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IS SPACE ABSOLUTE?

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Abstract

The hypothesis that empty space and particles are made up of four-dimensional (4D) spheres of space whose diameter is Planck's length provides a privileged frame of reference or ether, so that the relativistic effects are due to the true Lorentz contraction of physical objects. On the other hand, the fact that the particles are also formed by 4D spheres allows us to deduce the relativistic effects from the structure of the atoms.

Keywords: Space-time discrete; Fundamental length; Special relativity; Planck length; Fundamental inertial frame; Privileged frame of reference; Lorentz-Fitzgerald contraction; Time dilation.

1 Introduction

Space is a fundamental quantity in physics, because it cannot be defined through other fundamental physical quantities. In classical physics, time is an absolute fundamental quantity. For Newton, space and time are independent and absolute entities. For Einstein, space and time are instead united in a 4D structure called space-time. According to general relativity, this space-time is continuous and relative. Concepts such as the relativity of simultaneity, length contraction, time dilation, etc., apparently make no sense and collide with human experience. On the other hand, in quantum mechanics, energy, momentum, spin, and most of the properties of matter, are discrete.

In quantum gravity models, space-time is discrete, that is, it has a fundamental length that cannot be divided into smaller ones. The discrete space hypothesis, in principle, collides with Einstein's theory of special relativity, because for an observer moving at constant speed, this fundamental length would be shorter.

Can space and time be divided into smaller and smaller units, or is there a limit? Are space and time a continuum or are they composed of indivisible discrete units? These and similar questions were raised by Greek and medieval philosophers, such as Zeno of Elea presents in the Paradox of Plurality [1] and Maimonides [2] in the Guide for the Perplexed.

2 Planck length

According to general relativity, space-time is continuous. However, there is no experimental evidence for this. We're probably convinced of continuity as a result of education. In recent years however, both mathematicians and physicists have asked if it is possible that space and time are discrete? Smolin states that space is formed from atoms of space: *"If we could probe to size scales that were small enough, would we see atoms of space, irreducible pieces of volume that cannot be broken into anything smaller?"* that he calls *"Atoms of Space and Time"* [3].

Minimum values of volume, length and area are measured in Planck units [3]. The Planck scale combines gravity (G), quantum mechanics (h) and special relativity (c) [4]. Padmanabhan shows that the Planck length provides a lower limit of length in any suitable physical [5]. "*It is impossible to construct an apparatus which will measure length scales smaller than Planck length. These effects exist even in flat space-time because of vacuum fluctuations of gravity* [6].

Planck assumed that Newton's gravitational constant, Planck's constant and the speed of light were the most important universal constants. Using a dimensional analysis, he obtained the Planck mass, length, time and energy [7, 8]. There are several theories that predict the existence of a minimum length) [9,10]. These theories are related to quantum gravity, such as string theory and double special relativity, as well as black hole physics [11-13]. "*... a fundamental (minimal) length scale naturally emerges in any quantum theory in the presence* *of gravitational effects that accounts for a limited resolution of space-time. As there is only one natural length scale we can obtain by combining gravity (G), quantum mechanics (h) and special relativity (c), this minimal length is expected to appear at the Planck scale*" [4].

Messen showed that the minimum length a, is given by the total energy of the universe E_u in a four-dimensional space, $E_u = hc / 2a$. The different excitations of space-time give rise to different particles [14]. "*... we learned already from the development of relativity and quantum mechanics that Nature can impose restrictions on our measurements because of two universal constants: the velocity c and the quantum of action h. Could Nature impose a third restriction, resulting from the existence of a universally constant quantum of length a and a universally constant quantum of time a/c?*" [15].

Haug proposes different methods of measuring the Planck length independently of the gravitational constant *G*. The Planck length is both a physical measurement and the diameter of the true fundamental particle: "*The gravitational constant is a composite (derived) constant, while the Planck length represents something physical; it is the shortest reduced Compton wavelength possible. According to recent developments in mathematical atomism, there are also strong indications that the Planck length is the diameter of the only truly fundamental particle, namely an indivisible particle that together with void is making up all matter and energy*" [16].

On the other hand, Haug, raises the hypothesis that Hesisenberg's uncertainty principle collapses on the Planck scale [17, 18]. The search for a quantum theory of gravity leads to a generalisation of the Heisenberg uncertainty principle (GUP) on the Planck scale. Adler uses Newtonian and general relativistic gravity and modifies the uncertainty principle with an additional term "*In both theories it is clear that the extra term must be proportional to the energy or momentum of the photon, so on purely dimensional grounds the order of magnitude of the extra term is uniquely determined. As a consequence there is an absolute minimum uncertainty in the position of any particle such as an electron. Not surprisingly the minimum is of order of the Planck distance. In view of the absolute minimum position uncertainty one may plausibly question whether any theory based on shorter distances, such as a space-time continuum, really makes sense*" [19]. Other authors [20, 21], also conclude that, on the Planck scale, the fluctuations are of the same order of magnitude as the distances involved.

