-680\ GPa$. We select, in the capacity of a calculated magnitude, the characteristic value 654.9, which satisfies the metrological spectrum on the basis of the fundamental measure [22]:

$$E = 6.549 \cdot 10^{11}\ Pa. \qquad (6.3)$$

Let the remaining parameters be equal to

$$l = 2\pi r_0, \qquad \rho_l = m_e / l, \qquad S = \pi r_0^2. \tag{6.3a}$$

Under these conditions, a formula of the characteristic spectrum of frequencies takes the form

$$\omega = 4\omega_0 \cdot r_0 k, \tag{6.4}$$

where

$$\omega_0 = \frac{\pi}{2}\sqrt{\frac{Er_0}{2m_e}} = 6.85091084 \cdot 10^{15}\ s^{-1} \approx \frac{\omega_e}{272.88}. \tag{6.5}$$

The frequency ω_0 is bound up with the fundamental frequency ω_e by the following characteristic ratio:

$$\omega_e / \omega_0 = 272.8103045 \approx 272.8752708. \tag{6.5a}$$

The frequency of the fundamental tone ω_0 is the characteristic frequency of *H*-atomic level. If $l = n\lambda$, then $r_0 k = n$ and

$$\omega_n = 4\omega_0 \cdot n \approx \Delta \cdot 10^{16} n\ s^{-1}, \tag{6.6}$$

where $\Delta = 2\pi \lg e$ is the fundamental quantum-period [22, 23].
The spectrum of frequencies (6.6) defines the spectrum of associated masses of elementary particles with the elementary charge e:

$$M_n = \frac{e}{\omega_n} = \frac{e}{4\omega_0} \cdot \frac{1}{n} = \frac{68.5\, m_e}{n}. \tag{6.7}$$

If $l = n(\lambda/2)$, then $r_0 k = (1/2)n$ and

$$\omega_n = 2\omega_0 \cdot n, \quad M_n = \frac{e}{\omega_n} = \frac{e}{2\omega_0} \cdot \frac{1}{n} = \frac{137\, m_e}{n}. \tag{6.8}$$

At last, at $l = n(\lambda/4)$, it follows $r_0 k = (1/4)n$ and

$$\omega_n = \omega_0 \cdot n, \quad M_n = \frac{e}{\omega_n} = \frac{e}{\omega_0} \cdot \frac{1}{n} = \frac{274\, m_e}{n}. \tag{6.9}$$

Because at $n=1$, a particle of the mass $M_1 = 274 m_e$ is the π-meson, we will call the frequency ω_0 the meson frequency.

At $n=1, 2, 3, 4,$ we have

$$\begin{aligned} 274\, m_e &\Rightarrow \pi\text{-meson} \\ 137\, m_e &\Rightarrow \gamma\text{-quantum} \\ 91.3\, m_e &\Rightarrow \rho\text{-lepton} \\ 68.5\, m_e &\Rightarrow g\text{-lepton} \end{aligned} \qquad (6.10)$$

Two g-leptons form a γ-quantum, three g-leptons compose a μ-meson:

$$205.5\, m_e \Rightarrow \mu\text{-meson.} \qquad (6.11)$$

Particles are able to the mutual transformation. In particular, π-meson, as four g-leptons, can decay following the schemes:

$$\begin{aligned} \pi &\Rightarrow \gamma + \gamma \\ \pi &\Rightarrow \mu + g \\ \pi &\Rightarrow g + g + \gamma. \end{aligned} \qquad (6.12)$$

Evidently, in this series, the first decay is the most probable. The μ-meson and γ-quantum decay in a similar way:

$$\begin{aligned} \mu &\Rightarrow \gamma + g \\ \gamma &\Rightarrow g + g. \end{aligned} \qquad (6.13)$$

The g-lepton had no luck. Having a relatively big mass, it, nevertheless, was eliminated from a series of elementary particles because of the requirements of relativity theory. But afterwards, it was returned to this series under different names: muonic neutrino, electronic antineutrino, *etc.* Such names of g-lepton were stipulated by a concrete interpretation of the reaction. Following [1], we have to recognize that g-lepton exists in the four states:

$$+g, \quad -g, \quad +ig, \quad -ig. \qquad (6.14)$$

The mass of g-lepton is close to a quarter of the fundamental period Δ (in units of the electron mass) [22, 23]:

$$m_g = \frac{1}{4}(2\pi \lg e)\cdot 10^2 m_e \qquad (6.15)$$

that expresses the definite facets of the Eternity.

