HADRONIC JOURNAL 43, 363 - 461 (2020)
DOI 10.29083/HJ.43.04.2020

UNIVERSE AND TIME-REVERSAL ANTI-UNIVERSE AS ETERNAL CYCLE OF EVOLUTION WITH PHOTON REST ENERGY, HUBBLE "CONSTANTS" AND TIME-DEPENDENT COSMOLOGICAL "CONSTANT"

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Received March 25, 2020

Abstraet

In this work, we give a detailed review of the transition from the final state of the universe and the time-reversal anti-universe in the direction to the state of the big bang as a start of an eternal cyclic evolution, i.e. these two states are related to one another, so that we can estimate their parameters and the lifetime of the sterile neutrinos. The beginning of the universe and the anti-universe results from two equivalent energy uncertainties via one quantum fluctuation of the vacuum (origin), so that they expand in the opposite time directions. These Euclidian universes are based on the zero-point oscillations. The dark matter as well as the dark energy are described via the massive sterile breakup neutrinos as well as the massless sterile neutrino decay and breakup products, respectively. We estimate the rest energy of the photons, so that we can derive the quantum mechanical zero-point velocity. This rest energy of the photons, which must be identical with the rest energy of the gravitons, is confirmed by the intergalactic magnetic field. In the framework of the ΛCDM model, we derive various Hubble "constants" for the different epochs of evolution of the universes in excellent agreement with the most recent observations in 2019. Thus, in future, we observe a slow linear expansion of the universes instead of their accelerated expansion, i.e. their accelerated expansion is decelerated. Via a time-dependent cosmological "constant", we solve generally the discrepancy between the vacuum energy of the Planck scale and the present dark energy as a continuous transition in accordance with the quantum field theory. The same solution is obtained from the big bang to the final state of the universe, using again the Planck scale.

1 Introduction

According to Ref. [1, 2], we consider a total {"massless" ($0 \to R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) and massive ($R > R_{\mathrm{Pl}}$)} universe, where $R_{\mathrm{BB}} << R_{\mathrm{Pl}}$ and $R \leq R_{\mathrm{Pl}}$ describe the radius distance of the big bang (BB) and of the "massless" universe, whereas R_{Pl} and $R > R_{\mathrm{Pl}}$ are the Planck distance and the known scale factor, respectively. The denotation "massless" universe, we have applied because it contains mainly massless and nearly massless particles. It is determined by the gravitation [1, 2], whereas the massive universe is defined by a new inflation model [1-5] and the Friedmann-Lemaitre Equations [1-12]. In Refs. [2-5], we have shown that these universes have a Euclidian geometry.

In work [2], for the total ("massless" and massive) anti-universe, all assumptions derived from the known properties of the antiparticle, are completely correct. Unfortunately, in contrast to work [2], the work [1] gives a negative scale factor $-R$ as one incorrect assumption instead of the correct positive scale factor R for the anti-universe. Thus, we define once more the correct assumptions for the anti-universe in this work with starting point, universe, (see Sec. 3.10). We propose that the total ("massless" and massive)) anti-universe had its existence in the past, i.e. for a negative time direction $-\infty \leftarrow -t \leftarrow -t_{\mathrm{BB}} \leftarrow 0$. Thus, for the anti-universe, because of its negative time direction (from the big bang at $-t_{\mathrm{BB}}$ to the past), the Friedmann-Lemaitre Equations provide an expanding anti-universe with the scale factors greater than zero as well as the negative velocities $-c \leq -v \leq 0$ (see Table IV). For the universe, we have the positive time direction (from the origin (big bang at $+t_{\mathrm{BB}}$) to the future), so that the Friedmann-Lemaitre Equations yield an expanding universe with scale factors greater than zero as well as positive velocities $c \geq v > 0$. Thus, the Friedmann-Lemaitre Equations give a time reversal solution, in which anti-universe and universe result from two equivalent energy uncertainties via the uncertainty relation by one quantum fluctuation of the vacuum (origin).

In the works [1-4], we have uniquely estimated the rest energy of the light neutrinos and the supersymmetric grand unification particles. The works [1-4] show clearly that the massive universe (and anti-universe) are completely described by the (present), cosmological parameter values [1-12], which were also exactly estimated in Refs. [1, 2] via the light (anti)neutrino density parameters [1-5], multiplied by the different ratios of the relativistic energy and the rest energy of the supersymmetric grand unification (anti)particles [1-4]. On this way, it is also possible to calculate the rest energy and number density of the heavy and the sterile neutrinos [1-4], using the astronomical unit changing [1-5] and observed data of one sterile neutrino (as long-sought dark matter particle candidate [1-4, 13]).

At $R \leq R_{\mathrm{Pl}}$, for the "massless" universe and anti-universe, we must use the quantum cosmology or the quantum gravity. However, unfortunately, to this day, this quantum gravity is still highly incomplete and yields therefore no reliable predictions because the connection between the cosmological "constant" and the vacuum energy density is not clear [6]. Therefore, in the works [1, 2], for the "massless" universes, by aid of the virtual matter of the quantum vacuum, we have derived a simple solution for the long-sought four-dimensional quantum gravity. In works [1-3], we have derived generally the vacuum energy density for the total ("massless" and massive) universe and anti-universe, so that we solve generally the problem of the connection between the vacuum energy density and the cosmological "constant [1, 2]. These universes exist by the zero-point oscillations [1].

Thus, this four-dimensional quantum gravity, which is transferable to the massive universes, permits the determination of the parameters of the big bang and the evaluation of the lifetime of the sterile neutrinos via the transition from the final state of the universes in the direction to the big bang [1, 2]. Thus, we obtain a cyclic evolution of the total ("massless" and massive) universes as a result of the dark energy converted into photons and neutrino relics by the complete decay of the sterile neutrinos via the gravitation [1, 2]. Thus, this decay process must lead to an overheat of the final state of the universes because of the particle accumulation by deceleration, so that their heat death takes place [1]. Therefore, from these final states, we find to the "massless" universes and the big bang [1].

Using the expansion in opposite time directions from the big bang, the momentary beginning of the formation of the "massless" universe and anti-universe leads to a separation of the virtual particle-antiparticle pairs of the quantum vacuum into real particle pairs (matter) in the "massless" universe and antiparticle pairs (antimatter) in the "massless" anti-universe via the gravitational interaction. These separated particles and antiparticles are in a new thermal equilibrium with the photons, so that at the Planck length all real particles and antiparticles have the Planck energy $kT_{\mathrm{Pl}} = E_{\mathrm{Pl}}$ as relativistic energy for their start in the correspondingly separated massive universe and anti-universe [1, 2].

Consequently, at $R = R_{\mathrm{Pl}}$, all separated particles (universe) or antiparticles (anti-universe) have the Planck energy as relativistic energy, i.e. the non-separated particles and antiparticles can be formed only by interactions between the separated particles or antiparticles, so that these non-separated particles and antiparticles disappear again by annihilation. These interactions are determined by the known laws of nuclear physics or elementary particle physics. Because the statistical particle weights were derived by aid of these laws, the new thermal equilibrium has no influence on these statistical weights [1, 2]. In the old interpretation, where for the universe the non-separated particles and antiparticles are produced simultaneously by photons with the thermal energy of the sum of their rest energy in this old thermal equilibrium, so that this interpretation leads to difficulties at the explanation of the separation of matter and antimatter [1, 2].

In this work, for a better intelligibility, we give firstly a detailed review for the transition from the final state of the universes in the direction "big bang" as a start of an eternal cyclic evolution, so that these two states are bound with each other. The reason of this final state is a mean negative acceleration [1-5] interpreted as Pioneer anomaly [1-5, 14, 15]. We confirm the parameters of the big bang and the lifetime of the sterile neutrinos, so that we can corroborate the eternal cyclic evolution of the universes, whereat we derive a time reversal solution for anti-universe and universe, in which they begin their existence via two equivalent energy uncertainties by one quantum fluctuation of the vacuum. Secondly, we describe the dark matter and dark energy by the special properties of the sterile neutrinos. Thirdly, on the basis of all these results, we propose a solution of following important problems.

The existence of the new thermal equilibrium is supported by the light neutrinos.

We estimate the rest energy of the photons and the gravitons in accordance with their limiting values [16]. The existence of the rest energy of the photons, which is probably identical with the rest energy of the gravitons, is confirmed by the measured general galactic magnetic field [9, 17] and the theoretical magnetic neutrino moment [1-5, 10, 18].

Theoretical data, derived via the rest energy of the photons, permit far-reaching conclusions about the Hubble "constants". The Hubble expansion rate $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, which in Ref. [7] was assumed as the present Hubble "constant" of the universe, is interpreted in this work as the present "Hubble constant" of the cosmic microwave background (CMB), since it was derived by the Planck observations [12] from the measurement of the cosmic back ground radiation, which was formed at $3.72\cdot 10^5$ years after the big bang [7], so that this new Hubble "constant" $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ can be used further as basis for all hitherto existing considerations for the evolution of the universe, i.e. it must also determine all Hubble "constants" of the universe (see Sec. 7).

Thus, in the ΛCDM model, we assume that this present CMB value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ must also yield a new Hubble constant for the beginning of the "present" accelerated (acc) expansion of the universe, where we expect this new Hubble "constant" with a larger value $H_{\mathrm{acc}} > H_0$. After the accelerated expansion, we have a slow linear (lin) expansion [1, 2] with the new Hubble "constant" $H_{\mathrm{lin}} << H_0$, derived by aid of the astronomical unit changing [1-5, 19]. The accelerated expansion is decelerated, so that presently we must observe the present Hubble "constant" $H_{\mathrm{acc,0}}$ with the condition $H_{\mathrm{acc}} > H_{\mathrm{acc,0}} > H_0$. The calculated value $H_{\mathrm{acc,0}}$ is excellently confirmed by the most recent observations of Riess [20] in 2019.