"*We propose a GUP consistent with String Theory, Doubly Special Relativity and black hole physics, and show that this modifies all quantum mechanical Hamiltonians. When applied to an elementary particle, it implies that the space which confines it must be quantized*" [22]. The same authors solve the Klein-Gordon and Dirac equations corrected by GUP: "*We again arrive at quantization* *of box length, area and volume* and *an indication of the fundamentally grainy nature of space*" [23].

In doubly special relativity, a second parameter independent of the observer is introduced, in addition to the speed of light. It is postulated as the second invariant parameter: Planck length [24-27], mass [28, 29] or energy [30, 31].

3 Discrete space-time (DST)

One of the main objections to discrete space-time is that the existence of a discrete space-time atom is incompatible with the contraction of length and the time dilation of special relativity. However, it must be borne in mind that for lengths and times close to the Planck scale, the Pythagorean theorem is not verified. Therefore, some authors use a modified distance formula [32-35]. Specifically, Crouse and Skufca derive the relativistic phenomena of Lorentz-Fitzgerald contraction and time dilation using a modified distance formula that is appropriate for discrete spaces. They "*show that length contraction of the atom of space does not occur for any relative velocity of two reference frames. It is also shown that time dilation of the atom of time does not occur". "... It was shown that when applied to distances near the Planck scale, the new formula yields distances much different than those predicted by the Pythagorean theorem. But for larger length scales, the distances calculated with the new formula converge to those calculated using the Pythagorean theorem. When using the new distance formula in the otherwise typical derivations of time dilation and length contraction, one sees that the atom of space and atom of time are indeed immutable - true constants of nature and independent of the speed of any observer"* [36].

Quantum particles in discrete space-time are studied in relation to relativistic dynamics [37, 38]. Farrelly and Short studied the causal evolution of a single particle in discrete space-time [39]. There is evidence of discrete structures on the largest scales, for example superclusters and the redshift [40]. Cowan already said in 1969 that redshift can only occur with discrete values [41]. This was subsequently confirmed by Karlsson [42].

As early as 1930, Werner Heisenberg used discrete space-time to explain the electron's self-energy. For Werner Heisenberg, Henry Flint and Arthur Ruark, the discretisation of space-time is inherent in uncertainty relationships [1]. Interest in discrete space-time has increased in recent years due to the appearance of loop quantum gravity [43-45].

4 Ether or fundamental inertial frame

In the Michelson-Morley experiment to explain the constancy of the speed of light, Lorentz assumed that the arm of the interferometer contracts (Lorentz-Fitzgerald contraction) [46, 47] in the direction of the movement of the Earth, which also gives rise to time dilation [48] thus maintaining the absolute immobility of the ether [49], which is also the logical conclusion of the Michelson-Morley experiment.

From the time of Einstein's theory of special relativity, the ether theory was abandoned and Einstein's point of view was accepted: "*There is no room for ether in special relativity.*" During the 20th century it was taught that the Maxwell-Lorentz ether does not exist, there are only "fields" in a vacuum [50].

The reason for abandoning the Lorentz-FitzGerald hypothesis is illustrated in Einstein's words: "*The introduction of a 'luminiferous ether' will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties*" [51].
However, Einstein was convinced of the existence of the ether, even though there was no proof. Einstein introduced the concept of "new ether" in 1916, to refer to space-time, since it has physical properties. In a letter to Lorentz he says: "*I agree with you that the general theory of relativity is closer to the ether hypothesis than the special theory. This new ether theory, however, would not violate the principle of relativity, because the state of this $g_{\mu\upsilon}$= ether would not be that of rigid body in an independent state of motion, but every state of motion would be a function of position determined by material processes*" [52].

Again in 1920 writes: "*Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.*" [53].

Therefore Einstein accepts the existence of a static ether. The majority of physicists of the time (Newton, Faraday, Fizeau, Maxwell, Lorentz, Poincaré, Planck and many others) considered that ether was a real substance [50]. Dirac, Schwinger and other physicists also believe the existence of the ether is possible. Even today the idea of the ether is still valid and different studies are still being carried out [54-57].

For Isaev, 20th century physics is the physics of the ether. "*It is shown that there exists a new physical reality – the Ψ–ether. All the achievements of quantum mechanics and quantum field theory are due to the fact that both the theories include the influence of Ψ–ether on the physical processes occurring in the Universe. Physics of the XXth century was first of all the physics of Ψ–ether*" [50].

Ether is also used as a privileged frame of reference in the theory of gravitation. [58-68]. This gives rise to an alternative interpretation of special relativity, initiated by Lorentz [48] and Poincaré [69, 70].

