

**MASS PREDICTIONS AND LIFETIME FORMULAS OF HEAVY
FLAVOR HADRONS, AND SIMPLIFY OF SUPERSYMMETRY**

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Abstract

First, we discuss the modified accurate mass formula, and applied to heavy flavor hadrons, and derive some predictions, $m(\Omega_{cc}^+) = 3950.7$ or 3908 MeV , and $m(\Xi_{bb}) = 10396.8$ or 10348.9 MeV , etc. It is a quantitative and testable theory. Next, based on the new data, we propose various lifetime formulas of heavy flavor hadrons, which very agree with experiments. This is a new method on lifetime of hadrons described by quantum numbers, and can be unified for mass and lifetime. Finally, we discuss an approximate simplified supersymmetry theory based on the known symmetrical particles and their excited states.

Keywords: particle, mass formula, lifetime formula, heavy flavor hadron, supersymmetry.

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I. Introduction

It is well-known that the theoretical base of particles is the standard model:

$$q = \begin{pmatrix} (2/3)e \\ (c) \\ (t) \end{pmatrix} \quad \begin{pmatrix} -(1/3)e \\ (d) \\ (b) \end{pmatrix} \quad \begin{pmatrix} -e \\ (\mu) \\ (\tau) \end{pmatrix} \quad \begin{pmatrix} 0 \\ (\nu_e) \\ (\nu_\mu) \\ (\nu_\tau) \end{pmatrix}. \quad (1)$$

The total charge number of per generation quarks is $3(2/3-1/3)=1$, and the total charge number of per generation leptons is yet $(-1+0)=-1$. The total charge of per generation quark-lepton is 0. This is very symmetrical and beautiful theory.

After particles are classified, mass and lifetime are two main characters for any particles. In this paper, we discuss various mass and lifetime formulas, esp., for heavy flavor hadrons, and a simplified supersymmetry theory.

2. Accurate Mass Formula of Hadrons and Prediction

Based on two moving states of the emergence string: oscillation and rotation, we derived its quantum potential and the equation, whose energy spectrum is the GMO mass formula:

$$M = M_0 + AS + B[I(I+1) - \frac{S^2}{4}], \quad (2)$$

in which must assume that $M = m^2$ for mesons, and its modified accurate mass formula [1-3]:

$$M = M_0 + AS + B[I(I+1) - \frac{S^2}{2}]. \quad (3)$$

Based on the standard model and on the symmetry of s and c quarks in the same generation, we supposed that the hadrons, which made of u, d and c quarks, are also the SU(3) symmetry and are classified by octet and decuplet [4,2,3]. It is a subgroup of SU(4) of u, d, s and c quarks. Such we assume that these masses of the octet obey the corresponding simple mass formulas only by $S \rightarrow C$ in Eqs.(2) and (3):

$$M = M_0 + AC + B[I(I+1) - (C^2/4)], \quad (4)$$

$$\text{or} \quad M = M_0 + AC + B[I(I+1) - (C^2/2)]. \quad (5)$$

Since when $m(N)=939$, $m(\Lambda_c^+) = 2285$, $m(\Sigma_c) = 2453 \text{ MeV}$, from the two corresponding mass formulas (4) and (5) we predicted $m(\Xi_{cc}) = 3715$ or 3673 MeV [4,2,3]. In 10 July 2017 LHC announced to observe a new doubly charmed baryon $\Xi_{cc}^{++} = ucc$, whose mass is $3621.4 \pm 0.8 \text{ MeV}$, and decay mode is $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ [5,6]. New experimental data agree more on Eq.(5) and (3), whose error only is $(3673-3621)/3621=1.4\%$. Moreover, there should have $\Xi_{cc}^+ = dcc$, both form $I=1/2$ doublet with near mass, and one of decay modes is $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+ \pi^0$ [7]. In 1994 we predicted that "if the experiments derive this mass, it will show that our theory (hadrons made of u, d and c quarks are the SU(3) symmetry) is right"[4].

According new experimental data $m(p)=938.3$, $m(\Lambda_c^+) = 2286.5$, $m(\Sigma_c^+) = 2452.9$ and $m(\Xi_{cc}^{++}) = 3621.4 \text{ MeV}$ [6], it agrees with the mass relation

$$4(p + \Xi) = 7\Lambda + \Sigma \quad (18238.8 = 18458.4), \quad (6)$$

whose error is $(18458.4 - 18238.8) / 18458.4 = 1.19\%$.