Via time-dependent vacuum energy densities or cosmological "constants" (see, e.g., Ref. [21]), we solve generally the discrepancy between the vacuum energy of the Planck scale and the present dark energy as a continuous transition in accordance with the quantum field theory. The same solution is obtained from the big bang to the final state of the universe, using again the Planck scale. We demonstrate this behaviour from the big bang to the Planck scale as well as from Planck scale to the final state of the massive universes via the X, Y gauge bosons [1-5], the Higgs boson and the electron.

Thus, this work is organized as follows. In Sec. 2, we summarize the necessary equations and parameters. In Sec. 3, for a better intelligibility of this work, we give a detailed review for the universe and the time-reversal anti-universe as an eternal cycle of evolution, whereat we derive the transition from the final state of the massive universes in the direction to the big bang of the massless universe (Sec. 3.1), the lifetime of the sterile neutrinos (Sec. 3.2), the constant volume of the sterile neutrinos (Sec. 3.3), the heat accumulation in the final state of the universe as a result of the very high energy density (Sec. 3.4), the mean (maximum) energy of the "massless" and massive universe (Sec. (3.5), the zero-point oscillations as an existence form of the massless universes (Sec. 3.6), the particle horizon and the zero-point oscillations for the early and late massive universe (Sec. 3.7), the greatest possible gravitational energy and the hypothetical superforce of the particle interactions (Sec. 3.8), the very high temperature of the big bang (Sec. 3.9), the time reversal solution for anti-universe and universe (Sec. 3.10) as well as the eternal cyclic evolution for anti-universe and universe (Sec. 3.11). In Sec. 4, we explain the present dark matter and dark energy via the special properties of the sterile neutrinos. In Sec. 5, we give a reasonable argument for the new thermal equilibrium via the light neutrinos. In Sec. 6, we estimate the rest energy of photons and gravitons including conclusions. In Sec. 7, we calculate the different Hubble "constants" as a function of the cosmic evolution epochs. In Sec. 8, the time dependence of the cosmological "constant" is derived. In Sec. 9, we give a short summary. The values of physical constants, used in this work, are given in Refs. [7, 10].

2 The necessary equations and parameters

In Refs. [2-5], we have generally derived $\mathrm{k}=0$ by $\Omega_{\mathrm{tot}}=1$ and $\Omega_{\mathrm{tot}}(z)=1$, so that in the $\Lambda\mathrm{CDM}$ model the Friedmann-Lemaitre Equations are given by

$$H^2=\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G_N}{3}\rho+\frac{c^2}{3}\Lambda \tag{2.1}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda, \tag{2.2}$$

where $H = H(t)$ is the Hubble parameter, $R = R(t)$ is the scale factor, G_N is the gravitational constant, ρ is the mean mass density, Λ is the cosmological constant, P is the isotropic pressure and c is the speed of light in vacuum.

It is usual to introduce the mass density [1-3, 5-8, 10]:

$$\rho = \rho_\Lambda + \rho_\mathrm{m} + \rho_\mathrm{r} = \left[\Omega_\Lambda + \Omega_\mathrm{m}(R_0/R)^3 + \Omega_\mathrm{r}(R_0/R)^4\right]\rho_{0\mathrm{C}} =$$

$$= \left[\Omega_\Lambda + \Omega_\mathrm{m}(1+z)^3 + \Omega_\mathrm{r}(1+z)^4\right]\frac{3H_0^2}{8\pi G_N}, \tag{2.3}$$

where according to Table I $\rho_{0\mathrm{C}}$ is the present, critical density

$$\rho_{0\mathrm{C}} = \frac{3H_0^2}{8\pi G_N}. \tag{2.4}$$

The radiation density parameter Ω_r is defined by

$$\Omega_\mathrm{r} = \frac{1}{2}N(T)\,\Omega_\gamma, \tag{2.5}$$

where the statistical particle weights $N(T)$ are given in Refs. [1-3, 10]. For example, in Eq. (2.8), at temperatures $T << 0.5\,\mathrm{MeV}$, $N(T)$ has the value

$$N(T) = 3.362644 \quad \text{(see also Sec. 8)}, \tag{2.6}$$

since the only relativistic species are photons at temperature T as well as massless or nearly massless neutrinos and antineutrinos of three different types at the temperature $T_\nu = (4/11)^{1/3}T$ because of the entropy conservation [1-3, 6, 8], i.e. $N(T)$ of Eq. (2.5) has the value 3.362644 [6, 8] at the present CMB temperature $T = T_0 = 2.7255\,\mathrm{K}$ (see Table I) and increases at higher temperatures (see Refs. [1-3, 10]). Thus, the present, massive universe contains a relic background of light neutrinos [6] with the temperature

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.9454\,\mathrm{K}. \tag{2.7}$$

The radiation density parameter Ω_γ [1-3, 5-8, 10] is defined by

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0C}} = 5.46 \cdot 10^{-5} \tag{2.8}$$

with the radiation density [1-3, 5-8, 10, 11]:

$$\rho_0(\gamma) = \{\pi^2/15\,(\hbar^3 c^5)\}\,(kT_0)^4\,, \tag{2.9}$$

where $\hbar$ and k describe the reduced Planck constant and the Boltzmann constant, respectively.

In an expanding universe, the observed wavelength λ of the light emitted (e) from a distant source with λ_e, is shifted towards the red. Via the Friedmann-Robertson-Walker metric [1-3, 5-8, 10, 11], this so-called redshift z is defined for these photons by

$$1 + z = \frac{\lambda}{\lambda_e} = \frac{R_0}{R} = \frac{T}{T_0}\,, \tag{2.10}$$

where R_0 and R are the scale factors of absorption and emission of light, respectively. Then, by Eqs. (2.3) and (2.4), the relativistic formula (2.1) yields

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_m\left(\frac{R_0}{R}\right)^3 + \Omega_r\left(\frac{R_0}{R}\right)^4\right] \tag{2.11}$$

with the general solution

$$t = t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\,(\Omega_\Lambda + \Omega_m x^{-3} + \Omega_r x^{-4})^{½}}\,, \tag{2.12}$$

where $x \equiv (R/R_0) = (1/[1+z])$.

The most important parameter values, which were partly above introduced, are summarized in the Table I, which contains the critically analyzed observed (present) cosmological parameter values for the massive universe according to Ref. [7]. For better comparison of observed and estimated cosmological parameter values, we mention here also their measured Planck 2013 data [12], summarized in Table II. In Ref. [1, 2], we have shown that for the description of the massive anti-universe the (present), cosmological parameter values of Tables I and II are also valid, except the values H_0 and h, which are here negative. This behavior can be attributed to the known properties of particles and antiparticles according to Table III [1, 2]. Using these special properties for the Friedmann-Lemaitre Equations, we have also assumed the most important quantities in Table IV, which are necessary for the description of the "massless" and massive universe and anti-universe (see Sec. 1). In Refs. [1-4], we have estimated particle-defined (present) cosmological parameter values for universe and anti-universe, given in Table V. They agree well with the observed data in Table I.

It is usual to take apart Eq. (2.3) in the dominant density terms in dependence on the redshift evolution (2.10). Then, for example, if the universe expands adiabatically, the entropy per comoving volume is constant, so that we can describe the mass density of the radiation-dominated universe for $z \geq 10^5$ ($T \geq 3 \cdot 10^5$ K) by

$$\rho = \rho_{\mathrm{r}} = \tfrac{1}{2} N(T)\, \Omega_\gamma (1+z)^4 \rho_{0\mathrm{C}} = \tfrac{1}{2} N(T)\, (1+z)^4 \rho_0(\gamma)\,. \qquad (2.13)$$

By the new inflation model [1-5], for the redshift evolution $1+z$, we have

$$\widetilde{R} = \frac{R}{1+z_{\mathrm{M}}} = \frac{\widetilde{R}_0}{1+z} \qquad (2.14)$$

$$R = (1+z_{\mathrm{M}})\widetilde{R} = \frac{R_0}{1+z}\,, \qquad (2.15)$$

where $\widetilde{R}$ and R characterize the scale factors for the emission of light of prior to and after the inflation, whereas $\widetilde{R}_0$ and R_0 define the constant scale factors for the absorption of light before and after the inflation, respectively.

In Refs. [1-4], we have proved that these cosmological parameters of Tables I and II are excellently described by particle-defined, (present), cosmological parameter values, calculated by the light neutrino density parameters as well as the ratios of the relativistic energy to the rest energy of the SUSY GUT particles. In detail, we have summarized these particle-defined, (present), cosmological parameters in Table V (see above). In Ref. [1, 2], by aid of the data of Tables III and IV, we have shown that these particle-defined, (present), cosmological parameters are also valid for the anti-universe. Because of the excellent agreement between these observed (Tables I and II) and calculated (Table V) cosmological parameters, we apply always the cosmological parameter values of Table I in this work at the calculations except for the second half of Sec. 7.