On the basis of the g-lepton, we compile a sequential series of particles (n-multiple to g):

$$
\begin{array}{lcl}
2g = \gamma & \Rightarrow & g + g \\
3g = \mu & \Rightarrow & \gamma + g \\
4g = \pi & \Rightarrow & \mu + g \quad \text{or} \quad \gamma + \gamma \\
5g = \mathrm{K}_5 & \Rightarrow & \pi + g \quad \text{or} \quad \mu + \gamma \\
6g = \mathrm{K}_6 & \Rightarrow & \pi + \gamma \\
7g = \mathrm{K}_7 & \Rightarrow & \pi + \mu \\
8g = \mathrm{K}_8 & \Rightarrow & \pi + \pi \\
9g = \mathrm{K}_9 & \Rightarrow & \mathrm{K}_8 + g \\
10g = \mathrm{K}_{10} & \Rightarrow & 2\pi + \gamma \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \\
32g = \Lambda & \Rightarrow & n + \mathrm{K}_5 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & &
\end{array}
\qquad (6.16)
$$

Here, K_n is the symbol of the particle, expressed in g-leptons.

Experiments detect all these structures of the wave field (under either name or without it) in the form of energetic dispersion in a process of transformations. In 1931, Dirac showed [24] that a field theory could be constructed on the basis of a magnetic monopole g with the following elementary charge

$$q_g = \varepsilon_0 hc / e = 68.5e\,. \qquad (6.17)$$

The division of the charge q_g by the fundamental frequency ω_e gives its mass:

$$m_g = 68.5 m_e\,. \qquad (6.18)$$

Evidently, g-lepton and the Dirac monopole g are the same particle. At that time, the mass of the monopole was determined incorrectly, therefore, g-lepton was not rendered due attention. Knowing the associated mass of Dirac monopole, we obtain the radius of its sphere

$$r_g = \left(m_g / 4\pi\varepsilon_0\right)^{1/3} = 1.706 \cdot 10^{-9}\ cm \approx 4r_e\,. \qquad (6.19)$$

where r_e is the radius of the electron sphere.
We see that r_g is very close to the rational golden section of the fundamental metrological period Δ:

$$r_g \approx \frac{5}{8} 2\pi \lg e \cdot 10^{-9}\, cm. \qquad (6.20)$$

The result obtained gives us the reason to assume that *g*-lepton is a highly stable particle, which possibly is a constituent (like a nucleon for atoms) of protons, neutrons, and other elementary particles of this series. If only this is true, then on the basis of the monopole and the periodic law of space [1, 5], it is possible to compose the spectrum of elementary particles. In such a spectrum, *g*-lepton is a hydrogen analog, γ-quantum is a deuterium analog, μ-meson is a tritium analog, π-meson is a helium analog, *etc.*

7. The g-lepton structure of proton and neutron

Accepting the supposition that nucleons (protons and neutrons) consist of g-leptons, we must recognize that nucleons represent by themselves, by analogy with atoms of the nucleon level, the silicon of the g-lepton level of the atomic number 14 (having 14 nodes according to solutions of (1.2) presented in Fig. 1.1). Indeed, let the mass of g-lepton will be precisely multiple to a quarter of the fundamental period, with respect to the electron mass m_e,

$$m_g = (1/4)2\pi \lg e \cdot 10^2\, m_e = 68.21881769\, m_e. \qquad (7.1)$$

The masses of proton and neutron are, correspondingly,

$$m_p = 1836.1526675\, m_e \qquad \text{and} \qquad m_n = 1838.683645\, m_e. \qquad (7.2)$$

Hence, it is clear that the mass number of nucleons at the g-lepton level must be rather more than 27, because of the relation

$$m_n / m_g = 26.95273397 \qquad (7.3)$$

with taking into account an essential value of the binding energy of g-leptons influenced on the resulting mass of nucleons.

On this basis, we assume that protons and neutrons represent, respectively, at the g-lepton level, two stable isotopes analogous, in nodal structure, to the silicon isotopes, ${}^{28}_{14}Si$ and ${}^{29}_{14}Si$. Their nodal structure, in full agreement with solutions of the wave equation (1.2) (Fig. 1.1), is presented in Fig. 7.1. The

polar-azimuthal functions (according to Eq. (1.3)) of $_{14}Si$ and space disposition of its spherical shells and potential nodes are shown in Fig. 7.2.

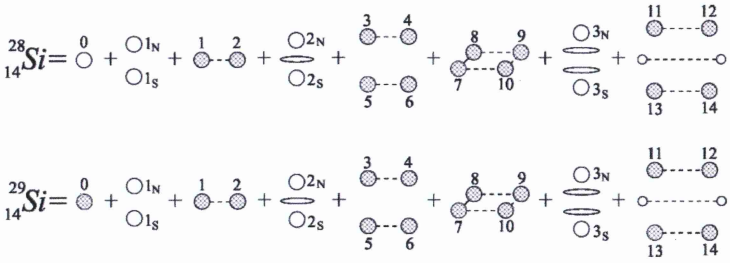


Fig. 7.1. A symbolic design of the shell-nodal structure of silicon, $^{28}_{14}Si$ and $^{29}_{14}Si$, in accordance with the shell-nodal structure of atoms shown in Fig. 1.2.

