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COMPARISON OF PROPORTIONAL TIME DILATION AND REMOTE NON-SIMULTANEITY: PROOF THAT THE LORENTZ TRANSFORMATION IS SELF-CONTRADICTORY

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Abstract

A review of the predictions of Einstein's Special Theory of Relativity (STR) shows that two of them, remote non-simultaneity and time dilation, are incompatible with each other. It is claimed thereby that two numbers, time differences for the same event that are measured by observers in different states of motion, always occur with a fixed ratio, but that one of them can be zero (simultaneous observation) without the other being so as well. It is impossible that both of these conditions can each be met in any given case, and this constitutes proof that the Lorentz transformation (LT), from which both effects are derived in STR, is not a physically valid set of space-time equations. It is further pointed out that a clock moving through space in the complete absence of unbalanced external forces, in accordance with Newton's Law of Inertia and the Law of Causality, must be expected to have a constant rate. As a consequence, elapsed times Δt and $\Delta t'$ measured by two such (inertial) clocks for the same event should always occur in a fixed ratio, as expressed by the following relation: $\Delta t'=\Delta t/Q$, where Q is a constant fully determined by the above ratio.

Keywords: Time dilation, remote non-simultaneity, Lorentz transformation (LT), Universal Time-dilation Law (UTDL), Global Positioning System-Lorentz transformation (GPS-LT)

1. Introduction

Two of the most significant predictions stemming from the Lorentz transformation (LT) of relativity theory are time dilation and remote non-simultaneity. Both have to do with the behavior of clocks in motion with respect to each other. The latter indicates that events which are simultaneous for one observer may not be so for another. This possibility was first discussed by Poincaré in 1898 [1]. He simply noted that such a phenomenon was an unavoidable consequence of the LT because of its mixing of space and time coordinates. He realized that this proposition ran counter to centuries of scientific opinion going back at the least to the work of Sir Isaac Newton, but he pointed out that there was no incontrovertible evidence which definitively ruled out such an occurrence of non-simultaneity in natural processes,

The phenomenon of time dilation seems to have been first discussed by Einstein in his landmark 1905 paper [2], in which he introduced his Special Theory of Relativity (STR) [2]. He pointed out that according to the LT, a moving clock always runs slower than a stationary one. More quantitatively, he derived a simple formula for a proportionality factor that specifies the ratio of the rates of two such clocks as a function of their speed relative to one another. As will be discussed in the following, however, it will be shown that the latter proportionality relationship is actually incompatible with remote non-simultaneity. Moreover, this circumstance proves that the LT itself is not a physically valid set of equations since both of these contradictory effects are derived from it.

2. Newton's Law of Inertia and its consequences for relativity theory

The derivations of the above two predictions both start with the LT equation given below:

$$\Delta t' = \gamma\left(\Delta t - vc^{-2}\Delta x\right) = \gamma\eta^{-1}\Delta t, \tag{1}$$

$\gamma = \left(1 - v^2c^{-2}\right)^{-0.5}$ and $\eta = \left(1 - vc^{-2}\dfrac{\Delta x}{\Delta t}\right)^{-1}$. It is assumed thereby

that two observers, O and O', are separating from each other along their mutual x,x' axis with relative speed v (c is the speed of light in free space, 299792458 ms^{-1}). They each observe two events such as lightning strikes and measure the time difference between them to be Δt and $\Delta t'$, respectively. The distance separating the events along the x axis is measured to be Δx by observer O. Reference to eq. (1) shows that if the events occur simultaneously for him, i.e. $\Delta t=0$, and both v and Δx are non-zero, the corresponding time difference for observer O' will not also be equal to zero ($\Delta t' \neq 0$). This LT prediction is referred to as *remote non-simultaneity*.

The phenomenon of time dilation is derived [3] by considering a different application of eq. (1). In this case, attention is centered on the stationary clocks in the two rest frames. The travel times for the clock of O' to travel between two fixed points in the rest frame of O are measured to be Δt and $\Delta t'$, respectively. The distance traveled by the latter clock is $\Delta x = v\Delta t$, since by construction it moves with constant speed v along the x axis from the vantage point of observer O. Substitution of this relation in eq. (1) gives:

$$\Delta t' = \gamma(v)\left(\Delta t - v^2 c^{-2} \Delta t\right) = \gamma^{-1} \Delta t. \qquad (2)$$

This equation states that the moving clock from the standpoint of observer O *always* runs slower than his by a factor of $\gamma(v)$. The proportionality of the two time differences is key in the present discussion, however. It clearly demands that if the lightning strikes in the first example occur simultaneously for one of the observers ($\Delta t=0$), they also must occur simultaneously for the other as well, i.e. $\Delta t'=0$. Multiplication of zero with any finite number, in this case $\gamma^{-1}(v)$, must give a product of zero. This prediction therefore runs contrary to the claim of non-simultaneity in the first example.