TABLE I. The most important, critically analyzed, (present), cosmological parameter values [7] of the massive universe. In this work, H_0 is interpreted as the present CMB Hubble "constant" (see also Sec. 1). [a] Calculated by us for $h = 0.673 \pm 0.012$. [b] Different fits [7]

Quantity	Symbol, equation	Value
present day CMB temperature	T_0	$2.7255(6)\ \mathrm{K}$
present day CMB Hubble expansion rate	H_0	$67.3 \pm 1.2\ \mathrm{km\ s^{-1}\ Mpc^{-1}} =$ $= h \times (9.777752\ \mathrm{Gyr})^{-1} =$ $= (2.181 \pm 0.039) \cdot 10^{-18}\ \mathrm{s^{-1}}$
scale factor for Hubble expansion rate	h	0.673 ± 0.012
critical present density of the universe	$\rho_{0C} = 3H_0^2 / 8\pi G_N$	$1.05375(13) \cdot 10^{-5} h^2 (\mathrm{GeV}/c^2)\ \mathrm{cm}^{-3} =$ $= 4.77(17) \cdot 10^3 (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}$
baryon (proton) density of the universe	$\Omega_b = \rho_b / \rho_{0C}$	$0.02207(27)\, h^{-2} = 0.0487(23)$ [a], $0.0499(22)$ [b]
(cold) dark matter density of the universe	$\Omega_{dm} = \rho_{dm} / \rho_{0C}$	$0.1198(26)\, h^{-2} = 0.265(15)$ [a], $0.265(11)$ [b]
dark energy density of the universe	$\Omega_\Lambda = \rho_\Lambda / \rho_{0C}$	$0.685^{+0.017}_{-0.016}$

TABLE I, continued.

Quantity	Symbol, equation	Value
pressureless matter density of the universe	$\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}} + \Omega_{\mathrm{dm}}$	$0.315^{+0.016}_{-0.017}$
CMB radiation density of the universe	$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0\mathrm{C}}$	$2.473 \cdot 10^{-5}(T/2.7255\ \mathrm{K})^4 h^{-2} =$ $= 5.46(19) \cdot 10^{-5}$
curvature	$\Omega_{\mathrm{tot}} = \Omega_{\mathrm{m}} + \Omega_{\Lambda} + \cdots$	$0.96^{+0.4}_{-0.5}$ (95% CL); 1.000(7) (95% CL; CMB + BAO)
sum of neutrino masses	$\sum m_{\nu} c^2$	< 0.23 eV
neutrino density of the universe	$\Omega_{\nu} = \rho_{\nu}/\rho_{0\mathrm{C}}$	$< 0.0025\, h^{-2}$; < 0.0055
redshift of matter-radiation equality	z_{eq}	3360 ± 70
redshift of photon decoupling	z_{dec}	1090.2 ± 0.7
redshift at half reionization	z_{reion}	11.1 ± 1.1
age of the universe	t_0	13.81 ± 0.05 Gyr = $= (4.358 \pm 0.016) \cdot 10^{17}$ s
Hubble length	$R_0 = c/H_0$	$0.9250629 \cdot 10^{26} h^{-1}$ m = $= (1.375 \pm 0.025) \cdot 10^{26}$ m [a)]

TABLE II. The measured, cosmological parameter values of Planck 2013 [12]. In column 1, the corresponding parameter symbols are given. The column 2 gives results for the Planck temperature data alone. Column 3, denoted as Planck+WP, combines these Planck data and the WMAP polarization data at low multipoles.

Parameter	Planck Best fit [68% limits]	Planck+WP Best fit [68% limits]
H_0 [km/s Mpc]	67.11 $[67.4 \pm 1.4]$	67.04 $[67.3 \pm 1.2]$
Ω_Λ	0.6825 $[0.686 \pm 0.020]$	0.6817 $[0.685^{+0.018}_{-0.016}]$
$100\,\Omega_b h^2$	2.2068 $[2.207 \pm 0.033]$	2.2032 $[2.205 \pm 0.028]$
$\Omega_{dm} h^2$	0.12029 $[0.1196 \pm 0.0031]$	0.12038 $[0.1199 \pm 0.0027]$
$\Omega_m h^2$	0.14300 $[0.1423 \pm 0.0029]$	0.14305 $[0.1426 \pm 0.0025]$
Age t_0 [Gyr]	13.819 $[13.813 \pm 0.058]$	13.8242 $[13.817 \pm 0.048]$
z_{eq}	3402 $[3386 \pm 69]$	3403 $[3391 \pm 60]$
z_{dec}	1090.43 $[1090.37 \pm 0.65]$	1090.48 $[1090.43 \pm 0.54]$
z_{reion}		11.37 $[11.1 \pm 1.1]$

TABLE III. The most important known properties of particles and antiparticles.

Property	Particle	Anti-particle
energy	E	E
mass	M	M
time	t	$-t$
momentum	p	$-p$
velocity	v	$-v$
elementary electric charge	q	$-q$

TABLE IV. The most important quantities of the "massless" and massive universe and anti-universe, derived via the known properties of the particles and the antiparticles (in connection with the time reversal). [a)]

Quantity	Universe	Anti-universe
energy	E	E
temperature	T	T
Boltzmann constant	k	k
mass density	ρ	ρ
gravitational constant	G_N	G_N
age	t_0	t_0
distance, scale factor	R	R
acceleration	$\ddot{R}$	$\ddot{R}$

TABLE IV, continued.

Quantity	**Universe**	**Anti-universe**
velocity including c	$\dot{R}$	$-\dot{R}$
reduced Planck constant	$\hbar$	$-\hbar$
time	t	$-t$
Hubble expansion rate	H	$-H$
present CMB Hubble constant	H_0	$-H_0$
scale factor of H_0	h	$-h$
ionized charges	q	$-q$

[a] In work [2], for the total (massive and "massless) anti-universe, all assumptions derived from the known properties of the antiparticle, are correct. Unfortunately, in contrast to Ref. [2], the work [1] gives a negative distance and scale factor $-R$ as one incorrect assumption instead of a correct positive scale factor R. We have here corrected this quantity.

Table V. The estimated particle-defined (present) cosmological parameter values for universe and anti-universe according to Refs. [1-4] (compare with Tables I and II). [a] Universe: "+"sign, anti-universe: "−"sign.

Symbol, equation	**Value for universe and anti-universe**
$\lvert\pm h\rvert$ [a]	$0.6736^{+0.0105}_{-0.0096}$
$\lvert\pm H_0\rvert$ [a]	$100\,h$ km s^{-1}Mpc^{-1} $= 67.36^{+1.05}_{-0.96}$ km s^{-1}Mpc^{-1} = $= h\times(9.777752\,\text{Gyr})^{-1} = (2.183^{+0.034}_{-0.031})\cdot 10^{-18}\,\text{s}^{-1}$

Table V, continued.

Symbol, equation	Value for universe and anti-universe
$R_0 = c/H_0$	$(1.373^{+0.020}_{-0.021})\cdot 10^{26}$ m
$\rho_{0C} = 3H_0^2/8\pi G_N$	$(4.78^{+0.15}_{-0.14})\cdot 10^3 (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}$
t_0	$13.82^{+0.06}_{-0.07}$ Gyr
Ω_{tot}	1
$\Omega_{\mathrm{b}} = \rho_{\mathrm{b}}/\rho_{0C}$	$0.02211^{+0.00089}_{-0.00091}\, h^{-2} = 0.0487 \pm 0.0020$
$\Omega_{\mathrm{dm}} = \rho_{\mathrm{dm}}/\rho_{0C}$	$0.1202^{+0.0059}_{-0.0051}\, h^{-2} = 0.265^{+0.013}_{-0.012}$
$\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{0C}$	$0.311^{+0.014}_{-0.013}\, h^{-2} = 0.686^{+0.020}_{-0.021}$
$\Omega_{\mathrm{m}} = \rho_{\mathrm{m}}/\rho_{0C}$	$0.1423^{+0.0059}_{-0.0052}\, h^{-2} = 0.314^{+0.013}_{-0.012}$
$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0C}$	$(2.4728 \pm 0.0025)\cdot 10^{-5}\, h^{-2} = (5.45^{+0.15}_{-0.17})\cdot 10^{-5}$
$\Omega_{\nu} = \rho_{\nu}/\rho_{0C}$	$(6.35^{+0.16}_{-0.14})\cdot 10^{-4}\, h^{-2} = (1.402^{+0.040}_{-0.044})\cdot 10^{-3}$
$\sum m_{\nu} c^2$	$(5.97^{+0.14}_{-0.13})\cdot 10^{-2}$ eV
$z_{\mathrm{eq}} = \dfrac{2\Omega_{\mathrm{m}}}{N(T)\Omega_{\gamma}} - 1$	3423^{+107}_{-94}
z_{dec}	1090.7
$z_{\mathrm{reion}}(\nu_{\tau})$	$11.60^{+0.31}_{-0.26}$

In Ref. [1], for the anti-universe, the negative sign of the scale factor is incorrect. In this work, using the time reversal of the time component of the 4-vectors and the tensors, we have corrected this inadmissibility, which is based on a careless application of the CPT conservation.

According to Refs. [1-5], via the present critical mass density (2.4), for the conditions (2.14) and (2.15), the quantities $\widetilde{R}_0$ and R_0 were found to

$$\widetilde{R}_0 = (R_{\text{Pl}}\, c/H_0)^{1/2} = (4.713 \pm 0.042) \cdot 10^{-5}\ \text{m} \tag{2.16}$$

and

$$R_0 = c/H_0 = (1.375 \pm 0.025) \cdot 10^{26}\ \text{m}\,. \tag{2.17}$$

The quantity

$$1 + z_{\text{M}} = \frac{T_{\text{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\text{Pl}}} = \frac{R_0}{\widetilde{R}_0} = (2.916 \pm 0.026) \cdot 10^{30} \tag{2.18}$$

enlarges the early (massive) universe ($R_{\text{Pl}} \le \widetilde{R} \le \widetilde{R}_0$) by the enormous factor $1 + z_{\text{M}} = 2.916 \cdot 10^{30}$ to the late (massive) universe ($\widetilde{R}_0 \le R$), whereat T_{M} is the temperature, which is defined by the magnetic monopoles (M) via their rest energy (see Eq. (2.58) or Refs. [1-5]).