As the result of the Michelson-Morley experiment was not completely null, but smaller than expected, Cahill and Kitto reinterpreted the experiment, taking into account the index of refraction of air. This gives rise to *"an absolute speed of the Earth of* v = 359±54 *km/s, which is in excellent agreement with the speed of* v = 365±18 *km/s determined from the dipole fitt, in 1991, to the NASA COBE satellite Cosmic Background Radiation (CBR) observations"* [71].

For Cahill, space is a quantum foam system and, in 2004, he analysed a total of seven experiments in relation to ether drift; they include the Michelson-Morley experiment and Cahill concludes that absolute motion has been detected in these experiments. "*...an analysis of date from seven experiments demonstrates that absolute motion relative to space has been observed by Michelson and Morley (1887), Miller (1925/26), Illingworth (1927), Joos (1930), Jaseja et al (1963), Torr and Kolen (1981), and by De Witte (1991)*" [72]. The speeds obtained in these experiments are in perfect agreement with the speed of the solar system obtained from the radiation of the cosmic microwave background. On the other hand, said radiation indicates that there is a privileged reference system that, in principle, is in contradiction with the theory of special relativity.

The privileged frame of reference has been used in quantum mechanics. *"In the context of modern quantum field theory we instead introduce the structured quantum vacuum, which fulfills the role that Einstein assigned to the non-material ether."* [73]. *"It is generally assumed that the physical vacuum of particle physics should be characterized by an energy momentum tensor in such a way to preserve exact Lorentz invariance. On the other hand, if the ground state were characterized by its energy-momentum vector, with zero spatial momentum and a non-zero energy, the vacuum would represent a preferred frame"* [74]. And is also used to explain the rotation curves of galaxies [75].

Finally, the characteristics of the Higgs field are reminiscent of those of the ether. Ultimately, the ether theory continues to be used today, although it is called the structured quantum field, quantum foam, fundamental inertial frame, privileged frame of reference, etc. All this clearly indicates that space-time has structure, is discrete and its length corresponds to Planck's length.

5 Gedanken experiment

5.1 Alice and Bob move away at relativistic speeds

Suppose Alice and Bob walk together at the speed of one step per second. At a given moment they decide to separate, so that their trajectories form an angle $\alpha = 90 - \varphi = 90$ - arc tag ¾ = 52.1°. Let us analyse the situation after five steps or five seconds. For Alice (Figure 1), Bob moves away at the speed of 0.8 steps per second along the x-axis, and at the speed of 0.6 steps per second along the y-axis. Therefore after five seconds Bob's coordinates will be (4,3). However, from Bob's point of view, the situation is different, since for Bob, it is Alice who moves away (Figure 2) and therefore it is Alice who has travelled four steps in the x direction, and three in the y direction.

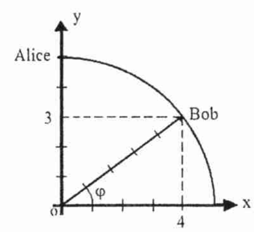

Figure 1. Alice and Bob's situation in Alice's framework

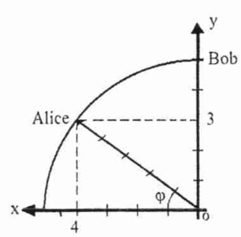

Figure 2. Alice and Bob's situation in Bob's framework.

Each observer chooses the direction of their movement as the vertical axis, so that the axes are turned at an angle of 52.1°, counter-clockwise. If we now change the speed of one step per second, by the speed of light in vacuum c, we have a symmetrical situation, equivalent to the paradox of the twins. On the y-axis we have seconds and, on the x-axis, light seconds. In Alice's frame (Figure 1), Bob moves away for five (t_A) seconds at the speed 0.8c, so Bob has moved away four light seconds ($x_0 = v\ t_A = 0.8c\ 5$), while the clock Bob will score:

$$t_B = t_A\sqrt{1 - v^2/c^2} = 3 \text{ seconds} \qquad (1)$$

However, in Bob's frame (Figure 2), Alice is the one that has drifted away for five seconds at speed of 0.8c, so Alice is within four light seconds of Bob and her watch will tick three seconds. Therefore, in Alice's frame of reference, Bob's clock runs slow, while in Bob's frame of reference it is Alice's clock that runs slow. Furthermore, special relativity implies that, since Alice takes three seconds (t_B) for Bob, it turns out that Bob has travelled a contracted Lorentz distance of:

$$x = x_0\sqrt{1-v^2/c^2} = vt_A\sqrt{1-v^2/c^2} = vt_B = v_x t_y = 2.4 \ light \ seconds \qquad (2)$$

The velocity v is in the x (v_x) direction, while Bob's time (t_B), in Alice's frame, is in the y direction (t_y). Obviously on the xy plane, the above equation is not applicable and neither on the xt plane, so there is no distance contraction.
In Alice's frame, Alice moves five seconds on the vertical axis t, and Bob moves five seconds in space-time (xt plane). In Bob's frame of reference, he moves five seconds on the vertical axis t, and Alice moves the same time but in the xt plane.