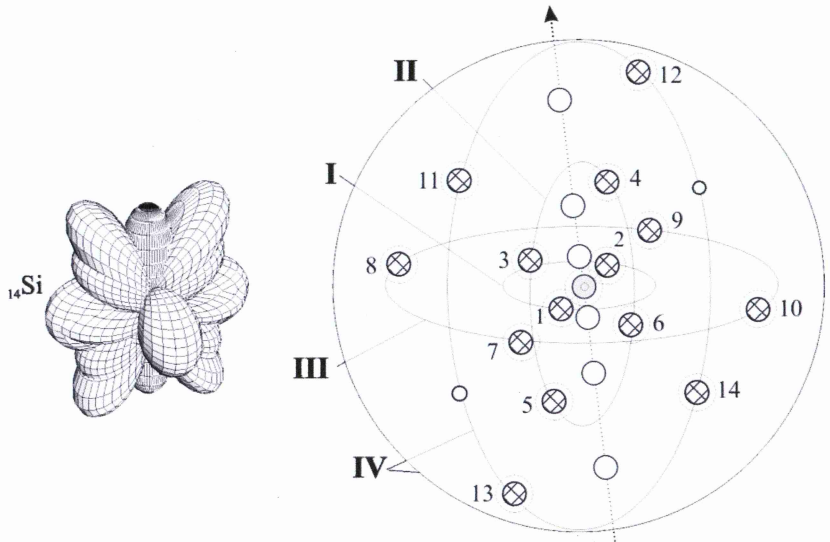


Fig. 7.2. The structure of polar-azimuthal functions (1.3) (at the left) and space arrangement of the 14 polar-azimuthal nodes on four (**I – IV**) spherical shells in $_{14}Si$.

Thus, we regard a neutron of the $^{29}_{14}Si$ structure, in the above meaning, as one of the isotopes of the simplest hydrogen atom $^{1}_{1}H$, protium. The neutron contains additionally one g-lepton in the central polar node (see Fig. 7.1) in

comparison with a proton (of the ${}^{28}_{14}Si$ structure). The neutron is a stable isotope in a bond state, like ${}^{29}_{14}Si$, and decays during $\tau = 1000\,s$ to a proton (the ${}^{28}_{14}Si$ analogous), an electron, and a neutral g-lepton (an antineutrino $\tilde{\nu}$ of nuclear physics):

$$n \rightarrow p + e^{-} + g\,. \qquad (7.4)$$

The shell-nodal structure of ${}^{28}_{14}Si$ (and ${}^{29}_{14}Si$) (Fig. 7.1 and 7.2) is more complicated than the shell-nodal structure of ${}^{12}_{6}C$ (Fig. 2.3), because it has two shells and eight nodes more (at $l = 2, m = \pm 2$ and $l = 3, m = \pm 1$). An internal shell (I) with two polar-azimuthal nodes (1, 2) is the shell of the helium atom (see Fig. 2.2). The second internal shell (II) is the external shell of the carbon atom. The third shell (III) is the external shell of the neon atom. The shell IV is the external shell characteristic for the silicon atom.

According to SNM, the unrepeatable (specific) structure of external shells mainly defines individual properties of atoms distinguishing them from each other. The external shell of ${}_{14}Si$ has two collateral nodes not completed by nucleons in the isotopes of silicon under consideration. Silicon is the first element of the periodic table with such nodes (unnumbered in Fig. 1.1 and other figures), which are metastable states judging from the fact that amplitudes of polar-azimuthal functions determining their positions on shells are essentially smaller that corresponding amplitudes of principal (numbered in presented figures) nodes. This feature provides the motion, in its internal space, not only of particles, which are much less than nucleons, but also the motion of nodal nucleons themselves. Quantum theory interprets this phenomenon as the motion of "holes".

A neutron has the magnetic moment of the value

$$\mu_n = -0.96623640(23) \cdot 10^{-26}\, J \cdot T^{-1} \qquad (7.5)$$

(according to the CODATA [25]). This magnitude is approximately in 1.46 times less in absolute value than the (positive) magnetic moment of a proton. As a system of a proton and an orbiting electron (*i.e.*, being an isotope of protium from the point of view of shell-nodal atomic model), the structure of the neutron with the surrounding field looks conditionally as is depicted graphically in Fig. 7.3.