In Refs. [1-5], by aid of the blueshift $1 + z(\nu_{\text{e}}) = 0.406^{+0.020}_{-0.025}$ of the light electron neutrino ν_{e}, for the final (f) state of the universe, we have found its scale factor to

$$R = R_{\text{f}} = R(\nu_{\text{e}}) = \frac{R_0}{1 + z(\nu_{\text{e}})} = (3.39^{+0.27}_{-0.23}) \cdot 10^{26}\ \text{m}\,. \tag{2.19}$$

In Eq. (2.4), according to Refs. [1-3, 5], the influence of the dark mass density $\Omega_\Lambda \rho_{0\text{C}}$ begins at

$$1 + z_\Lambda = 18.90^{+0.48}_{-0.49}\,, \tag{2.20}$$

so that by Eq. (2.15) we find its minimum scale factor

$$R = R_\Lambda = \frac{R_0}{1 + z_\Lambda} = (7.28 \pm 0.32) \cdot 10^{24}\ \text{m}\,. \tag{2.21}$$

Because of the expressions (2.19) and (2.21), which define the range of influence of the dark energy (Ω_Λ), for the mean, negative acceleration (MNA), we must assume the average of their redshift conditions [1-3, 5]:

$$1+z_{\text{MNA}} = \frac{R_0}{\sqrt{R_\Lambda R(\nu_e)}} = \left([1+z_\Lambda][1+z(\nu_e)]\right)^{1/2} = 2.77^{+0.10}_{-0.12}, \quad (2.22)$$

so that we obtain the corresponding scale factor [1-3, 5]:

$$R = R_{\text{MNA}} = \frac{R_0}{1+z_{\text{MNA}}} = (4.96^{+0.31}_{-0.27})\cdot 10^{25}\ \text{m}. \quad (2.23)$$

Consequently, according to Refs [1-3, 5], by Eq. (2.2), for $\Lambda = 0$, at the pressure $P \cong 0$, in the matter-dominated universe, we obtain this mean negative acceleration to

$$\ddot{R}_{\text{MNA}} = a = -\frac{1}{2}H_0^2\left[\Omega_m(R_0/R_{\text{MNA}})^3 + \Omega_\Lambda\right]R_{\text{MNA}} =$$
$$= (-8.71^{+0.76}_{-0.87})\cdot 10^{-8}\ \text{cm s}^{-2}. \quad (2.24)$$

The value (2.24) agrees excellently with the Pioneer anomaly [14, 15]. Then, by Eq. (2.2), for $\Lambda = 0$ (wCDM model), according to $\rho = \rho_{\text{vac},\Lambda} = \Omega_\Lambda \rho_{0C}$ (see Eq. (2.3)) and $P = -\rho_{\text{vac},\Lambda}c^2 = -\Omega_\Lambda \rho_{0C} c^2$, the acceleration of the "present" accelerated expansion (see below) is positive [1, 2] because of

$$\ddot{R} = \Omega_\Lambda H_0^2 R. \quad (2.25)$$

Thus, for the end of this "present" accelerated expansion, the effective (eff) equilibrium condition $\ddot{R} + \ddot{R}_{\text{MNA}} = 0$ yields the effective scale factor [1, 2]:

$$R_{\text{eff}} = \frac{-\ddot{R}_{\text{MNA}}}{\Omega_\Lambda H_0^2} = 2.67\cdot 10^{26}\ \text{m}. \quad (2.26)$$

Because the "present" accelerated expansion [1, 2] is defined by

$$R = R_0\, e^{\Omega_\Lambda^{1/2} H_0\,(t-t_0)} = R_0\, e^{H(t-t_0)} \quad \text{at} \quad H = \Omega_\Lambda^{1/2} H_0, \quad (2.27)$$

the effective equilibrium takes place at the time

$$t_{\text{eff}} = t_0 + \frac{1}{\Omega_\Lambda^{1/2} H_0}\ln\frac{R_{\text{eff}}}{R_0} = 8.034\cdot 10^{17}\ \text{s} = 25.46\ \text{Gyr}, \quad (2.28)$$

where t_0 describes the present age of the universe (see Table I).

Consequently, after this effective equilibrium, according to Refs. [1, 2], we have only still a slow linear expansion. By this slow linear expansion, in Refs. [1, 2], we find the provisional age of the final state of the universe to

$$t_{\mathrm{f}} = 2.136 \cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}\,. \tag{2.29}$$

This age $6768\,\mathrm{Gyr}$ is much greater than the age $t = t_{\exp}(\nu_{\mathrm{e}}) = 29.63\,\mathrm{Gyr}$ in Refs. [3-5], which is alone defined by the "present" exponential expansion of the universe, i.e. for $R = R_{\mathrm{f}} = R(\nu_{\mathrm{e}}) = 3.39 \cdot 10^{26}\,\mathrm{m}$ in Eq. (2.27).

However, to the result (2.29), where we must still take into account the influence of the general vacuum energy density or the corresponding cosmological "constant" of the massive universe, we will return in Sec. 3.2.

According to Refs. [1, 2], the general vacuum energy density or the cosmological "constant" of the massive universe were found to

$$\rho_{\mathrm{vac}}(T)\, c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \tag{2.30}$$

or

$$\Lambda = \widetilde{\Lambda} = \frac{8\pi G_N}{c^2}\rho_{\mathrm{vac}}(T) = \frac{8}{45}\pi^3 \frac{(kT)^4}{E_{\mathrm{Pl}}^2(\hbar c)^2}\frac{1}{N(T)}\,, \tag{2.31}$$

respectively. Using Eqs. (2.30) and (2.31), we can introduce the two conditions

$$\rho_{\mathrm{vac}}(T)\, c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \geq \rho_{\mathrm{vac},\Lambda}\, c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \tag{2.32}$$

and

$$\widetilde{\Lambda} = \frac{8}{45}\pi^3 \frac{(kT)^4}{E_{\mathrm{Pl}}^2(\hbar c)^2}\frac{1}{N(T)} \geq \Lambda = \Lambda_\Lambda = \frac{3\Omega_\Lambda}{R_0^2}\,, \tag{2.33}$$

where on the right-hand side the terms describe the corresponding values of the "present" accelerated expansion of the massive universe [1, 2].

Using $N(T) = 3.362644$ (see Eq. (2.6)), by the two conditions (2.32) and (2.33), we get the lower limiting temperature

$$T = 51.41\,\mathrm{K}\,. \tag{2.34}$$

This temperature agrees excellently with the value $T = (1 + z_\Lambda) T_0 \cong 51.45\,\mathrm{K}$, where for $1 + z_\Lambda \cong 18.90$ the influence of the vacuum mass density $\Omega_\Lambda \rho_{0\mathrm{C}}$

begins according to Eq. (2.20). Thus, we assume that the vacuum energy density (2.30) influences the "present" exponential expansion, which therefore is considered as follows.

According to Refs. [1, 2], at the redshift condition $1+z_{\rm acc}=1.632$, the "present" accelerated expansion began after the big bang at the time

$$t(z_{\rm acc})=2.43\cdot10^{17}\,{\rm s}=7.70\ {\rm Gyr}\,. \tag{2.35}$$

Then, by Eq. (2.15)), we find the corresponding scale factor

$$R=R(z_{\rm acc})=R_0/(1+z_{\rm acc})=8.43\cdot10^{25}\,{\rm m}\,. \tag{2.36}$$

Using Eqs. (2.27) and (2.36), the fictitious beginning of the "present" accelerated expansion takes place before the time

$$t_{\rm acc}=t_0-t=2.72\cdot10^{17}\,{\rm s}=8.62\ {\rm Gyr}\,, \tag{2.37}$$

seen backward from the present age $t_0=4.358\cdot10^{17}\,{\rm s}$ of the universe (see Table I). We can solve the discrepancy between Eqs. (2.35) and (2.37) if we use the cosmological "constant" (2.31) in the following expression

$$(t_0-\tilde{t})=(t_0-t)\,\frac{\Omega_\Lambda^{1/2}H_0}{(c^2\tilde{\Lambda}/3)^{1/2}}=$$

$$=(t_0-t)\,\frac{\Omega_\Lambda^{1/2}H_0}{(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)\,E_{\rm Pl}^2\,\hbar^2)^{1/2}}\,, \tag{2.38}$$

derived in Refs. [1, 2].

Thus, we can solve the discrepancy between Eqs. (2.35) and (2.37) if in Eq. (2.38) we select $\tilde{t}=t(z_{\rm acc})=2.43\cdot10^{17}\,{\rm s}$, so that for $t_0-t=2.72\cdot10^{17}\,{\rm s}$ the result (2.38) provides $(c^2\tilde{\Lambda}/3)^{1/2}=2.55\cdot10^{-18}\,{\rm s}^{-1}$ with $\tilde{\Lambda}=2.16\cdot10^{-52}\,{\rm m}^{-2}$ or $\rho_{\rm vac}(T)\,c^2=6.50\cdot10^3\,{\rm eV\,cm}^{-3}$ for $N(T)=3.362644$ at $T=61\,{\rm K}$.