The twin paradox is resolved by keeping in mind that the receding twin has to accelerate, decelerate, and turn around, so it cannot say that it is at rest in its frame of reference. "*...the accelerated twin cannot say that he is at rest because the gravitational field he experiences has no source. It is an ad hoc gravitational field introduced into the description when we say that twin A is at rest and B travels*" [76].

5.2 Alice and Bob approach at relativistic speeds

Now suppose Alice and Bob are approaching with a speed of 0.8c. Bob has a chronometer that will start when both intersect, while Alice has made an isosceles triangle using 3 mirrors, as shown in Figure 3.

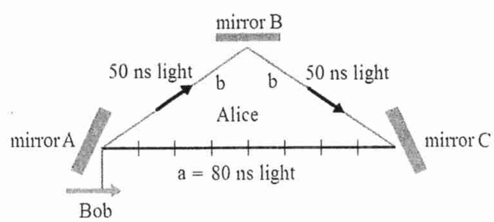

Figure 3. Start of timing according to Alice.

Alice´s frame. When Alice and Bob meet, Bob sends a photon or pulse of light to Mirror A, while starting the stopwatch. The time Bob takes to travel the distance between mirrors A and C will be:

$$t_a = \frac{a}{v} = \frac{a}{0.8c} = 100 \ ns \qquad (3)$$

We choose segments b, so that the photon that hits mirror A and that is reflected in mirror B, reaches mirror C at the same time that Bob, results:

$$t_a = \frac{2b}{c} \Rightarrow b = 50 \ ns \qquad (4)$$

this way, the photon and Bob reach mirror C at the same time, therefore, the photon will stop the stopwatch (Figure 4). Obviously, the distance between Bob's stopwatch and mirror A or C must be negligible compared to segment b or included in said segment.

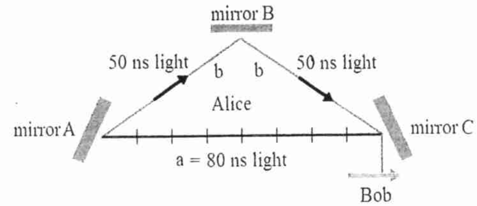

Figure 4. End of timing according to Alice. Stopwatch stopped.

Bob's frame. Like before, Bob has a chronometer that will start when both intersect (Figure 5).

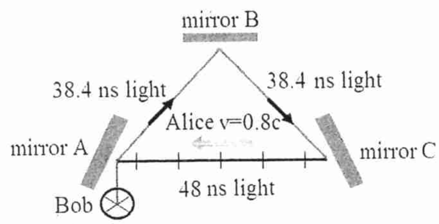

Figure 5. Start of timing according to Bob.

In this case, Alice gets closer to Bob and therefore the mirror triangle will contract in the direction of movement, according to special relativity. In this way the distance Bob has to travel is:

$$a' = a\sqrt{1 - v^2/c^2} = 48 \ ns \ luz \qquad (5)$$

and it will take a time:

$$t_b = \frac{a'}{v} = t_a\sqrt{1 - v^2/c^2} = 60 \ ns \qquad (6)$$

Since the height of the triangle does not contract, its value will be the same in any reference frame, then:

$$h = \sqrt{b^2 + (a/2)^2} = 30 \ light - ns \qquad (7)$$

Instead, the distance the photon must travel is:

$$d = 2b' = 2\sqrt{(a'/2)^2 + h^2} = 76.8 \ light - ns \qquad (8)$$

Therefore, the light pulse will reach mirror C after Bob has passed (Figure 6) and consequently the stopwatch will not stop.

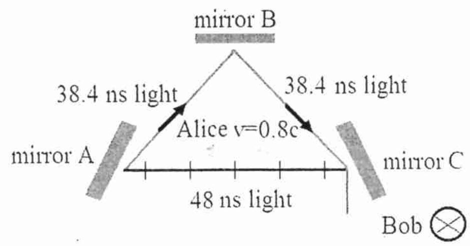

Figure 6. End of timing according to Bob. Stopwatch running.

In short, depending on the frame of reference, the stopwatch will be stopped or will continue to run. It is a thought experiment that can be studied by speeding up Alice, speeding up Bob, or both. In any case, it is a paradox, since the watch cannot be both stopped and working at the same time.

6 Lorentz contraction

For Lorentz, Poincare, Amelino-Camelia and other authors, what contracts is the object, not the space. *"..... according to FitzGerald-Lorentz length contraction, different inertial observers would attribute different values to the same physical length. The idea that the Planck length should play a truly fundamentall role in the structure of space-time appears to be in conflict with the combined implications of the Relativity Principle and Fitgerald-Lorentz length contraction"..."The Planck length could play a similar role in fundamental physics, i.e. it could reflect the properties of a background, but then the presence of such a background would allow to single out a "preferred" class of inertial frames for the description of the short-distance structure of space-time"* [24].