The fact that the LT is responsible for both the predictions of *proportional* time dilation and remote non-simultaneity proves unequivocally that this set of equations is not a valid space-time transformation. At this point in the discussion, it is not possible to say if either of the predictions is false, only that they can't both be true for the same set of circumstances. The question is therefore whether there is another space-time transformation that is not self-contradictory, but one that at the same time satisfies the other

constraints put on relativity theory by virtue of experimental observations.

To consider this goal it is helpful to take a careful look at the characteristics of an earlier transformation introduced by Voigt in 1887 [4]:

$$\Delta t' = \Delta t - vc^{-2}\Delta x = \eta^{-1}\Delta t \tag{3a}$$

$$\Delta x' = \Delta x - v\Delta t \tag{3b}$$

$$\Delta y' = \gamma^{-1}\Delta y \tag{3c}$$

$$\Delta z' = \gamma^{-1}\Delta z. \tag{3d}$$

His main accomplishment was to adjust the classical (Galilean) transformation in such a way that it becomes consistent with experimental observations which seemed to indicate that the speed of light in free space is independent of the state of motion of the observer. The transformation in eqs. (3a-d) succeeds in this goal, as can be seen by forming the following relationship between the squares of its various quantities:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \gamma^{-2}(\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2). \tag{4}$$

It shows that if the speed of a light pulse is equal to c in the rest frame of observer O', it is also equal to c in the rest frame of another observer O moving with speed v relative to him.

To arrive at this transformation, Voigt first added a distance-dependent term, $vc^{-2}\Delta x$, to the classical equation which assumes that the clocks of both observers run at exactly the same rate, i.e. $\Delta t'=\Delta t$. In addition, in eqs. (3c-d) he added a factor of γ^{-1} to the classical relations for motion in directions that are perpendicular to the relative velocity of the two observers. The mixing of space and time coordinates in eq. (3a) amounts to a clear break with accepted views of the time. *It was based on a conclusion that such mixing is essential for arriving at a set of space-time equations that is consistent with light-speed constancy.*

There is nonetheless something unacceptable about the Voigt transformation. This is because it does not satisfy the conditions required by the Relativity Principle (RP) and its assertion that the laws of physics are the same in all inertial systems. One can simulate the exchange of observers in these equations by interchanging the primed and unprimed coordinates and reversing the sign of the relative speed v of the two observers.

The RP requires that when this procedure is employed, which will henceforth be referred to as *Galilean inversion*, the resulting set of equations must be the exact inverse transformation of the original. This means, for example, that eq. (3c) is changed to $\Delta y=\gamma^{-1}\Delta y'$ (note that changing the sign of v has no effect on the value of γ). Substitution of the latter relation back into eq. (3c) gives the nonsensical result of $\Delta y'=\gamma^{-2}\Delta y'$.

Lorentz [5] subsequently provided a means of improving upon Voigt's transformation. He pointed out there is a degree freedom in any such transformation because of the fact that multiplying the right-hand sides of all four equations by the same factor has no effect on the *ratio* of its space and time intervals. The condition of light-speed constancy can therefore be satisfied with the choice of any finite value for this factor. This conclusion thus leads to a more general version of the Voigt transformation which is given below, where the aforementioned factor is designated as φ:

$$\Delta t' = \varphi(\Delta t - vc^{-2}\Delta x) = \varphi\eta^{-1}\Delta t \tag{5a}$$

$$\Delta x' = \varphi(\Delta x - v\Delta t) \tag{5b}$$

$$\Delta y' = \varphi\gamma^{-1}\Delta y \tag{5c}$$

$$\Delta z' = \varphi\gamma^{-1}\Delta z. \tag{5d}$$

In particular, the factor of γ^{-2} in eq. (4) is thus changed to $\varphi^2\gamma^{-2}$ based on the new transformation, without therefore affecting the light-speed constancy condition in any way.