Therefore, in the matter-dominated universe, by this constant temperature $T=61\,{\rm K}$, the influence of the dark energy (vacuum energy) begins a little earlier for the "present" accelerated expansion

$$R=R(z_{\rm acc})=R_0\,{\rm e}^{(c^2\tilde{\Lambda}/3)^{1/2}(\tilde{t}-t_0)}\,. \tag{2.39}$$

Finally, we summarize still the results of the quantum gravity [1, 2] for the massless universe ($R\le R_{\rm Pl}$). According to Refs. [1, 2], we have obtained here the following expressions for the connection between distance R as well as thermal energy kT or particle (P) rest energy $E_0(\mathrm{P})$:

$$R = \frac{G_N}{c^4} kT = \frac{\hbar c}{E_{\mathrm{Pl}}^2} kT = \frac{G_N}{c^4} E_0(\mathrm{P}), \tag{2.40}$$

for the vacuum energy density:

$$\rho_{\mathrm{vac}}(R)\, c^2 = \widehat{\rho}_{\mathrm{vac}}\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c\, R^2}, \tag{2.41}$$

for the cosmological "constant":

$$\Lambda = \widehat{\Lambda} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R^2}, \tag{2.42}$$

for the distance:

$$R = ct, \tag{2.43}$$

for the thermal particle number density:

$$n_{\mathrm{P}}(R) = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{kT} = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{E_0(\mathrm{P})} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}, \tag{2.44}$$

for the kinetic photon energy:

$$E_{\mathrm{K}}(\gamma) = 2.701178\, kT = 2.701178 \frac{E_{\mathrm{Pl}}^2}{\hbar c} R \tag{2.45}$$

and for the thermal photon number density:

$$n_\gamma(R) = 2.701178 \frac{\rho_{\mathrm{vac}}(R)\, c^2}{E_{\mathrm{K}}(\gamma)} = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{kT} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}. \tag{2.46}$$

Consequently, we have $n_{\mathrm{P}}(R) = n_\gamma(R)$.

The usefulness of the expressions (2.40) to (2.46) is demonstrated at the big bang, for example, in Sec. 3.1.

In Refs. [1-3, 5], for the three light (left handed) neutrinos (electron neutrino ν_{e}, muon neutrino ν_μ and tauon neutrino ν_τ), we have estimated the following rest energies for these neutrinos:

$$E_0(\nu_{\mathrm{e}}) \cong (1.589^{+0.078}_{-0.098}) \cdot 10^{-3}\,\mathrm{eV}, \tag{2.47}$$

$$E_0(\nu_\mu) \cong (8.85^{+0.14}_{-0.16}) \cdot 10^{-3}\,\mathrm{eV}, \tag{2.48}$$

$$E_0(\nu_\tau) \cong (4.93^{+0.12}_{-0.10}) \cdot 10^{-2}\,\mathrm{eV}. \tag{2.49}$$

Then, the sum of the neutrino rest energies (see Eqs. (2.47) to (2.49)) gives the approximate solution

$$\sum_i E_0(\nu_i) = E_0(\nu_e) + E_0(\nu_\mu) + E_0(\nu_\tau) \cong (5.97^{+0.14}_{-0.13}) \cdot 10^{-2}\,\mathrm{eV}, \qquad (2.50)$$

where the subscript $i = \mathrm{e}, \mu, \tau$ characterizes the e, μ and τ neutrino.

According to Refs. [1-3, 5], we have derived the following general expression for the light neutrino density parameters:

$$\Omega_\nu(\nu_i) = (429.889 \pm 0.095)\, E_0(\nu_i)\, \Omega_\gamma =$$

$$= (0.010630 \pm 0.000011)\, E_0(\nu_i)' h^{-2}, \qquad (2.51)$$

where

$$\Omega_\gamma = (2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2}. \qquad (2.52)$$

Then, using the rest energies (2.47) to (2.49) together with the radiation density $\Omega_\gamma = (5.46 \pm 0.19) \cdot 10^{-5}$ (see Table I), by Eqs. (2.51) and (2.52), we get the following neutrino density parameters:

$$\Omega_\nu(\nu_e) \cong (0.683^{+0.034}_{-0.042})\, \Omega_\gamma =$$

$$= (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} h^{-2} = (3.73^{+0.32}_{-0.36}) \cdot 10^{-5}, \qquad (2.53)$$

$$\Omega_\nu(\nu_\mu) \cong (3.805^{+0.061}_{-0.070})\, \Omega_\gamma =$$

$$= (9.41^{+0.16}_{-0.18}) \cdot 10^{-5} h^{-2} = (2.08 \pm 0.11) \cdot 10^{-4}, \qquad (2.54)$$

$$\Omega_\nu(\nu_\tau) \cong (21.19^{+0.52}_{-0.44})\, \Omega_\gamma =$$

$$= (5.24^{+0.13}_{-0.11}) \cdot 10^{-4} h^{-2} = (1.157^{+0.069}_{-0.064}) \cdot 10^{-3}. \qquad (2.55)$$

The sum of Eqs. (2.53) to (2.55) determines the total neutrino density parameter as follows

$$\Omega_\nu = \sum_i \Omega_\nu(\nu_i) \cong (25.68^{+0.62}_{-0.55})\, \Omega_\gamma =$$

$$= (6.35^{+0.16}_{-0.14}) \cdot 10^{-4} h^{-2} = (1.402^{+0.083}_{-0.079}) \cdot 10^{-3}. \qquad (2.56)$$

In Refs. [1-3, 5], we have estimated the rest energy of the X and Y gauge bosons as well as the magnetic monopoles to

$$E_0(\mathrm{X}) = E_0(\mathrm{Y}) = (2.675^{+0.058}_{-0.063}) \cdot 10^{16}\,\mathrm{GeV} \qquad (2.57)$$

and

$$E_0(\mathrm{M}) = (6.849 \pm 0.063) \cdot 10^{17}\ \mathrm{GeV}, \qquad (2.58)$$

respectively. According to Refs. [1-3, 5], the correctness of the estimations of the rest energies of these particles of the supersymmetric grand unification theories (SUSY GUTs) (see Eqs. (2.57) and (2.58)) can be proved by their coupling constant α_{GUT}, which is defined by

$$\alpha_{\mathrm{GUT}} = \frac{E_0(\mathrm{X})}{E_0(\mathrm{M})} = 0.0391^{+0.0012}_{-0.0013} \qquad (2.59)$$

in excellent agreement with the value $\alpha_{\mathrm{GUT}} \approx 0.04$ obtained by a completely different way via the extrapolation of the gauge couplings constants of the standard model to very high energies [7].

By the Gunn-Peterson effect [1-6], the intergalactic neutral hydrogen (HI) gas, which has the density $n_{\mathrm{HI}}(z) \approx 2.42 \cdot 10^{-11} \tau(z)\, h\, E(z)\ \mathrm{cm}^{-3}$ (see Ref. [6]) {using $E(z) = \left[\Omega_\Lambda + \Omega_{\mathrm{m}}(1+z)^3 + \Omega_{\mathrm{r}}(1+z)^4\right]^{1/2}$ (see Refs. [1-3, 5])}, can be estimated according to Ref. [6] by the measurement of its optical depth $\tau(z)$ from flux decrement in quasar spectra at the wavelength $\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$ in $\mathrm{Ly}\alpha$ absorption (of the neutral hydrogen), which has a very large cross-section for

$$T_{\mathrm{IGM}} << T_{\mathrm{Ly}\alpha} = \frac{2\pi \hbar c}{k\, \lambda_{\mathrm{Ly}\alpha}} = 1.183 \cdot 10^5\ \mathrm{K}. \qquad (2.60)$$

The observations show that at a redshift of order $z \approx 5\text{-}6$ the neutral hydrogen, left over from the time of recombination, becomes reionized by this ultraviolet light ($\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$) from the massive stars (quasars) [6, 8].

Consequently, according to Refs. [1-3, 5], via Eqs. (2.54) to (2.56), at $N(T) = 3.362644$ (see Eq. (2.6)), for this reionization, we assume that the redshifts z_{reion} of the neutrinos

$$z_{\mathrm{reion}}\left(\sum_i \nu_i\right) = \frac{2\sum_i \Omega_\nu(\nu_i)}{N(T)\,\Omega_\gamma} - 1 = 14.27^{+0.37}_{-0.33}, \qquad (2.61)$$

$$z_{\mathrm{reion}}(\nu_\tau) = \frac{2\,\Omega_\nu(\nu_\tau)}{N(T)\,\Omega_\gamma} - 1 = 11.60^{+0.31}_{-0.26} \qquad (2.62)$$

and

$$z_{\text{reion}}(\nu_\mu) = \frac{2\,\Omega_\nu(\nu_\mu)}{N(T)\,\Omega_\gamma} - 1 = 1.263^{+0.036}_{-0.042} \tag{2.63}$$

define the beginning, the half and the end of the reionization, respectively. This assumption is supported by the neutrino temperatures

$$T(\textstyle\sum_i \nu_i) = \left\{ 1 + z_{\text{reion}}(\sum_i \nu_i) \right\} T_0 = 41.62\ \text{K}\,, \tag{2.64}$$

$$T(\nu_\tau) = \left\{ 1 + z_{\text{reion}}(\nu_\tau) \right\} T_0 = 34.34\ \text{K} \tag{2.65}$$

and

$$T(\nu_\mu) = \left\{ 1 + z_{\text{reion}}(\nu_\mu) \right\} T_0 = 6.168\ \text{K}\,, \tag{2.66}$$

since they fulfil the condition (2.60).