Therefore, it is necessary to somehow justify this contraction of the object, for which the structure of the elements that make up the atoms must be known. It is enough, for this, to focus on the hydrogen atom, since from it all the others are obtained.

6.1 Contraction of the particles

The hypothesis is that the universe is made up of four-dimensional (4D) space spheres whose diameter is Planck's length $l_p = \sqrt{G\hbar / c^3}$. Each of the spheres has two possible states, state at rest and movement of rotation. Rest spheres are empty space, and the rotational motion of the spheres gives rise to different properties of the particles. Of the four dimensions, three are observed as space (x, y, z) and the fourth (*u*=*ct*) spatial dimension is observed as time. Planck's four-dimensional spheres are atoms of space and time that Smolin comments [3].

In addition the 4D Planck sphere has two rotations, one in three-dimensional space and one in the fourth dimension. Rotation in the fourth dimension (ω_u) rotates the u-axis and another spatial axis around any two axes. For example, the u and y axes spinning around the x and z axes. In the rotation in space (ω_e) it is rotated around the u-axis and another spatial axis. For example, the x and z axes spinning around the u and y axes.

Each Planck 4D sphere can rotate both in 3D space and in the fourth dimension (u = ct, Figure 7), resulting in the following possible combinations [77-81]:

- zero rotations (vacuum space);
- one spatial rotation, ω_e (photons);
- one rotation in the fourth dimension, ω_u (neutrinos);
- two rotations i.e. one spatial rotation, ω_e, and one rotation in the fourth dimension, ω_u (first-generation electrons and quarks).

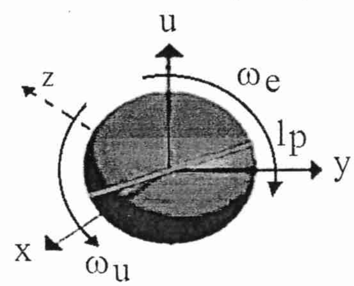

Figure 7. Rotations of a 4D Planck sphere

Static spatial spheres are not observed; it is what we call empty space. We can observe the spheres that rotate on themselves as elementary particles, such as electrons, photons and the first generation of quarks and neutrinos. The energy of rotation in the fourth dimension gives rise to the mass at rest and the period of rotation in the fourth dimension gives rise to the electric charge.

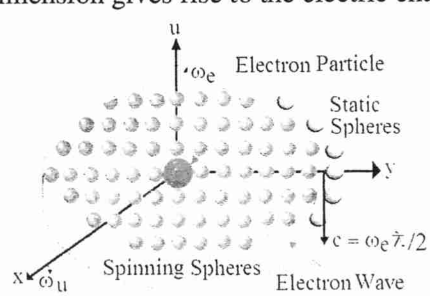

Figure 8. 2D representation of an electron

The 4D Planck spheres are linked by Planck's force, so that spinning one of them will drag it to adjacent spheres. The linear velocity of rotation (Figure 8) will increase as we move away from the rotating sphere, until the speed of light c is reached at a distance r, then

$$v = \omega_u l_p \quad (9)$$

$$c = \omega_u r \quad (10)$$

The resting mass of the particle is due to the energy of the rotation [77, 79-83].

$$E = mc^2 = \frac{1}{2}\hbar\omega_e = \hbar\omega_u = \frac{\hbar c}{\lambda\!\!\!^-} \quad (11)$$

Where the reduced Compton wavelength $\lambda\!\!\!^-$, is the diameter of the particle $c = \omega_e \lambda\!\!\!^- / 2$. The mass can also be considered as the space in the fourth dimension of the 4D Planck sphere, projected onto the 3D sphere that we observe as a particle [81].

The equation that determines the characteristics of the particle is $c = \omega r$. An equation with two unknowns ω and r. In the same way that a skater increases his rotation by shrinking his arms, particles increase their rotation with increasing energy and at the same time decrease their radius r. Therefore, the value of the mass is not determined except for the maximum (Planck mass) and minimum (mass at rest) values [79, 81, 83].

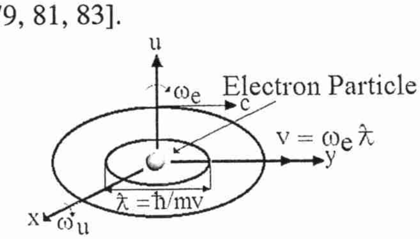

Figure 9. Rotation and displacement velocity of the electron

The rotation ω_e in the space of the Planck 4D sphere determines the speed of translation of the particle in 3D space; the greater the energy, the greater the speed. Speed corresponding to the De Broglie wavelength.

$$\lambda = h / mv \quad (12)$$

Where $v = \omega_e \lambda / 2\pi$ is the speed of displacement in 3D space, and the wavelength λ, is the distance that the particle travels while to rotate a complete round.

The rotation ω_e is the one that generates the De Broglie wavelength. The rotation ω_u is what generates the Compton wavelength. Since both rotations are perpendicular, their moments also will be (Figure 10).