Further, for the $J^P = 3^+ / 2$ baryons form also a decuplet:

$$\Delta^{++}, \Delta^+, \Delta^0, \Delta^- \quad (I=3/2); \Sigma_c^{++}, \Sigma_c^+, \Sigma_c^0 \quad (I=1); \Xi_{cc}^{++}, \Xi_{cc}^+ \quad (I=1/2)$$

$$\text{and } \Omega_{ccc}^{++} = ccc \quad (I=0).$$

Their masses are possibly an equal-spacing rule, i.e.

$$M = M_0 + aC. \quad (7)$$

Of course, these baryons obey probably the more accurate mass formulas

$$M = M_0 + aC + bC^2, \quad M = M_0 + a'I + b'I^2, \quad (8)$$

which correspond to Eqs.(2) and (5).

The $J^P = 0^-$ octet of heavy flavor mesons are $\pi^{+-}, \pi^0 \quad (I=1); D^+ = c\bar{d}$, $D^0 = c\bar{u} \quad (I=1/2)$ and their antiparticles; $\eta'_c = a(u\bar{u} + d\bar{d}) + b(c\bar{c})$. If their mass relation is:

$$4m(D) = m(\pi) + 3m(\eta'_c) \text{ or } 8m(D) = m(\pi) + 7m(\eta'_c), \quad (9)$$

so $m(\eta'_c) = 2444$ or 2114 MeV since $m(\pi) = 137$, $m(D) = 1867 \text{ MeV}$. For the $J^P = 1^-$ octet, $m(\rho) = 770$, $m(D^+) = 2010 \text{ MeV}$, so $m(\eta'_c) = 2423$ or 2187 MeV .

For the $J^P = 1^+ / 2$ baryons, $m(\Xi_c^+(usc)) = 2467.7 \text{ MeV}$, $m(\Xi_c^0(dsc)) = 2471.7 \text{ MeV} \quad (I=1/2)$, $m(\Omega_c(ssc)) = 2695.2 \text{ MeV}$.

These octets and decuplet are a certain cross section of the diagrams of the SU(4) multiplets, respectively. For the $J^P = 3^+ / 2$ baryons, probably, the masses of the triplet $\Sigma^+, \Sigma^0, \Sigma^- \quad (I=1)$, the doublet $\Xi_c^+(usc), \Xi_c^0(dsc) \quad (I=1/2)$, and the singlet $\Omega_{cc}^+ = scc \quad (I=0)$ are approximately an equal-spacing rule, it is the second series of series of heavy flavor baryons. Then the masses of $\Omega^- = sss, \Omega_c^0 = ssc, \Omega_{cc}^+$ and Ω_{ccc}^{++} are only four hadrons for mixtures of second generation, and their masses are should be equal-spacing too. It is the third series of heavy flavor baryons. For the $J^P = 3^+ / 2$ baryons, $m(\Sigma_c) = 2518$, $m(\Xi_c) = 2646$, so $m(\Omega_c(ssc)) = 2766 \text{ MeV}$, these masses obey equal-spacing rule. Such based on the known masses of $3^+ / 2$ baryons including c quark [6], other masses of baryons will be able to be estimated. Because $\Sigma_c(uuc, 2518) - \Delta(uuu, 1232) = 1286 \cong m(c) - m(u)$, so $m(\Xi_{cc}) = 3804$ and $m(\Omega_{ccc}^{++}) = 5090$. From the third series, we obtain $m(\Omega^-) = 1672.1$, $m(\Omega_c^0) = 2811.4$, $m(\Omega_{cc}^+) = 3950.7$ and $m(\Omega_{ccc}^{++}) = 5090$. But, from the second series the known $m(\Sigma) = 1385$ and $m(\Xi_c) = 2646.6$, so $c - u \cong 1261.6$ and $m(\Omega_{cc}^+) = 3908.2$. Both $m(\Omega_{cc}^+)$ are different due to different of spin. Of course, these baryons obey probably the more accurate mass formulas (8).

Such any one of masses of $3^+ / 2$ baryons including c quark is known again, for example, for $\Sigma_c(2518.1) \text{ MeV} \quad (J^P = 3^+ / 2)$ [6], then other five masses of other baryons will be able to be estimated.

baryon	Ω^-	Ξ_c^-	Δ^-	Σ_c^-	Ξ_{cc}^-	Ω_c^-	Ω_{cc}^+	Ω_{ccc}^{+++}
$m(\text{exp})$	1672	2646	1232	2518				
$m(\text{est})$	input	2646	input	input	3804	2811	3951	5090

Therefore, by using different ways we predict different $m(\Omega_{cc}^+)=3950.7$, or 3908MeV , etc.