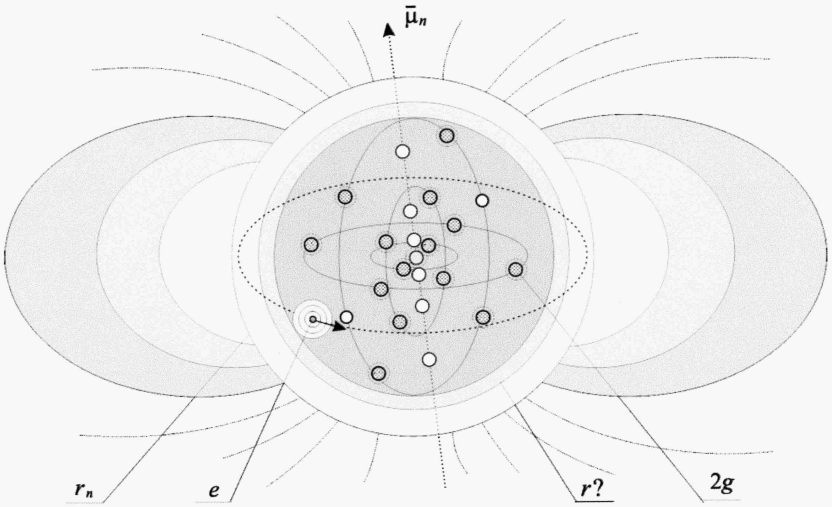


Fig. 7.3. A neutron as an analogous of the silicon atom ${}^{29}_{14}Si$ with the surrounding field; r_n is the neutron outer shell; e is an orbiting electron; $r?$ is the inner radius of the neutron shell; $2g$ is the condition designation of 14 principal polar-azimuthal *potential* nodes completed with coupled g-leptons (the 29th g-lepton is in the central polar *potential-kinetic* node, on the z-axis); $\bar{\mu}_n$ is the magnetic moment of the neutron.

According to the formula of mass defect (4.1), the binding energy of a proton, consisted of 28 g-leptons, is to be

$$\Delta E(p) = c2\Delta m = c^2(28m_g - m_p) = 62.79769638\,MeV\,, \qquad (7.6)$$

so that the binding energy per g-lepton is

$$\varepsilon(p) = \Delta E(p)/A(p) = 2.242774871\,MeV\,, \qquad (7.7)$$

where $A(p) = 28$ is the mass number of a proton at the g-lepton level.

The corresponding values for a neutron ($A(n) = 29$) are

$$\Delta E(n) = c^2(29m_g - m_n) = 71.36715712\,MeV\,, \qquad (7.8)$$

$$\varepsilon(n) = \Delta E(n)/A(n) = 2.460936452\,MeV\,. \qquad (7.9)$$

Now, resting on the shell-nodal g-lepton structure of nucleons, we can proceed to derive the binding energy of deuterium and tritium regarding them as the junction, respectively, of two and three g-lepton systems (pairs)

8. The binding energy of deuterium ${}^{2}_{1}H$ and tritium ${}^{3}_{1}H$

At the joining of two H-atoms, of a neutron and the hydrogen atom ${}^{1}_{1}H$ (protium), the deuterium atom ${}^{2}_{1}H$ is formed. The process of joining results in the penetration of spaces of one nucleon into another, so that the partial overlapping of spherical shells of both nucleons occurs. With this, all g-lepton nodes (Fig. 7.3), filled with coupled g-leptons, of one nucleon and corresponding nodes of another nucleon draw together at the distance r defined by solutions of the wave equation (1.2) (*i.e.*, by the roots of Bessel functions). As a result, 28 helium structures on the basis of binding of approached pairs of coupled g-leptons, like that one shown in Fig. 8.1, are formed.

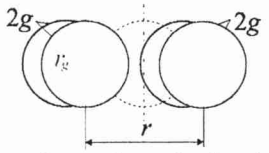


Fig. 8.1. The helium structure formed on the basis of binding of two g-lepton nodes.

The distance r between the nodes is defined by roots of Bessel functions

$$z_{m,n} = kr. \qquad (8.1)$$

The unknown value in this expression is the wave number k equal to the inverse value of the wave radius $\lambda\!\!\!^{-}$,

$$k = 1/\lambda\!\!\!^{-}. \qquad (8.2)$$

The wave radius $\lambda\!\!\!^{-}$ defines the characteristic radii of elementary spherical and cylindrical surfaces described by Bessel functions (8.1) with zero and extremal values. As we saw above, at the *nucleon level*, $k = 1/\lambda\!\!\!^{-}_{e}$, where $\lambda\!\!\!^{-}_{e} = 1.603886538 \cdot 10^{-8}\ cm$ originates from the DM (3.12). The wave radius of

the value $\lambda\!\!\!^{-}_e$ is responsible for the arrangement of *nucleons* in atoms, and hence in crystals, molecules, *etc.* at the definite absolute distances. Accordingly, it is not a random coincidence that the wave diameter of

$$2\lambda\!\!\!^{-}_e \approx 3.2 \cdot 10^{-8} cm \qquad (8.3)$$

is equal, in average, to lattice parameters of crystals. Thus, the wave radius at the nucleon level $\lambda\!\!\!^{-}_n$ is equal to (coincides with) the fundamental wave *radius of exchange*

$$\lambda\!\!\!^{-}_n = \lambda\!\!\!^{-}_e = c/\omega_e, \qquad (8.4)$$

it defines the principal parameters of atomic spaces. Note that

$$\omega_e = e/m_e \qquad (8.5)$$

is the fundamental *frequency of exchange* at the atomic and subatomic levels (the frequency of the "electrostatic field").