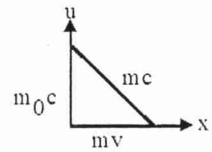

Figure 10. Particle momentums

$$mc = \sqrt{(mv)^2 + (m_0c)^2} \qquad (13)$$

From where

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \qquad (14)$$

At small speeds, m and m_0 coincide, but at relativistic speeds the mass m increases due to the increase in the rotation energy.

The electron is a Planck sphere in the minimum energy state (m_0). The maximum energy will be the Planck energy ($m_p c^2$). This gives us a maximum speed for the electron.

Haug [16, 84] has suggested that there is a maximum velocity for any particle with a rest mass, as given by the previous equation para m igual a la masa de Planck. Haug has calculated this maximum speed with 50 decimal places [18].

4D Planck spheres are always spheres regardless of the movement of the observer. As energy increases, the rotation of the particles increases and therefore the size decreases. The relativistic effects are due to the contraction of physical objects as they move through 4D space. 4D space can be considered a fundamental frame of reference or "ether" according to the alternative interpretation of the special relativity of Lorentz [48], Poincaré [69, 70] and others [58-60]. "*The "relativistic" effects, which essentially follow from the Lorentz transformation, are all due to the "true" Lorentz contraction of physical objects as they are moving through the "ether" or fundamental inertial frame*" [66].

6. 2 Contraction of atoms

Each atom is made up of a nucleus and one or more electrons rotating around the nucleus. In turn, the nucleus is made up of protons and neutrons that are called nucleons. Nucleons are made up of a triad of up and down quarks with positive and negative electric charges, respectively.

While the mass is the energy of the rotation of the fourth dimension, the electric charge is the period of that rotation. Therefore the mass and the electric charge are related by the Planck constant.

$$E = mc^2 = \frac{2\pi^2 h}{q} \qquad (15)$$

Obviously, in the previous equation, the electric charge is in seconds. Just multiply by an ampere to have the electric charge in coulombs, an arbitrary unit of electrical charge.

Rotation ω_u generates the electric charge, and rotation ω_e of the electric charge generates a magnetic field that will have two components: a spatial component and another in the temporal or fourth dimension direction. The spatial component originates the anomalous magnetic moment of the electron [79, 81] while the component in the temporal direction causes the electrons to attract each other. The union or collision of three electrons gives rise to the quark down, which will have a rotation equal to three times the rotation of the electron and therefore its charge will be 1/3 of the electric charge of the electron. In the same way, the shock of three positrons gives rise to two quarks up, which will have a rotation equal to 3/2 of the rotation of the positron and therefore its charge will be 2/3 of the electrical charge of the positron [79, 82].

The energy of the rotation of the quarks, originates the mass of the quarks. By joining three quarks, a new rotation is generated, that originates the mass of the protons and neutrons that make up the nucleus [79]. Therefore, as the energy of the object increases, the nuclei that make up the atoms contract, as a consequence of the increased rotation of the constituent quarks.

In addition to the nucleus, we have electrons. Next, let's look at the electron in the hydrogen atom. In 1913, Bohr drew the hydrogen atom with a proton in the nucleus and an electron that spins in circle orbits around the nucleus (Figure 11). In a circle orbit, the electrostatic force of attraction (F_e) is equal to the centripetal force (F_C), then:

$$K\frac{q^2}{r^2} = \frac{mv^2}{r} \qquad (16)$$

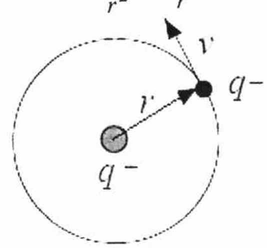

Figure 11. Bohr atomic model

If we consider that the speed of the electron in the free state is $v = \alpha c$ [83], it results in:

$$K\frac{q^2}{v} = mvr = \hbar \qquad (17)$$

Then the electron has a minimum angular momentum equal to $\hbar$. The total energy results:

$$E = \frac{1}{2}mv^2 - K\frac{q^2}{r} = \frac{p^2}{2m} - K\frac{q^2}{\hbar}p \qquad (18)$$

The system will be stable in the state of minimum energy, thus annulling the first derivate:

$$\frac{dE}{dp} = \frac{p}{m} - K\frac{q^2}{\hbar} = 0 \quad \rightarrow \quad p = K\frac{q^2}{\hbar}m \qquad (19)$$

Therefore, in the hydrogen atom the electron is in the free state, with minimal energy and with a minimum moment equal to $\hbar$, so it cannot radiate energy.

In addition, it must be taken into account that Coulomb's law is only valid for charges at rest, so the effect of speed must be taken into account. That makes the orbit that the electron describes open, so it moves around a spherical surface of radius:

$$r = \frac{\hbar}{mv} = a_0 \qquad (20)$$

Which is the Bohr atomic radius.