For the doublet $\Xi_c^+ = usc, \Xi_c^0 = dsc$ ($I=1/2, S=-1, C=1$) and both masses are 2467.9 and 2471.9MeV , the singlet $\Omega_c^0 = ssc$ ($I=0, S=-2, C=1$) and its mass is $2695.2 \pm 1.7\text{MeV}$, and the singlet $\Omega_{cc}^+ = scc$ ($I=0, S=-1, C=2$), we can assume that the simplest mass formula is:

$$M = M_0 + A(C - S) + B[I(I + 1) - (C - S)^2 / 2], \quad (10)$$

$$\text{or } M = M_0 + A(C - S) + B[I(I + 1) - (C^2 + S^2) / 2]. \quad (11)$$

Some similar multiplets and mass formulas exist possibly in baryons and mesons including b or t quarks. For instance, both mass spectra of $\psi = c\bar{c}$ and $Y = b\bar{b}$ are similar; as in the neutral kaon system, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ mixings should exist.

Further, this method will be able to be extended to other potentials and other heavy flavor hadrons. For example, based on the symmetry of quarks model, we may suppose that the hadrons, which made of u, d and b quarks, and of u, d and t all are the SU(3) symmetry. It is a subgroup of SU(4) of u, d, b and t quarks. Such the eight $J^p = 1^+ / 2$ baryons form also an octet:

$p=uud, n=udd$ ($I=1/2$); $\Lambda_b^0 = udb$ ($I=0$); $\Sigma_b^+ = uub, \Sigma_b^0 = udb, \Sigma_b^- = ddb$ ($I=1$); and $\Xi_b^0 = ubb, \Xi_b^- = dbb$ ($I=1/2$).

Such the corresponding mass formulas are:

$$M = M_0 + aB + b[I(I + 1) - (B^2 / 4)], \quad (12)$$

$$\text{or } M = M_0 + aB + b[I(I + 1) - (B^2 / 2)]. \quad (13)$$

It is known the $m(\Lambda_b)=5620$, $m(\Sigma_b)=5811.5(5811.3, 5815.5)\text{MeV}$ [6]. From the two corresponding mass formulas, we may predict $m(\Xi_{bb})=10396.8$ or 10348.9MeV , whose error is probably bigger due to weaker symmetry between first (u,d) and third generations (t,b).

Generally, based on the symmetry of quark model we can suppose that the hadrons, which made of s, c and b quarks are the SU(3) symmetry. Similar, the hadrons made of s, c and t quark are also the SU(3) symmetry. Both are two subgroups of SU(4) of s, c, b and t quarks, but these quarks are all $I=0$ and very unstable. These mass formulas are possibly:

$$M = M_0 + aS + a'C + a''B + bS^2 + b'C^2 + b''B^2, \quad (14)$$

$$\text{or } M = M_0 + aS + a'C + a''B + b(S + C + B)^2. \quad (15)$$

The mixtures of three generations are $m(\Xi_b^0 = usb)=5791.9\text{MeV}$, $m(\Xi_b^- = dsb)=5797.0\text{MeV}$ ($I=1/2$). Other hadrons are $m(\Lambda_b^0(udb))=5619.6\text{MeV}$, $m(\Omega_b^-(ssb))=6046.1\text{MeV}$, etc[6].

In a word, our research based some symmetries among different generations is a quantitative and testable theory [8].

3. Lifetime Formulas of Heavy Flavor Hadrons

Based on the Y-Q and I-U symmetries between mass and lifetime on the general SU(3) theory, we obtained the lifetime formulas of hyperons and mesons [1-3]:

$$\tau = A[2U(U+1) - Q/2], \quad (16)$$

$$\text{and } \tau = A'[(1/2) + 2U(U+1) - Q/2 - Q^2/3]. \quad (17)$$

They agree better with experiments [6].

Generally, lifetime of various hadrons can be classified by different times:

Lifetimes of mesons K_L^0, π^-, K^+ are 10^{-8} sec.

Lifetimes of hyperons $\Xi^0, \Xi^-, \Sigma^-, \Sigma^+$ and K_S^0, Ω^- are 10^{-10} sec.

Lifetimes of heavy flavor hadrons mainly are 10^{-13} sec.

Lifetime of π^0 is 8.4×10^{-17} sec.

Lifetime of Σ^0 is 7.4×10^{-20} sec.

Lifetimes of J/ψ and Y are KeV.

Lifetimes of other hadron-resonances mainly all are MeV.