It is a simple matter to take advantage of the above degree of freedom in order to satisfy the RP. One merely has to choose φ to be equal to γ in eqs. (5a-d). The result is the LT, with the adjusted eq. (5a) becoming identical to eq. (1). The LT does satisfy the RP. For example, it removes the problem with the result of applying Galilean inversion to eq. (3c). With $\varphi=\gamma$, eq. (5c) simply becomes $\Delta y'=\Delta y$, which relation is obviously unchanged by interchanging the primed and unprimed subscripts.

Nevertheless, as has been shown above, the LT is also unsatisfactory because its eq. (1) leads to completely incompatible predictions of remote non-simultaneity and time dilation. The present discussion shows that the two conditions of light-speed constancy and the RP, which are Einstein's two postulates of relativity [2], are not sufficient to determine the true space-time transformation.

It also indicates that another characteristic of time measurements would be helpful in this respect. Consider, for example, the role of inertial clocks in the above discussion. According to Newton's First Law of Motion (Law of Inertia), all such clocks will continue moving indefinitely in a straight line with constant speed. Because of the assumed complete absence of unbalanced external forces, it can be assumed with great confidence based on the Law of Causality that the *properties of these clocks* will be unchanged for the duration of their flight. In particular, one should expect *that their rates remain constant* under these circumstances. Just as the speeds and directions of the various clocks do not have to be the same, it seems equally reasonable to conclude that the rates of the clocks can also be different. The key point in the present discussion, however, is that the *ratio of the rates of any two inertial clocks will also be constant* and that the same holds true for their respective elapsed times for the same event. These considerations lead unambiguously to the following simple relation, which should hold true as long as no change in the states of motion of either occurs:

$$\Delta t' = \frac{\Delta t}{Q}, \tag{6}$$

where Q is a constant proportionality factor.

The question clearly arises whether eq. (6) is compatible with the other two conditions of light-speed constancy and the RP. To answer it, one need only go back to the general space-time transformation in eqs. (5a-d). To begin with, it is necessary to choose a suitable value for the degree-of-freedom parameter φ. A solution is readily found from eq. (5a):

$$\Delta t' = \varphi\eta^{-1}\Delta t = \frac{\Delta t}{Q}, \tag{7}$$

from which one obtains the following value for φ, namely

$$\varphi = \frac{\eta}{Q}. \tag{8}$$

Substitution of this value in each of eqs. (5a-d) then leads to the following transformation:

$$\Delta t' = \eta \frac{\Delta t - vc^{-2}\Delta x}{Q} = \frac{\Delta t}{Q} \tag{9a}$$

$$\Delta x' = \eta \frac{\Delta x - v\Delta t}{Q} \tag{9b}$$

$$\Delta y' = \left(\frac{\eta}{\gamma Q}\right)\Delta y \tag{9c}$$

$$\Delta z' = \left(\frac{\eta}{\gamma Q}\right)\Delta z. \tag{9d}$$

Because of its relation to the general transformation in eqs. (5a-d), it is clear that the new transformation satisfies the light-speed constancy condition. It is also clear that it satisfies the proportional time condition of eq. (8) since this appears directly as its eq. (9a). It remains to be shown that it also satisfies the RP, however.

To this end, it is helpful to consider the effect of applying Galilean inversion to its various equations. First of all, the inverse of eq. (9a) must be $\Delta t=\Delta t'/Q'$, where Q' is not defined initially. It is easy to satisfy this condition, however, by requiring that $Q'=1/Q$. The corresponding condition for eqs. (9c-d) is $\eta\eta'/\gamma^2QQ'=1$, or simply $\eta\eta'=\gamma^2$ because of the reciprocal relationship of Q an Q' already determined. Proof of this equality is given below [7]. It also can be used to show that Galilean inversion leads to the inverse of eq. (9b). In summary, the transformation of eqs. (9a-d) satisfies all three of the required conditions, unlike either the LT or the original Voigt transformation [4].

The same transformation can be obtained by another route that does not involve explicit consideration of the degree of freedom discussed above in eqs. (5a-d). The transformation of the velocity components $u_x=\Delta x/\Delta t$, $u_x'=\Delta x'/\Delta t'$ etc. for the two observers results from division of each of eqs. (5b-d) by eq. (5a):

$$u_x' = \left(1 - vu_x c^{-2}\right)^{-1}\left(u_x - v\right) = \eta\left(u_x - v\right) \tag{10a}$$

$$u_y' = \gamma^{-1}\left(1 - vu_x c^{-2}\right)^{-1} u_y = \eta\gamma^{-1}u_y \tag{10b}$$

$$u_z' = \gamma^{-1}\left(1 - vu_x c^{-2}\right)^{-1} u_z = \eta\gamma^{-1}u_z. \tag{10c}$$

Exactly the same velocity transformation (RVT) is obtained by carrying out the analogous divisions for the original Voigt

transformation of eqs. (3a-d) as well as for the LT. Multiplication of each of these three equations by the Newtonian proportional time relation of eq. (6) leads directly to eqs. (9b-d).