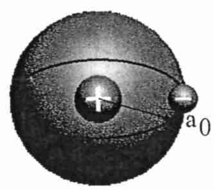

Figure 12. Electron orbital sphere

The electron moves at all times over a_0 radius sphere (figure 12) until it is observed, as the electron absorbs the energy of the observation and then modifies its angular momentum, still, the Heisenberg's uncertainty principle is always verified in a way that the quotient between the energy it has and the acquired energy as a result of the observation is the wave function. From that function it is easy to deduce the Schrodinger equation [78, 79].

Einstein was right when he said: *"I think that a particle must have a separate reality independent of the measurements. That is an electron has spin, location and so forth even when it is not being measured. I like to think that the moon is there even if I am not looking at it"* [85].

The electron, in the hydrogen atom, rotates around the nucleus with a minimum energy and a minimum angular momentum $\hbar$. In any other atom, the energy of the electron will be greater and the angular momentum will be greater than or equal to ħ. Therefore, by applying energy to the object, the electrons will increase their momentum mv, bringing them closer to the nucleus due to the principle of conservation of angular momentum. If mv increases r decreases ($mvr = n\hbar$).

Therefore, the movement relative to the ether affects the nucleus and the distance of the electrons from the nucleus, causing the contraction of the object. Or in FitzGerald's words: "*We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently*" [47]. The contraction of the object is due to the decrease in the number of 4D Planck spheres in the core, it does not change the size of the spheres. It also decreases the number of 4D Planck spheres between the nucleus and the electrons.

Obviously, in the paradox of the isosceles triangle of mirrors, the contraction of the object undoes the paradox, because if the sides of the triangle are chosen properly (equation (4)), the chronometer stops in both frames of reference.

Ultimately, due to the movement relative to Planck's 4D spheres that make up empty space, the particles increase their rotation and consequently the object contracts. Planck's 4D spheres constitute a privileged frame of reference or "ether" that remains motionless. The size of these spheres depends on the Planck length and the speed of light, which are the two constants used in the doubly special theory. Planck's 4D spheres originate the space-time structure proposed by Amelino-Camelia. *"I propose a general class of space-times whose structure is governed by observer independent scales of both velocity (c) and length (Planck length), and I observe that these space-times can naturally host a modification of FitzGerald-Lorentz contraction such that lengths which in their inertial rest frame are bigger than a "minimum length" are also bigger than the minimum length in all other inertial frames"* [24].

7 Time dilation

The time dilation has been verified on many occasions: in airplanes [85, 86] and in satellites of global positioning systems [87]. Even today, experiments are carried out to increase the precision of the measurements made [88, 89] and recently by an international group of physicists by accelerating lithium ions used as a clock in motion [90].

When we apply energy to the electron, its wavelength decreases. In the same way, when applying energy to an atom, the distances of the different energy levels decrease and therefore the frequency of each transition increase.
Combining equations (11) and (14)

$$E = \frac{hc}{\lambda} = \hbar\omega = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \frac{hc}{\lambda_0\sqrt{1 - v^2/c^2}} = \frac{\hbar\omega_0}{\sqrt{1 - v^2/c^2}} \quad (21)$$

Therefore, by decreasing the period of the transition, the time for the same number of transitions will decrease. Hence, the atomic clock in motion indicates a shorter time than the atomic clock at rest. The equation (21), can be put in function of the period of rotation.

$$t = t_0\sqrt{1 - v^2/c^2} \quad (22)$$

On the other hand, in the International System the second is defined as the duration of 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom. The two hyperfine levels of the Cesium atom get closer, it is not the space that contracts and pulls them. This approach results in shorter transitions. The size of the atoms is reduced, due to conservation of angular momentum, as a consequence of increasing the momentum *mv*, as the speed increases with respect to Planck's 4D spheres, which constitute empty space.

We need the concept of inertial system to be able to know which object moves (twin paradox). However, the particles know perfectly what particle is moving, because their energy in space ($\hbar\omega$) is greater than their energy at rest ($\hbar\omega_0$). Therefore the period t, which corresponds to the rotation ω_u does not vary with the speed. What varies is the time that the particle moves in space (Figure 13).

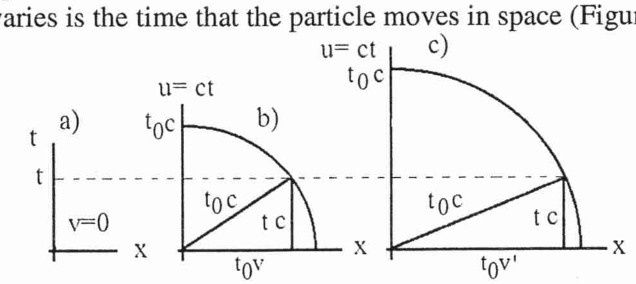

Figure 13. a) observer at rest. b) observer in motion at speed v. c) observer in motion at the speed v'> v

8 Expansion of the universe

8.1 Flat space-time

The equation that determines the characteristics of the particle is $c = \omega r$, it is the same that determines the expansion of the universe r (x, y, z) = c t. As space expands, the fourth dimension $u = c\,t$ expands.