For heavy flavor hadrons we propose their lifetime formulas. For Ξ_c^+ (usc),

Λ_c^+ (udc), Ξ_c^0 (dsc) and Ω_c^0 (ssc), it is:

$$\tau = [1.4(2I - C^2) - S + 3.4Q] \times 10^{-13}. \quad (18)$$

Then $\tau(\Xi_c^+) = 4.4$, $\tau(\Lambda_c^+) = 2$, $\tau(\Xi_c^0) = 1$ and $\tau(\Omega_c^0) = 0.6$, and the experimental data are (4.42 ± 0.26) , (2.00 ± 0.06) , (1.12 ± 0.13) and $(0.69 \pm 0.12) \times 10^{-13}$ [6].

They all agree within the range of error. Further, for $\tau(\Xi_{cc}^{++}, ucc) = 2.6$ by this formula (18), it agrees accurately with the experimental data $(2.56 \pm 0.37) \times 10^{-13}$ [9].

For Λ_b^0 (udb), Ξ_b^- (usb), Ξ_b^0 (dsb) and Ω_b^- (ssb), we propose the lifetime formula:

$$\tau = [14.8B^2 - (I + Q - \frac{1}{2}S)] \times 10^{-13}. \quad (19)$$

Then $\tau(\Lambda_b^0) = 14.8$, $\tau(\Xi_b^-) = 15.8$, $\tau(\Xi_b^0) = 14.8$ and $\tau(\Omega_b^-) = 16.8$, and the experimental data are (14.70 ± 0.10) , (15.71 ± 0.40) , (14.79 ± 0.31) and $(16.4 \pm 1.8) \times 10^{-13}$ [6]. They all agree within the range of error.

For $D^+(c\bar{d})$, $D^0(c\bar{u})$ and $D_s^+(c\bar{s})$, we propose the lifetime formula:

$$\tau = [4.1 + 6.3(2I - C^2 + Q) - S] \times 10^{-13}. \quad (20)$$

Then $\tau(D^+) = 10.4$, $\tau(D^0) = 4.1$ and $\tau(D_s^+) = 5.1$, and the experimental data are (10.4 ± 0.07) , (4.101 ± 0.015) and $(5.04 \pm 0.04) \times 10^{-13}$ [6]. They all agree within the range of error.

For $B^+(u\bar{b})$, $B^0(d\bar{b})$, $B_s^0(s\bar{b})$ and $B_b^+(c\bar{b})$, we propose the lifetime formula:

$$\tau = [15 + 5.5(2I - C^2 - B^2 - S) + Q] \times 10^{-13}. \quad (21)$$

Then $\tau(B^+) = 16$, $\tau(B^0) = 15$, $\tau(B_s^0) = 15$ and $\tau(B_b^+) = 5$, and the experimental data are (16.38 ± 0.04) , (15.20 ± 0.04) , (15.09 ± 0.04) and $(5.07 \pm 0.09) \times 10^{-13}$ [6]. They all very agree.

We may unify these lifetime formulas for the heavy flavor hadrons to:

$$\tau = [\tau_0 + a(2I - C^2 - B^2) - bS + cQ] \times 10^{-13}. \quad (22)$$

It is a new method on lifetime of hadrons described by quantum numbers. They are symmetrical with the corresponding mass formulas, and can be unified for mass and lifetime.

4. Supersymmetry and Its Simplify

Supersymmetry is a very beautiful theory, and it combines string, and derives the superstring. But so far any particles of supersymmetry are not observed. We derived some new representations of the supersymmetric transformations, and introduced the supermultiplets. Based on these representations, Graded Lie Algebras and various formulations (equations, commutation relations, propagators, Jacobi identities, etc.) of bosons and fermions may be unified. On the one hand, the mathematical characteristic of particles is proposed: bosons correspond to real number, and fermions correspond to imaginary number, respectively. Such fermions of even (or odd) number form bosons (or fermions), which is just consistent with a relation between imaginary and real number. The imaginary number is only included in the equations, forms, and matrixes of fermions. It is connected with relativity. On the other hand, the unified forms of supersymmetry are also connected with the statistics unifying Bose-Einstein and Fermi-Dirac statistics, and with the possible violation of Pauli exclusion principle; and a unified partition function is obtained [10,11]. Moreover, three quarks may be described by the Borromean rings [12]. We discussed some unifications in particle physics. The quantum statistics is unified by the nonlinear equations. Based on the gauge groups, various unifications of interactions are researched. A developed direction of particle physics and modern science is possibly the higher dimensional complex space [10-12].

Now we discuss an approximate simplified supersymmetry theory based on the known symmetrical particles and their excited states, and the Regge trajectory formula $S = AJ + B$.