The RVT satisfies the RP, as can be seen by applying the identity relation already discussed. The proof of the latter is given below in terms of the velocity components:

$$\begin{aligned}\eta\eta' &= \left[\left(1-u_x vc^{-2}\right)\left(1+u_x' vc^{-2}\right)\right]^{-1} = \left[\left(1-u_x vc^{-2}\right)\left(1+\eta vc^{-2}\left(u_x - v\right)\right)\right]^{-1} \\ &= \left[\left(1-u_x vc^{-2}\right)\frac{1-u_x vc^{-2}+vc^{-2}u_x - v^2c^{-2}}{1-u_x vc^{-2}}\right]^{-1} \qquad (11) \\ &= \left(1-v^2c^{-2}\right)^{-1} = \gamma^2 .\end{aligned}$$

Applying Galilean inversion to eq. (9c), for example, and substituting this result for the u_y component leads back directly to the value of u_y' required to satisfy the RP by making use of eq. (11).

3. Comparison with experiment

The transformation in eqs. (9a-d) differs in a number of significant ways from the LT. The proportionality relationship between measured elapsed times given in its first equation clearly supports the original Newtonian view of absolute simultaneity for all events throughout the universe. At the same time, it leaves open the possibility that the rates of clocks depend on their state of motion. While it is necessary to reject the LT as a valid space-time transformation because of its two predictions of remote non-simultaneity and proportional time dilation, the same cannot therefore be said about that in eqs. (9a-d).

Recognition of this point opens up the broader question of whether the latter transformation is consistent with all available experimental findings. It also remains to be shown how the constant Q in eq. (9a) can be determined, since lack of a concrete means of accomplishing this goal would obviously severely limit the transformation's potential advantages in practical applications.

A good place to begin in this regard is the experiment with circumnavigating atomic clocks carried out by Hafele and Keating in 1971 [8,9]. It was found that the rates of the clocks decreased as their speed v relative to the earth's center of mass (ECM) increased.

A correction based on the gravitational red shift was also applied to account for differences in the altitudes of the clocks. Specifically, the authors found that the rates of the clocks are inversely proportional to $\gamma(v)\approx 1+0.5v^2c^{-2}$. As a result the relationship between measured elapsed times Δt and $\Delta t'$ for any given portion of the flight of two clocks with respective speeds u and u' relative to the ECM is given by the following equation:

$$\Delta t'\gamma(u') = \Delta t\gamma(u). \tag{12}$$

Experiments carried out a decade earlier with high-speed rotors [10-12] can be described by eq. (12) as well. In this case the absorber and detector of an x-ray source were mounted on the rotor and it was found that the rate/frequency of each such clock decreased with its speed u relative to the rotor axis [11]. A key aspect of eq. (12) is that a definite rest frame needs to be designated from which to compute the speeds u and u' to be inserted into it. It is the rotor axis in the x-ray frequency study and the ECM in the case of the Hafele-Keating experiment, for example. In previous work [13], this reference frame has been referred to as the objective rest system (ORS). Einstein mentioned a related application in his 1905 paper [2] according to which a clock located at the Equator was expected to run at a slower rate that an identical counterpart at one of the earth's Poles. More generally, the ORS is the rest frame from which an object undergoes an applied force which causes it to be accelerated to a given speed.

The developers of the Global Positioning System (GPS) have made use of eq. (12) in order to adjust the rates of atomic clocks located on orbiting satellites so that they are equal to those of identical clocks located on the earth's surface. The ratio of the rates of two such clocks is accordingly computed on the basis of their respective speeds relative to the ECM. A pre-correction procedure [14,15] is applied to the satellite clock prior to launch so that its rate is increased artificially by the above ratio. The effect of time dilation on this clock counter-balances the latter adjustment, with the desired result that it runs at nearly the same rate as earthbound clocks after it reaches its prescribed orbit.