Since the fourth dimension u is perpendicular to the other three, it is.

$$r^2(x, y, z) + u^2 = 2c^2t^2 \quad (23)$$

Deriving twice with respect to t, it results:..

$$r\frac{dr}{dt}+u\frac{du}{dt}=2c^2t \qquad (24)$$

$$\left(\frac{dr}{dt}\right)^2+r\frac{d^2r}{dt^2}+\left(\frac{du}{dt}\right)^2+u\frac{d^2u}{dt^2}=2c^2 \qquad (25)$$

What we can put in the form:

$$\left(\frac{dr}{dt}\right)^2+\left(\frac{du}{dt}\right)^2=c^2 \qquad (26)$$

$$r\frac{d^2r}{dt^2}+u\frac{d^2u}{dt^2}=c^2 \qquad (27)$$

Equation (26) can be put:

$$v^2+v_u^2=c^2 \qquad (28)$$

Being v the speed in space and v_u the speed in the fourth dimension. That is, the vector sum of the speed in space and in the fourth dimension is constant and equal to the speed of light. Therefore, the movement in space-time is reduced to a constant movement at the speed of light c in the xt or ru plane, as seen in section 5. At rest (v = 0) we move in the fourth dimension at the speed of ligh.

Clearing the speed in the fourth dimension, it turns out:

$$v_u=\sqrt{c^2-v^2} \qquad (29)$$

Dividing by c and multiplying by t, we obtain:

$$t'=\frac{v_u}{c}t=t\sqrt{1-v^2/c^2} \qquad (30)$$

The equation above is the relativistic formula of time dilation. Furthermore, the previous formula indicates that we can consider time as two-dimensional. There is a time in space (t), which we observe as space, and there is a time in the fourth dimension (t ') that we observe and measure as time. The vector sum of both times is constant and independent of the speed of the observer.

The expansion of the universe implies that the entire universe moves at the speed of light, so that when applying energy to an object, to move it with respect to another, it changes the direction of its movement. This change in the direction of movement is what produces the contraction of the moving object and the time dilation.

8.2 Curved space-time

We assume that the universe is a 4D hypersphere, formed by Planck's 4D spheres, which expands at the speed of light, resulting:

$$r^2(x,y,z)+u^2=R^2=c^2t^2 \qquad (31)$$

Deriving twice with respect to t, it results

$$\left(\frac{dr}{dt}\right)^2 + r\frac{d^2r}{dt^2} + \left(\frac{du}{dt}\right)^2 + u\frac{d^2u}{dt^2} = c^2 \qquad (32)$$

What we can put in the form:

$$\left(\frac{dr}{dt}\right)^2 + \left(\frac{du}{dt}\right)^2 = c^2 \qquad (33)$$

$$r\frac{d^2r}{dt^2} + u\frac{d^2u}{dt^2} = 0 \qquad (34)$$

As before, we obtain, that the movement in space at speed v, is reduced to a movement in the xt or ru plane at constant speed equal to the speed of light c. In any case, what cannot be done is multiply the time ($t\,' = t_u$) elapsed at speed v_u by the speed in space v, since both speeds are perpendicular, except for $v << c$. This would be equivalent to calculating the distance traveled in the x direction, in the form $x = t$ and v_x, instead of $x = t\ v_x$, when an object moves at constant speed v, in the xy plane, during time t.

9 Conclusion

The cosmic microwave background indicates that there is a privileged reference system. This privileged system, according to the hypothesis of this paper, must be formed by Planck 4D spheres that remain immobile and whose size is independent of the speed of the observer. Under this hypothesis, space does not contract, the object formed by Planck's 4D spheres contracts, so that as the energy applied to the object increases, it reduces its size with increasing frequency of transition between the two levels hyperfines of the cesium atom. This results in a temporary dilation of the moving watch.

The twin paradox, in today's physics, is solved by claiming that there is no symmetry because the travelling twin has to accelerate and is therefore not an inertial system. However, acceleration causes a change in velocity in the travelling twin, and according to this study, that increased velocity causes object contraction and temporal dilation.

It seems that the old idea that something should be at absolute rest is correct.

Note Added in Proof

Since this communication was submitted, I have had knowledge of an article published by Professor Santilli in 1956. For Santilli, space must be a solid and incompressible medium. Being also the means of transmission of waves and forces. Matter is a dynamic modification of space.

"Space, that must transmit waves and forces, must be full, and matter, which must be a dynamic state of this space – because it interferes and generates forces – must be 'empty in relation to common concepts'. If we could stop all its

movements for a moment, matter would disappear completely, as it actually does, whenever corpuscular radiation interferes" [91].

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