We see that the value (2.62) agrees excellently with the redshifts of the half reionization in the Tables I and II, which were found on completely other way. This fact is a strong argument for the correctness of the assumptions (2.61) and (2.63) as the beginning and the end of the reionization, respectively.

At the electron neutrino, its blueshift [1-3, 5] is defined by

$$1 + z(\nu_e) = \frac{2\,\Omega_\nu(\nu_e)}{N(T)\,\Omega_\gamma} = 0.406^{+0.020}_{-0.025}\,, \tag{2.67}$$

so that the scale factor of the final state of the massive universe is determined via the expression (2.19).

In this work, the results of the most recent analysis of the observed cosmological parameters and the 3 neutrino oscillation parameters of Ref [16] are not applied, since within the error limits these most recent data agree well with the corresponding cosmological parameters of Ref. [7] and the 3 neutrino oscillation parameters of Ref [10]. Therefore, a renewed calculation of all astrophysical parameters, determined in Refs [1-5], where they were derived by aid of the cosmological parameters of Ref. [7] and the 3 neutrino oscillation parameters of Ref. [10], is first necessary, if new better measurements are available, since within the error limits the application of the cosmological parameters and 3 neutrino oscillation parameters from Ref. [16] gives similar results. Thus, in this work, we use always as basis the cosmological parameters from Table I, given by Ref. [7]. We will return to this problem in Sec. 4, where we have exceptionally derived correspondingly new cosmological parameters

(Table VI) as a result of a new interpretation of the dark matter and dark energy, which are needed in the second half of Sec. 7.

Using the data of Tables III to V, these considerations, derived in this Sec. 2 for the universe, are also valid for the anti-universe.

3 Final state and big bang as well as the time reversal solution and the eternal cyclic evolution of universe and anti-universe

In this Sec. 3, via results of Refs. [1, 2], we give a detailed review, whereat we show the relationship between the final state of the universe and the big bang.

3.1 The final state of the universe and the big bang

According to the results of Refs. [1, 2], in this chapter, via the relationship between the final state of the universe and the big bang (see also Sec. 3.11), we calculate their parameters.

At the time $t_{\text{eff}} = 8.034 \cdot 10^{17}\ \text{s} = 25.46\ \text{Gyr}$ (see Eq. (2.28)) or the scale factor $R_{\text{eff}} = 2.67 \cdot 10^{26}\ \text{m}$ (see Eq. (2.26)), we have the end of the "present" accelerated expansion because of the effective equilibrium (see Eq. (2.26)), so that here the final value of the dark (d) energy E_{d} is found via the vacuum energy density $\rho_{\text{vac},\Lambda}\, c^2 = \Omega_\Lambda \rho_{0\text{C}}\, c^2 = 3.27 \cdot 10^3\ \text{eV cm}^{-3}$ (see Eq. (2.32)) to

$$E_{\text{d}} = \Omega_\Lambda \rho_{0\text{C}} c^2\, {}^4\!/\!_3\, \pi\, d_{\text{eff}}^3\,, \qquad (3.1)$$

where the proper distance d_{eff} is defined by

$$d_{\text{eff}} = R_{\text{eff}}\, r_{\text{eff}} = R_{\text{eff}}\, r \qquad (3.2)$$

with $r = r_{\text{eff}}$ as the dimensionless, time-independent, comoving coordinate distance (see Refs. [1, 2, 6, 8, 11]).

Using the hypothesis of the joint origin of the dark matter and dark energy by the three sterile, neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{\text{dm}}$ and $\hat{\nu}_{\text{b}}$ (see, e.g., Ref. [1]), we can assume that this dark energy E_{d} must be distributed among the decay products of these sterile neutrinos ($\hat{\nu}$). Assuming initially the decay condition of these sterile neutrino types via the gravitation into one photon with the energy of their half rest energy $\frac{1}{2}E_0(\hat{\nu})$ and one sterile neutrino relic with the energy of their half rest energy $\frac{1}{2}E_0(\hat{\nu})$ (see Refs. [1-4]), all these photons $kT_\gamma = \frac{1}{2}E_0(\hat{\nu})$ must define in average the half dark energy $\frac{1}{2}E_\mathrm{d}$ as the greatest possible thermal photon energy kT according to

$$kT = \tfrac{1}{2}E_\mathrm{d} = \sum_\gamma \langle kT_\gamma \rangle = \sum_\gamma \langle \tfrac{1}{2}E_0(\hat{\nu}) \rangle . \qquad (3.3)$$

Therefore, applying the new thermal equilibrium [1, 2] between the photons $kT_{\frac{1}{2}\hat{\nu}}$ and the sterile neutrino relics $\frac{1}{2}E_0(\hat{\nu})$, because of the energy conservation, all these sterile neutrino relics $\frac{1}{2}E_0(\hat{\nu}) = kT_{\frac{1}{2}\hat{\nu}}$ must also form in average the other half of the dark energy $\frac{1}{2}E_\mathrm{d}$ as the greatest possible thermal energy kT according to

$$kT = \tfrac{1}{2}E_\mathrm{d} = \sum_{\frac{1}{2}\hat{\nu}} \langle kT_{\frac{1}{2}\hat{\nu}} \rangle = \sum_{\frac{1}{2}\hat{\nu}} \langle \tfrac{1}{2}E_0(\hat{\nu}) \rangle , \qquad (3.4)$$

since the corresponding energy $kT_{\frac{1}{2}\hat{\nu}}$ of the sterile neutrino relics must be equivalent to kT_γ of the corresponding photons. The nature of the decay product $\frac{1}{2}E_0(\hat{\nu}) = kT_{\frac{1}{2}\hat{\nu}}$ is explained by Eq. (3.135).

The gravitational potential energy $V_\mathrm{gr}(d_\mathrm{eff})$ of these decay products can be defined by

$$V_\mathrm{gr}(d_\mathrm{eff}) = \frac{G_N}{c^4}\frac{(\frac{1}{2}E_\mathrm{d})^2}{d_\mathrm{eff}} = \frac{\hbar c}{E_\mathrm{Pl}^2}\frac{(\frac{1}{2}E_\mathrm{d})^2}{d_\mathrm{eff}} . \qquad (3.5)$$

Using $V_\mathrm{gr}(d_\mathrm{eff}) = \frac{1}{2}E_\mathrm{d}$, we obtain

$$d_\mathrm{eff} = \frac{G_N}{c^4}\tfrac{1}{2}E_\mathrm{d} = \frac{\hbar c}{E_\mathrm{Pl}^2}\tfrac{1}{2}E_\mathrm{d} . \qquad (3.6)$$

By Eqs. (3.1) and (3.6), we find the condition

$$d_\mathrm{eff} = \frac{\hbar c}{E_\mathrm{Pl}^2}\tfrac{1}{2}\Omega_\Lambda \rho_{0C} c^2 \tfrac{4}{3}\pi d_\mathrm{eff}^3 \qquad (3.7)$$

with the solution of the proper distance

$$d_{\text{eff}} = \left(\frac{E_{\text{Pl}}^2}{\hbar c\, \frac{1}{2} \Omega_\Lambda \rho_{0C} c^2\, \frac{4}{3}\pi} \right)^{\frac{1}{2}} = 3.321 \cdot 10^{26}\ \text{m}\,, \tag{3.8}$$

so that Eq. (3.2) gives

$$r = r_{\text{eff}} = \frac{d_{\text{eff}}}{R_{\text{eff}}} = 1.2438\,. \tag{3.9}$$

Using Eqs. (3.1) and (3.8), we get quantitatively the dark energy

$$E_{\text{d}} = \Omega_\Lambda \rho_{0C} c^2\, \frac{4}{3}\pi d_{\text{eff}}^3 = 5.017 \cdot 10^{89}\ \text{eV}\,. \tag{3.10}$$

By the result (3.10), the assumptions (3.3) and (3.4) are trivially defined by

$$kT = \frac{1}{2} E_{\text{d}} = \sum_\gamma \langle kT_\gamma \rangle = \sum_\gamma \langle \frac{1}{2} E_0(\hat{\nu}) \rangle = 2.509 \cdot 10^{89}\ \text{eV} \tag{3.11}$$

and

$$kT = \frac{1}{2} E_{\text{d}} = \sum_{\frac{1}{2}\hat{\nu}} \langle kT_{\frac{1}{2}\hat{\nu}} \rangle = \sum_{\frac{1}{2}\hat{\nu}} \langle \frac{1}{2} E_0(\hat{\nu}) \rangle = 2.509 \cdot 10^{89}\ \text{eV}\,: \tag{3.12}$$

Then, according to Refs. [1, 2], via Eq. (3.11), the thermal photon number density $n_\gamma(R_{\text{eff}})$ is found to

$$n_\gamma(R_{\text{eff}}) = 2.701178\, \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 1.353 \cdot 10^{282}\ \text{cm}^{-3}\,, \tag{3.13}$$

whereas because of Eqs. (3.4) or (3.12) for the sterile neutrino relics their thermal number density $n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}})$ must be equivalent to $n_\gamma(R_{\text{eff}})$, i.e. according to Eq. (3.13) we have the condition

$$n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = n_\gamma(R_{\text{eff}}) = \frac{\pi^2}{15} \frac{(\frac{1}{2} E_{\text{d}})^3}{(\hbar c)^3} = 1.353 \cdot 10^{282}\ \text{cm}^{-3}\,. \tag{3.14}$$

Thus, at $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see above), in the massive universe, because of $n_\gamma(R_{\text{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}})$, we have a stable equilibrium between the photons and the sterile neutrino relics, since still no complete decay of the sterile neutrinos has taken place.