There is also a gravitational effect which needs to be considered in order to achieve the desired level of accuracy for GPS distance measurements. The fact that eq. (12) is applicable to all the above situations, and especially that there are no known exceptions to it, indicates that it is a fundamental law of physics. It is therefore deserving of the designation: Universal Law of Time Dilation (UTDL [16,17]). Because of the application of eqs. (9a-d) to the adjustment of the rates of satellite clocks, they have been designated as the Global Positioning System-Lorentz Transformation (GPS-LT) [18-20].

Moreover, eq. (12) can be used directly to quantitatively determine the value of the parameter Q in the Newtonian elapsed time proportionality of eq. (6), namely as:

$$Q = \frac{\gamma(u')}{\gamma(u)}. \tag{13}$$

This parameter is also used in all four equations of the alternative space-time transformation of eqs. (9a-d). It is also important to apply the Galilean inversion procedure to eq. (13) to see how the corresponding relationship is perceived by O'. The result is:

$$Q' = \frac{\gamma(u)}{\gamma(u')} = \frac{1}{Q}, \tag{14}$$

in agreement with the requirement mentioned in the previous section: it is essential to have the inverse transformation of eqs. (9a-d) be obtained by simply reversing the roles of the two observers, in accordance with the RP.

It is helpful to look upon Q and Q' as conversion factors for the different units of time employed in the two rest frames. The parameter Q is needed in order to convert elapsed times measured by O' to the corresponding unit employed by observer O. The conversion factor in the reverse direction is simply the reciprocal of Q, analogous to the conventional case in which cm are to be converted to m and vice-versa.

4. Conclusion

The Lorentz transformation (LT) is not a valid component of relativity theory because it leads to two predictions which are

hopelessly incompatible with each other: remote non-simultaneity and time dilation. The latter requires that time differences for the same event that are measured by two observers in constant relative motion with speed v always occur in the same proportion [$\Delta t' = \gamma(v)\Delta t$], whereas the non-simultaneity prediction claims that one of the time differences can be zero without the other being so as well. To believe in both relationships requires that one disregard the axiom of algebra which states that the product of any finite number with zero is itself equal to zero, and that is clearly unacceptable for any theory of physics.

The space-time mixing character of eq. (1) of the LT is responsible for the above conflict. Consideration of Newton's Law of Inertia also indicates that space and time are not mixed. A straightforward extension indicates that any clock which is moving under the absence of external forces should not only move at constant velocity, but also that its rate should remain the same indefinitely as well. On this basis the only reasonable conclusion is that the ratio of the rates of any two such clocks should be constant as well, which leads to a simple alternative to eq. (1), namely the proportionality relation of eq. (6). The latter also leaves open the possibility of time dilation, but in contrast to the LT, it removes any chance that observers could each find that it is the other's clock that is running slower. Remote non-simultaneity is also ruled out by this relation.

Experimental tests of time dilation have always been perfectly consistent with eq. (6). Their results can be formulated in another proportionality relationship, the Universal Time-dilation Law (UTDL) of eq. (12). The latter allows for a straightforward prediction of the constant Q in eq. (6) in terms of the speeds of any two such clocks relative to a specific frame of reference. The latter is the earth's center of mass in the study of circumnavigating atomic clocks carried out in 1971 [8,9], for example. Specifically, the value of Q is given by eq. (13). It shows that the clock which runs faster relative to the above rest system has a slower rate than its counterpart. A useful means of describing the role of the constant Q is as a conversion factor between elapsed times measured on different clocks. The asymmetry of the above relationship again stands in stark contrast to the LT version of time dilation.

The constant ratio of inertial clock rates expressed in eq. (6) serves as a third postulate of relativity. In particular, the support it has received from experiment in the form of the UTDL of eq. (12) makes it quite difficult to argue for a version of the theory which ignores it. Such a postulate goes along with the two Einstein used in his derivation of the LT [2], the Relativity Principle and the constancy of the speed of light in free space. The GPS-LT of eqs. (9a-d) succeeds in incorporating all three. This set of equations is perfectly self-consistent, unlike the LT, and also leads directly to the same relativistic velocity transformation (RVT) as has long been accepted by the physics community. The latter is sufficient by itself to explain the occurrence of the aberration of starlight at the zenith and the characteristics of the Fresnel light-drag experiment, for example, so this characteristic of the GPS-LT shows that *the LT is not essential for the description of these effects*. The concrete indications from both experiment and theory of the validity of the GPS-LT suggest that it is highly desirable to carry out new experiments in future work to further test the accuracy of eq. (12).

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