Obviously, spaces of the g-lepton level (we mean the internal spaces of nucleons) have another absolute value of the wave radius. The spherical space of a nucleon, like the spherical space of an atom, is also a system of wave shells, but its own whose relative size is defined by the *relative* radius $\rho = kr = z_{m,n}$. The internal proper shells of nucleons with its own nodes, where g-particles are localized, form the superfine discrete structure of atoms.

Thus, we stressed again that solutions of the wave equation (1.2) give only the *relative* radius ρ, and hence, the *relative* value of the corresponding wave radius $\lambda\!\!\!^{-}$. The a*bsolute* value of the latter one must seek from some conditions general for wave processes at different levels.

We will define $\lambda\!\!\!^{-}$ from the scale analogy which exists between wave processes at any levels and, in particular, which must exist between ones at the nucleon and g-lepton levels. The matter is that the fundamental relations existed between the main wave parameters in both scales must keep. One of the fundamental relations exists between the radius of the wave spherical shell of a proton r_p and the fundamental wave radius $\lambda\!\!\!^{-}_e$ of exchange of the proton with other particles and the surrounding field. The theoretical radius of the wave shell of the proton (proton's radius for short), obtained from the formula (3.2) at the condition $(k_e r_p)^2 << 1$ and $\varepsilon_r = 1$, is

$$r_p(th) = (m_p / 4\pi\varepsilon_0)^{1/3} = 0.510578616 \cdot 10^{-8}\, cm. \qquad (8.6)$$

The fundamental wave radius is

$$\lambda\!\!\!^{-}_e = c / \omega_e = 1.603886538 \cdot 10^{-8}\, cm. \qquad (8.7)$$

The ratio of both magnitudes is equal, with some accuracy, to the fundamental constant π,

$$\lambda\!\!\!^{-}_e / r_p(th) = 3.141311617 \approx \pi. \qquad (8.8)$$

This ratio shows that the wave radius $\lambda\!\!\!^{-}_e$, in value, is a half of the length of the equatorial circumference of the wave spherical shell of a proton. Obviously, the same ratio must be valid for the radius of the wave spherical shell of g-lepton, r_g, and the wave radius of the g-lepton level, $\lambda\!\!\!^{-}_g$, so that we have the right to assume that

$$\lambda\!\!\!^{-}_g / r_g(th) = \pi. \qquad (8.9)$$

Hence, for

$$r_g(th) = (m_g / 4\pi\varepsilon_0)^{1/3} = 0.170370509 \cdot 10^{-8}\, cm \qquad (8.10)$$

with $m_g = 6.214420763 \cdot 10^{-26}\, g$ (see (7.1) and (6.15)), the wave radius of the g-lepton level $\lambda\!\!\!^{-}_g$ is

$$\lambda\!\!\!^{-}_g = \pi r_g(th) = 0.534 \cdot 10^{-8}\, cm. \qquad (8.11)$$

We see that the value of $\lambda\!\!\!^{-}_g$ obtained on the basis of a series of approximations is close to the Bohr radius $r_0 = 0.529 \cdot 10^{-8}\, cm$. It is quite possible that more accurate derivations will lead to the equality $\lambda\!\!\!^{-}_g = r_0$. Thus, we cannot exclude the equality of the above parameters: of the Bohr radius and the wave radius of g-lepton level $\lambda\!\!\!^{-}_g$, which both are the basic parameters of the wave sphere atomic space.

Hence, taking the root of Bessel functions, $z_{m,n} = y_{0,1} = 0.89357697$, as in the case of the helium atom, we arrive at the following distance r between two pairs of g-leptons (see Fig. 8.1) in coupling nucleons:

$$r = y_{0,1}\lambda\!\!\!^{-}_{g} = 0.477 \cdot 10^{-8}\, cm\,. \qquad (8.12)$$

It means that wave spherical shells of two H-atoms in the deuterium ${}^{2}_{1}H$ are partially overlapped as is shown in Fig. 8.2 (where $r_p = 0.51 \cdot 10^{-8}\, cm$ (8.6)). Centers of masses of two constituent H-atoms are at the distance $r = 0.477 \cdot 10^{-8}\, cm$, which is some less than the Bohr radius, $r < r_0$.