Taking the result (3.8), the proper lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos can be assumed to

$$\tau_{\hat{\nu}} = \frac{d_{\text{eff}}}{c} = 1.108 \cdot 10^{18}\ \text{s} = 35.11\ \text{Gyr}. \tag{3.15}$$

Consequently, for $t_{\text{eff}} = 8.034 \cdot 10^{17}\ \text{s} = 25.46\ \text{Gyr}$ (see above), if we use the denotation $n(\hat{\nu})$ for the total number density of the sterile neutrinos, because of their constant volume $V(\hat{\nu})$ (see Sec. 3.3), by the universal decay law

$$n_{\hat{\nu}}(t_{\text{eff}}) = n(\hat{\nu})\, \text{e}^{-t_{\text{eff}}/\tau_{\hat{\nu}}}, \tag{3.16}$$

we have the decay of the half of all sterile neutrinos, since we observe

$$n_{\hat{\nu}}(t_{\text{eff}}) \approx \tfrac{1}{2} n(\hat{\nu}). \tag{3.17}$$

Therefore, via Eqs. (3.13) and (3.14), we assume

$$n_{\hat{\nu}}(t_{\text{eff}}) = n_{\gamma}(R_{\text{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = 1.353 \cdot 10^{282}\ \text{cm}^{-3}, \tag{3.18}$$

so that for the total number density of the sterile neutrinos we get

$$n(\hat{\nu}) = 2n_{\gamma}(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) =$$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3}. \tag{3.19}$$

Then, for the final state of the massive universe, at $R_{\text{f}} = 3.39 \cdot 10^{26}\ \text{m}$ (see Eq. (2.19)) or $t_{\text{f}} = 2.136 \cdot 10^{20}\ \text{s} = 6768\ \text{Gyr}$ (see Eq. (2.29)), we can assume a complete decay of all sterile neutrinos with $n(\hat{\nu}) = 2.706 \cdot 10^{282}\ \text{cm}^{-3}$, so that because of the complete conversion of the dark energy we get correspondingly an unstable equilibrium (see Eq. (3.135)) between the photons with

$$n_{\gamma}(R_{\text{f}}) = 2n_{\gamma}(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3} \tag{3.20}$$

and the sterile neutrino relics with

$$n(\hat{\nu}) = n_{\gamma}(R_{\text{f}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}}) = 2n_{\gamma}(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) =$$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3} \tag{3.21}$$

This unstable equilibrium leads to a transition from the final state of the massive universe in the direction of the big bang of the massless universe.

Therefore, for example, by Eqs. (2.46) and (3.20), because of particle conservation, we assume the condition

$$n_\gamma(R_{\mathrm{BB}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{BB}}^3} =$$

$$= n_\gamma(R_{\mathrm{f}}) = 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3 \qquad (3.22)$$

with the solution for the big bang distance

$$R_{\mathrm{BB}} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{\frac{1}{3}} \frac{\hbar c}{\frac{1}{2}E_{\mathrm{d}}} = 2.069\cdot 10^{-98}\ \mathrm{m}\,. \qquad (3.23)$$

Then, the remaining parameters of the big bang are defined as follows. By Eq. (2.43), the big bang has taken place at the time

$$t_{\mathrm{BB}} = \frac{R_{\mathrm{BB}}}{c} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{\frac{1}{3}} \frac{\hbar}{\frac{1}{2}E_{\mathrm{d}}} = 6.901\cdot 10^{-107}\ \mathrm{s}\,. \qquad (3.24)$$

According to Eqs. (2.41) and (2.42), for the big bang, the vacuum energy density or the cosmological "constant" are given by

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2 = \widehat{\rho}_{\mathrm{vac}}\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c\,R_{\mathrm{BB}}^2} =$$

$$= \frac{2}{45}\frac{\Omega_\gamma^{\frac{1}{3}}}{\pi^{\frac{2}{3}}}(292.2273)^{\frac{2}{3}}\frac{E_{\mathrm{Pl}}^2}{(\hbar c)^3}\left(\tfrac{1}{2}E_{\mathrm{d}}\right)^2 = 4.227\cdot 10^{247}\ \mathrm{eV\,cm^{-3}} \qquad (3.25)$$

or

$$\Lambda = \widehat{\Lambda} = \Lambda_{\mathrm{BB}} = \frac{16}{45}\pi^3\Omega_\gamma\frac{1}{R_{\mathrm{BB}}^2} =$$

$$= \frac{16}{45}(\pi\,\Omega_\gamma)^{\frac{1}{3}}(292.2273)^{\frac{2}{3}}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^2 = 1.406\cdot 10^{192}\ \mathrm{m^{-2}}\,, \qquad (3.26)$$

respectively.

Then, using Eq. (2.40) and the uncertainty relation, the smallest possible particle energy kT_{BB} and the energy uncertainty $\widetilde{E}_{\mathrm{BB}}$ (see Eqs. (3.128) and (3.129)) for the big bang in the universe are correspondingly found to

$$kT_{\rm BB} = \frac{E_{\rm Pl}^2}{\hbar c} R_{\rm BB} = \left(\frac{\pi^4 \Omega_\gamma}{292.2273}\right)^{1/3} \frac{E_{\rm Pl}^2}{\tfrac{1}{2} E_{\rm d}} = 1.563 \cdot 10^{-35}\,{\rm eV} \tag{3.27}$$

and

$$\widetilde{E}_{\rm BB} = k\widetilde{T}_{\rm BB} = \frac{\hbar}{2t_{\rm BB}} = \frac{\hbar c}{2R_{\rm BB}} = \frac{E_{\rm Pl}^2}{2kT_{\rm BB}} = 4.769 \cdot 10^{90}\,{\rm eV}\,. \tag{3.28 a}$$

Then, Eq. (3.28 a), multiplied by a factor 2, gives the greatest possible relativistic energy $E_{\rm BB} = 2\widetilde{E}_{\rm BB}$ by the origin (see also Eq. (3.130)) to

$$E_{\rm BB} = 2\widetilde{E}_{\rm BB} = \frac{\hbar}{t_{\rm BB}} = \frac{\hbar c}{R_{\rm BB}} = \frac{E_{\rm Pl}^2}{kT_{\rm BB}} = 9.537 \cdot 10^{90}\,{\rm eV}\,. \tag{3.28 b}$$

In Eq. (3.28 b), the expression $\hbar c/R_{\rm BB}$ is interpreted as the potential energy of a new attractive force, similar to the gravitational force (see Eq. (3.110)). To this new force, we will return in Eqs. (3.114) and (3.115).

With that, we have shown the relationship between the final state of the universe and the big bang (see also Sec. 3.11), since the transition, from the final state of the massive universe ($R_{\rm f} \geq R \geq R_{\rm Pl}$) in the direction to the big bang of the massless universe ($0 \rightarrow R_{\rm BB} \leq R \leq R_{\rm Pl}$), means the start of an eternal cyclic evolution of the total [massless ($R_{\rm BB} \leq R \leq R_{\rm Pl}$) and massive ($R_{\rm f} \geq R \geq R_{\rm Pl}$)] universe (see Sec. 3.11). More generally, when the universes begin by a hot big bang, they must have also an end and a fresh start.

Using the data of Tables III to V, all results, derived in this Sec. 3.1 for the total (massless and massive) universe, are also valid for the total (massless and massive) anti-universe and its cyclic evolution.

3.2 The lifetime of the sterile neutrinos in the total universe

According to Refs. [1, 2], in this chapter, we estimate the lifetime of the sterile neutrinos in the total universe. For this goal, we must still determine the initial sterile neutrino number density.

Therefore, we use the fact that the present day universe contains a relic neutrino (ν) background with the temperature $T_{\nu,0} = (4/11)^{1/3} T_0 = 1.945\,\text{K}$ (see Eq. (2.7)), where ν characterizes the light neutrinos, which were assumed as nearly massless.

For the formation of the particle-defined cosmological parameters (see Refs. [1-4]), which were identified as the cosmological parameters of the heavy neutrinos (see Refs. [1, 2]), the necessary energy for the light neutrinos was taken from the early universe, so that the light and the heavy neutrinos have the same number density, i.e. $n(\nu) = n(\tilde{\nu}) = 112\,\text{cm}^{-3}$ (see Refs. [1, 2]).

Consequently, the light (ν) and the heavy ($\tilde{\nu}$) neutrino background must possess the same temperature $T_{\nu,0} = T_{\tilde{\nu},0} = 1.945\,\text{K}$ according to the ideal gas law because of the constant pressure.

At the pressure P, between the energy E and the volume V, for the expansion of the massive universe, the kinetic theory of gases yields

$$dE = -PdV\,. \tag{3.29}$$

At the formation of sterile neutrinos, the necessary energy is taken from the relic neutrino ($\tilde{\nu}$) background, whereat the number density of the sterile neutrinos ($\hat{\nu}$) decreases to $n(\hat{\nu}) = 0.178\,\text{cm}^{-3}$ (see Refs. [1, 2]).