It is known that baryons ($J=1/2, 3/2$) and mesons ($J=0, 1$) possess symmetry.

For $SU(3)$ octet of u,d,s and their excited states:

J=1/2	$p, n; \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$ (in octet baryons first generation 2, second generation 6)
J=0	$\pi^0, \pi^\pm; K^\pm, K^0; \eta$ (in five mesons first generation 2, second generation 3)
J=3/2	$\Delta; \Sigma^*; \Xi^*; \Omega(sss)$ (in decuplet baryons first generation 4, second generation 6)
J=1	$\rho(770), \omega(782); (s\bar{s})$

For leptons there are:

J=1/2	$e, \nu; \mu, \tau, \nu_\mu, \nu_\tau$ (six leptons first generation 2, second-third generation 4)
J=1	$\gamma; W^\pm, Z^0$ (1 and 2)

J=2, graviton.

In a word, bosons are all the degenerate states.

For SU(3) octet of u,d,c and their excited states possess structures of complete symmetry:

J=1/2	$p, n; \Lambda_c^+, \Sigma_c^{++}, \Sigma_c^+, \Sigma_c^0; \Xi_c^+, \Xi_c^0$ (in octet baryons first generation 2, second generation 6)
J=0	$\pi^0, \pi^\pm; D^0, D^\pm; \eta_c(c\bar{c})$ (in five mesons first generation 2, second generation 3)

Second generation has the mixtures $\Omega_c^0(ssc)$, and (scc), (ccc) for J=1/2;

$D_s^+(c\bar{s})$ for J=0.

For SU(3) octet of u,d,b and their excited states possess structures of complete symmetry:

J=1/2, known baryons are $p, n; \Lambda_b^0(udb); \Xi_b^0(usb), \Xi_b^-(dsb)$; should yet have $\Sigma(ubd, udb, ddb)$. (in octet baryons first generation 2, third generation 6);

J=0, $\pi^0, \pi^\pm; B^+(u\bar{b}), B^0(d\bar{b})$ (in four mesons first generation 2, second generation 2).

Moreover, J=1, $Y(b\bar{b})$.

We combine supersymmetry and the standard model (1). Suppose the second and third generations of quark-lepton are the different excited states of the first generation. This will be the simplest particle model.

First generation

J=1/2	p(uud), n(udd) are stable	J=3/2	$\Delta^{++}(uuu), \Delta^-(ddd)$
J=0	$\pi^+(u\bar{d}), \pi^0(u\bar{u}, d\bar{d})$		

Second generation

J=1/2	$\Omega_c^0(ssc), (scc)$	J=3/2	$\Omega^-(sss), X^{++}(ccc)$
J=0	$D_s^+(c\bar{s}), (s\bar{s}), \eta_c(c\bar{c})$	J=1	$J/\psi(c\bar{c})$

Third generation

J=1/2	(bbt), (btt)	J=3/2	They are probably (bbb), (ttt)
J=0	$(t\bar{b}), (b\bar{b}), (t\bar{t})$	J=1	$Y(b\bar{b})$

Therefore, the second and third generations are completely similar to the first generation. Three quarks are the same, probably all J=3/2.

From the second generation begins, some mixing states of first and second generations exist:

J=1/2	(uus), (uds), (dds); (uss), (dss)	And (uuc), (udc), (ddc); (ucc), (dcc)
J=0	$K^+(u\bar{s}), K^0(d\bar{s})$	And $D^+(c\bar{d}), D^0(c\bar{u})$

From this begins existence of the substable states.

Further, some mixing states of first and third generations exist:

J=1/2	$(uub), \Lambda_b^0(udb), (ddb); (ubb), (dbb)$	And $(uut), (udt), (ddt); (utt), (dtt)$
J=0	$B^+(u\bar{b}), B^0(d\bar{b})$	And $(u\bar{t}), (d\bar{t})$

And there are some mixing states of second and third generations:

J=1/2	$(ssb), (scb), (ccb); (sbb), (cbb)$	And $(sst), (sct), (cct); (stt), (ctt)$
J=0	$B_s^0(s\bar{b}), B_c^+(c\bar{b})$	And $(s\bar{t}), (c\bar{t})$

Moreover, there are some mixtures of first-second-third generations:

$\Xi_b^0(usb), \Xi_b^-(dsb)$, etc.

Quarks have three generations, gluons should have three generations. Or second and third generation quarks are excited states of the first generation quarks, so the corresponding second and third generation gluons will also be the first generation excited states.

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