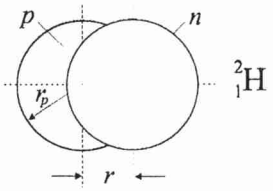


Fig. 8.2. The relative disposition of two nucleons in the deuterium atom ${}^{2}_{1}H$; $r < r_0$.

At such conditions (distance r of the value (8.12)), the binding energy of internodal g-leptons pairs (Fig. 8.1) is

$$E_g = \frac{q_g^2}{8\pi\varepsilon_0 r} = 0.070246848 \cdot 10^6\, eV\,, \qquad (8.13)$$

where

$$q_g = m_g \omega_e = 1.161576228 \cdot 10^{-7}\, g \cdot s^{-1}$$

is the exchange charge of the g-lepton, which is an elementary quantum of exchange at the g-lepton level.

According to the above definition (model), a proton has 28 g-leptons (14 nodes filled every with 2 coupled g-leptons). A neutron, in comparison with the proton, has one more g-lepton located in the central polar node (see Fig. 7.1 and 7.2). Thus, because of all g-leptons take part in the exchange (interaction), we have 28.5 pairs of interacting g-leptons in ${}^{2}_{1}H$. Hence, the resulting binding energy, related to the internodal exchange (interaction) of all g-leptons belonging to two interacting nucleons, is

$$E_{g,exch} = 28.5 \cdot E_g = 2.002\, MeV\,. \qquad (8.14)$$

The obtained value is close to the known value of 2.224 *MeV* (4.3) for the binding energy of a deuteron. It is the main (1st) but not alone constituent of the total binding energy of ${}^{2}_{1}H$ (as in the case of helium and carbon atoms considered above). By accepted analogy between wave processes at two levels under consideration (nucleon and g-lepton), we must take also into account (2nd) the energy of coupling of two g-leptons in their nodes and (3rd) the binding energy of g-lepton nodes with the shells where these nodes are located.

However, we will not derive the rest (2nd and 3rd) constituents here. The derivation of the third one was carried out for helium and carbon atoms. A rough estimate of these constituents of the binding energy on the basis of the analogy between two-nodal structure of helium and two-nodal g-lepton helium structure (Fig. 8.1), will be quite sufficient here.

In this connection, let us assume that the ratio existed between the total binding energy of helium ${}^{4}_{2}He$, 28.3 *MeV*, and its second constituent, the binding energy of coupled nucleons in its nodes (*i.e.*, the binding energy of deuterium), 2.224 *MeV*, keeps the same and for the corresponding g-lepton helium structure shown in Fig. 8.1. In this case, because the total binding energy of all 28 g-lepton helium structures must be equal to 2.224 *MeV* (according to (4.3)), the binding energy of all g-lepton "deuterons" in all g-lepton nodes must be

$$E_{g(2)} = 0.175\,MeV\,. \qquad (8.15)$$

And the binding energy of one g-lepton "deuteron" is

$$\varepsilon_{2g} = 6.25\,keV\,. \qquad (8.15a)$$

Hence, finally, we arrive at the following binding energy of ${}^{2}_{1}H$:

$$E({}^{2}_{1}H) = E_{g,exch} + E_{g(2)} + E_{g(3)} = 2.177\,MeV + E_{g(3)}\,. \qquad (8.16)$$

Obviously, the contribution of the third constituent $E_{g(3)}$, corresponding to the binding energy of all 28 g-nodes with their wave spherical shells will be less than the contribution of the second constituent estimated above. Therefore, we assume that after adding of $E_{g(3)}$ to the total energy we will closer approach to the value 2.224 *MeV*, which follows from the formula on mass defect (4.3).

In addition, let us proceed now to the derivation of the binding energy of tritium. The shell-nodal structure of three joined g-lepton nodes in tritium (belonging to three interacting nucleons), on the g-lepton level, recalls the nodal structure of helium isotope ${}^{6}_{2}He$ (Fig. 8.3). Appearance of two coupled g-leptons in the central polar node slightly changes (increases) the former equilibrium distance r existed between outmost pairs of g-leptons in the g-lepton helium structure shown in Fig. 8.1.

The nearest to the $r = 0.477 \cdot 10^{-8}\, cm$ equilibrium distance between g-lepton nodes, admitted by solutions of the wave equation (1.2), is the distance equal to the wave radius of the g-lepton level, $\lambda\!\!\!^{-}_{g} = 0.534 \cdot 10^{-8}\, cm$. Therefore, we accept this value of the distance between the outermost g-lepton nodes in tritium (Fig. 8.3) for further calculations, so that we have

$$r = \lambda\!\!\!^{-}_{g} = 0.534 \cdot 10^{-8}\, cm,$$
$$r_1 = r_2 = \lambda\!\!\!^{-}_{g} / 2 = 0.267 \cdot 10^{-8}\, cm. \qquad (8.17)$$

We also assume that the exchange interaction in the presented structure exists between every two partially overlapped pairs as is shown conditionally by two arrows in Fig. 8.3.

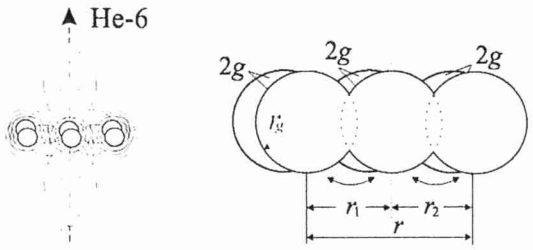


Fig. 8.3. The nodal structure of helium isotope ${}^{6}_{2}He$, and the local structure formed under the joining of three g-lepton nodes in tritium regarded as the *p-n-n* system.

The main constituent of the binding energy in this case, the energy of internodal exchange between two nearest nodes, is

$$E_g = \frac{q_g^2}{8\pi\varepsilon_0 r_1} = 0.140493696\,MeV\,. \qquad (8.19)$$

Hence, the total binding energy of internodal exchange, with allowance for all g-lepton bonds in tritium, is

$$E_{g,exch} = N_{g,bonds} \cdot E_g = 8.07838752\,MeV\,, \qquad (8.20)$$

where $N_{g,bonds} = 57.5$ is the number of internodal g-lepton bonds (*p-n* and *n-n*, 28.5+29) in tritium consisted of two neutron and one proton.

The second constituent, the total binding energy of all g-lepton "deuterons" in tritium, is (according to (8.15))

$$E_{g(2)} = N_{g-nodes} \cdot E_{2g} = 0.2625\,MeV\,, \qquad (8.21)$$

where $N_{g-nodes} = 3 \cdot 14 = 42$ is the number of completed polar-azimuth g-lepton nodes (or the number of coupled g-leptons).

Without the smallest in value contribution of the third constituent $E_{g(3)}$ (related to the binding energy of g-lepton nodes with the shells of their localization), we obtain finally the following magnitude

$$E({}^3_1H) = E_{g,exch} + E_{g(2)} = 8.34088752\,MeV\,. \qquad (8.22)$$

For comparison, the binding energy of tritium, originated from the formula (4.1), is

$$\Delta E = c^2 \Delta m = 8.481821\,MeV\,. \qquad (8.23)$$

Thus, we have an approximate coincidence in the resulting data obtained by two ways different of principle.

9. Conclusion

The obtained results justify in favor of the validity of the new foundations used in this work for the derivation of the binding energies: the shell-nodal atomic model and the dynamic model of elementary particles. Considering hierarchy of particles beginning from an electron up to a nucleon, we can add the following [1].

It is possible to suppose that at the g-level the electron is a very miniature nucleon. Then a g-lepton, judging from its reference mass m_g=68.22m_e, represents a composite atom-molecule of the electron level with the ordinal number $z \approx 32$ (if to rely on the wave equation in the g-lepton space). Indeed, an atom of the periodic table at the nucleon level with the mass number more than 68 ("more", because we must take into account the binding energy of g-leptons) corresponds rather to $_{32}Ge$, than $31Ga$, with the mass numbers of the stable isotopes within 70-76. Moreover, the germanium atom is in the same 4th group of the periodic table as the silicon atom, which is an analogous of nucleons at the g-lepton level.

In that case, it is possible to say that all elementary particles consist finally of electrons. The relation of radii of the electron and g-lepton spheres makes it possible to give one more prediction: the spectrum of particles with measures beginning from the electron to g-quantum masses (the constituent of the vast variety of e-class particles) also exists in nature. The last is the most probable.

By radioactive atomic decay, the rebuilding of atoms occurs, helium the most important fragment of nucleonic shells of atoms is rejected in this case. Two outer shell nodes of helium lost their own electrons. Of course, for all that, the definite modification both on the part of nucleons and on the part of g-particles, runs its course. As a result, fine fractions in the form of γ-rays and miniature nucleons-electrons of right and left polarization are rejected. The latter, in the form of the flow of positive and negative electrons, is also experimentally recorded.

Further, during bombarding of targets by fast protons, decay of nucleons takes place and nucleonic "helium", in the form of π-mesons, is thrown out. In turn, π-mesons decay into two γ-quanta, each of which generates a pair of g-lepton of right and left polarization. In addition, these g-leptons can eject electrons.

The above-considered picture of decay corresponds to reality. Therefore, it is possible to state that in the hierarchy of elementary particles the electrons are at the end of the hierarchy chain of e-class microobjects.

References

1. L. Kreidik and G. Shpenkov, *Atomic Structure of Matter-Space*, Geo. S., Bydgoszcz, 2001, 584 p.
2. L. Kreidik and G. Shpenkov, *The Wave Equation Reveals Atomic Structure, Periodicity and Symmetry*, Kemija u Industriji, **51**, 9, 375-384, (2002).

3. G. Shpenkov, *Shell-Nodal Atomic Model,* Hadronic Journal Supplement, Vol, 17, No. 4, 507-567, (2002).
4. G. P. Shpenkov, *The Role of Electrons in Chemical Bonds Formations* (*In the Light of Shell-Nodal Atomic Model*), Molecular Physics Reports **41**, 89-103, (2005).
5. G. P. Shpenkov, *An Elucidation of the Nature of the Periodic Law*, Chapter 7 in "*The Mathematics of the Periodic Table*", edited by Rouvray D. H. and King R. B., Nova Science Publishers, NY, pp. 119-160, 2006.
6. G. P. Shpenkov, *The nodal structure of standing spherical waves and the Periodic Law: What is in common between them*? Physics Essays, Vol. 18, No. 2, (2005)
7. L. Kreidik and G. Shpenkov, *Dynamic Model of Elementary Particles and the Nature of Mass and 'Electric' Charge*, "Revista Ciencias Exatas e Naturais", Vol. 3, No 2, 157-170, (2001); http://shpenkov.com/pdf/masscharge.pdf
8. F.W.J. Olver, ed., *Royal Society Mathematical Tables*, Vol. 7, Bessel Functions, part. III, Zeros and Associated Values, Cambridge, 1960.
9. D. I. Blokhincev, *Foundations of Quantum Mechanics* (in Russian), Nauka, Moscow, 1983.
10. L. Kreidik and G. Shpenkov, *Important Results of Analyzing Foundations of Quantum Mechanics*, Galilean Electrodynamics & QED-East, Special Issues 2, **13**, 23-30, (2002); http://shpenkov.com/pdf/QM-Analysis.pdf
11. L. Kreidik and G. Shpenkov, *Schrödinger's Errors of Principle*, Galilean Electrodynamics; 3, 16, 51-56, (2005) http://shpenkov.com/pdf/Blunders.pdf
12. G. Shpenkov and L. Kreidik, *Discrete Configuration of Probability of Occurrence of Events in Wave Spaces*, Apeiron, Vol. 9, No. 4, 91-102, (2002); http://www.redshift.vif.com/JournalFiles/V09NO4PDF/V09N4shp.pdf
13. G. P. Shpenkov, *Conceptual Unfoundedness of Hybridization and the Nature of the Spherical Harmonics*, Hadronic Journal (sent for publication).
14. R.F. Harrington, *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, 1961.
15. G. Shpenkov, *Generalized Table of the Elements*, http://shpenkov.com/periodic_table.html
16. G. Shpenkov, *Relative Atomic Masses of the Elements*, http://shpenkov.com/isotopes_table.html

17. G. Shpenkov and L. Kreidik, *Potential-Kinetic Parameters of Oscillations*, Hadronic Journal, Vol. 26, No 2, 217-230, (2003).
18. G. Shpenkov and L. Kreidik, *Conjugated Parameters of Physical Processes and Physical Time*, Physics Essays, Vol. 15, No. 3, (2002).
19. L. Kreidik and G. Shpenkov, *Alternative Picture of the World*, Vol. 1-3, Geo. S., Bydgoszcz, 1996.
20. L. G. Kreidik and G. P. Shpenkov, *Philosophy of Contents-Form and Coulomb's Law*, Proceedings of The Twentieth World Congress of Philosophy, Copley Place, Boston, Massachusetts, USA, 10-16 August, 1998, http://www.bu.edu/wcp/Papers/Scie/ScieShpe.htm
21. G. P. Shpenkov and L. G. Kreidik, What the electric charge is; http://shpenkov.janmax.com/Elec-Charge.pdf
22. L. Kreidik and G. Shpenkov, *Interrelation of Values of Base Units and Fundamental Constants with the Fundamental Quantum of Measures*, in *Foundations of Physics: 13.644...Collected Papers*, Geo. S., Bydgoszcz, 1998, 55-68 pp; http://shpenkov.janmax.com/Metrology.pdf
23. L. Kreidik and G. Shpenkov, *Philosophy and the Language of Dialectics and the Algebra of Dialectical Judgments*, Proceedings of The Twentieth World Congress of Philosophy, Copley Place, Boston, Massachusetts, USA, 10-16 August, 1998; http://www.bu.edu/wcp/Papers/Logi/LogiShpe.htm
24. P.A.M. Dirac, Proc. Roy. Soc., A133, 60, 1931; Phys. Rev, 74, 817, 1948.
25. Fundamental Physical Constants from NIST; http://physics.nist.gov/cuu/Constants/