Therefore, according to Eq. (3.29), at constant pressure, for the heavy neutrino relic of $T_{\tilde{\nu},0} = 1.945\,\text{K}$, we must here assume the small volume changing $-1/n(\hat{\nu})$, so that we get a cooling of this relic neutrino background from $T_{\tilde{\nu},0} = 1.945\,\text{K}$ to $T_{\hat{\nu},0}$ in the large volume changing $-1/n(\tilde{\nu})$, so that the ideal gas law yields

$$T_{\hat{\nu},0} = \frac{n(\hat{\nu})}{n(\tilde{\nu})} T_{\tilde{\nu},0} = 3.09 \cdot 10^{-3}\,\text{K}\,, \tag{3.30}$$

i.e. the corresponding thermal photon energy is given by

$$kT = k\,(11/4)^{1/3} T_{\hat{\nu},0} = 3.73 \cdot 10^{-7}\,\text{eV}\,. \tag{3.31}$$

Then, using Eqs. (2.40) and (3.31), for the quantum gravity of the massless universe ($R \le R_{\text{Pl}}$), we obtain the distance

$$R = R_{\hat{\nu}} = \frac{\hbar c}{E_{\text{Pl}}^2} k\,(11/4)^{1/3} T_{\hat{\nu},0} = 4.938 \cdot 10^{-68}\,\text{cm}\,, \tag{3.32}$$

so that Eqs. (2.44) or (2.46) yield the initial thermal number density of the sterile neutrinos, which must be identical with that of the sterile neutrino relics $n_{\hat{\nu}}(R_{\hat{\nu}})$ or of the massless photons $n_\gamma(R_{\hat{\nu}})$ from the decay of these initial sterile neutrinos in the massless universe, i.e. we have

$$n_{\hat{\nu}}(R_{\hat{\nu}}) = n_\gamma(R_{\hat{\nu}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\hat{\nu}}^3} = 1.989\cdot 10^{197}\ \text{cm}^{-3}. \tag{3.33}$$

Therefore, by the universal decay law, for $n_\gamma(R_{\hat{\nu}}) = 1.989\cdot 10^{197}\,\text{cm}^{-3}$ (see Eq. (3.33)) and $n_\gamma(R_\text{f}) = 2.706\cdot 10^{282}\ \text{cm}^{-3}$ (see Eq. (3.20)), using a constant sterile neutrino volume $V(\hat{\nu})$ for $n_\gamma(R_{\hat{\nu}})$ and $n_\gamma(R_\text{f})$, we can assume

$$n_\gamma(R_{\hat{\nu}}) = n_\gamma(R_\text{f})\, \text{e}^{-t_\text{f}/\tau_{\hat{\nu}}}\,, \tag{3.34}$$

where t_f yields the age of the final state of the massive universe, whereas $\tau_{\hat{\nu}}$ describes the lifetime of the sterile neutrinos. Their constant volume $V(\hat{\nu})$ is determined in Sec. 3.3. Now, we have two possibilities.

Firstly, in the variant 1, we can thus calculate $\tau_{\hat{\nu}} = \tau_{\hat{\nu}1}$, using the age $t_\text{f} = t_\text{f1} = 2.136\cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}$ (see Eq. (2.29)) for the final state of the massive universe.

Secondly, in the variant 2, we can estimate the age $t_\text{f} = t_\text{f2}$ for the final state of the massive universe, taking $\tau_{\hat{\nu}} = \tau_{\hat{\nu}2} = 1.108\cdot 10^{18}\,\text{s} = 35.11\,\text{Gyr}$ (see Eq. (3.15)).

Then, taking the results (3.20) and (3.33), via Eq. (3.34), the variant 1 yields

$$\tau_{\hat{\nu}1} = t_\text{f1}\left(\ln\{n_\gamma(R_\text{f})/n_\gamma(R_{\hat{\nu}})\}\right)^{-1} = 1.090\cdot 10^{18}\ \text{s} = 34.52\,\text{Gyr}\,, \tag{3.35}$$

whereas the variant 2 provides

$$t_\text{f2} = \tau_{\hat{\nu}2}\ln\{n_\gamma(R_\text{f})/n_\gamma(R_{\hat{\nu}})\} = 2.172\cdot 10^{20}\ \text{s} = 6883\,\text{Gyr}\,. \tag{3.36}$$

Now, we show that the variant 2 is correct. For this goal, we introduce the proper distances

$$d_\text{f1} = R_\text{f} r_\text{f1} = ct_\text{f1} = 6.404\cdot 10^{28}\ \text{m} \tag{3.37}$$

and

$$d_\text{f2} = R_\text{f} r_\text{f2} = ct_\text{f2} = 6.512\cdot 10^{28}\ \text{m}\,, \tag{3.38}$$

where r_{f1} and r_{f2} describe again the dimensionless time-independent comoving coordinate distances r (see Eqs. (3.2) and (3.9)). Thus, via Eqs. (3.1) and (3.8), because of the dark energy of Eq. (3.10), we can form its energy densities

$$\rho_{f1}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{\text{eff}}}{d_{f1}} \right)^3 = 4.560 \cdot 10^{-4}\,\text{eV cm}^{-3} \tag{3.39}$$

and

$$\rho_{f2}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{\text{eff}}}{d_{f2}} \right)^3 = 4.337 \cdot 10^{-4}\,\text{eV cm}^{-3}, \tag{3.40}$$

so that because of the proper volumes V_{f1} and V_{f2} we have the ratios

$$\frac{\rho_{f1}}{\rho_{f2}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}} \right)^3 = \left(\frac{t_{f2}}{t_{f1}} \right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}} \right)^3 \cong 1.052. \tag{3.41}$$

The discrepancy between $t_f = t_{f1} = 2.136 \cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}$ (see Eq. (2.29)) and $t_{f2} = 2.172 \cdot 10^{20}\,\text{s} = 6883\,\text{Gyr}$ (see Eq. (3.36)) has the plausible reason that the derivation of $t_f = t_{f1}$ is alone based on the vacuum energy density $\rho_{\text{vac},\Lambda} c^2 = \Omega_\Lambda \rho_{0C} c^2 = 3.27 \cdot 10^3\,\text{eV cm}^{-3}$ (see Eq. (2.32) and Refs. [1, 2]) or the cosmological "constant" $\Lambda_\Lambda = 3\,\Omega_\Lambda / R_0^2 = 1.087 \cdot 10^{-52}\text{m}^{-2}$ (see Eq. (2.33) and Refs. [1, 2]), i.e. the limiting conditions (2.32) and (2.33) yield here the temperature $T = T_{f1} = 51.41\,\text{K}$ (see Eq. (2.34)).

However, this discrepancy can be explained uniquely by Eq. (2.38), using the assumptions for the times $\tilde{t} = \tau_{\hat{\nu}2} \ln 2 = 7.680 \cdot 10^{17}\,\text{s}$ ($\tau_{\hat{\nu}2}$ see Eqs. (3.15)) and $t = t_{\text{eff}} = 8.034 \cdot 10^{17}\,\text{s}$ (see Eq. (2.28)), so that the result (2.38) provides

$$\left(\frac{c^2 \tilde{\Lambda}}{3} \right)^{1/2} = \frac{t_{\text{eff}} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} \Omega_\Lambda^{1/2} H_0 = 1.997 \cdot 10^{-18}\,\text{s}^{-1} \tag{3.42}$$

as semi-empirical present limiting value. By Eq. (3.42), we obtain

$$\tilde{\Lambda} = 1.331 \cdot 10^{-52}\,\text{m}^{-2}, \tag{3.43}$$

so that the temperature $T = T_{f2} = 54.07\,\mathrm{K}$ results from Eqs. (2.31) and (3.43) for $N(T) = 3.362644$ {see Eq. (2.6) or Refs. [6, 8]}.

For the constant pressure $P = -\Omega_\Lambda \rho_{0C} c^2$, the ideal gas law yields following connection between the temperatures $T = T_{f2} = 54.07\,\mathrm{K}$ (see above) and $T = T_{f1} = 51.41\,\mathrm{K}$ (see also above) as well as the corresponding volumes V_{f2} and V_{f1} for the state changing according to Eq. (3.41):

$$\frac{T_{f2}}{T_{f1}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052\,. \tag{3.44}$$

Thus, we get

$$t_{f2} = t_{f1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 6883\ \mathrm{Gyr} \tag{3.45}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 35.11\ \mathrm{Gyr}\,, \tag{3.46}$$

i.e. $t_{f2} = 6883\,\mathrm{Gyr}$ (see Eq. (3.36)) and $\tau_{\hat{\nu}2} = 35.11\,\mathrm{Gyr}$ (see Eq. (3.15)) are correct.

Using the data of Tables III to V, the results, derived in this Sec. 3.2 for the total (massless and massive) universe, are also valid for the total (massless and massive) anti-universe.

3.3 The constant volume of the sterile neutrinos

In this chapter, we derive the constant volume $V(\hat{\nu})$ of the sterile neutrinos (see also Sec. 3.2) in the total ("massless" and massive) universe.

For this goal, we assume that in the final state of the massive universe by the sterile neutrinos of the dark energy $E_d = 5.017 \cdot 10^{89}\,\mathrm{eV}$ (see Eq. (3.10)) the volume V_f is occupied, i.e. we have

$$V_{\mathrm{f}} = \frac{E_{\mathrm{d}}}{\Omega_{\Lambda} \rho_{0\mathrm{C}} c^2} = 1.534 \cdot 10^{86}\,\mathrm{cm}^3 . \tag{3.47}$$

Using the number density $n(\hat{\nu}) = 0.178\,\mathrm{cm}^{-3}$ of the sterile neutrinos (see Sec. 3.2), we can introduce their constant volume $V(\hat{\nu})$ to

$$V(\hat{\nu}) = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})} , \tag{3.48}$$

so that for the final state of the massive universe the number N_{f} of the sterile neutrinos is given by

$$N_{\mathrm{f}} = \frac{V_{\mathrm{f}}}{V(\hat{\nu})} = \frac{n(\hat{\nu})\, V_{\mathrm{f}}}{\mathrm{const}(\hat{\nu})} = \frac{2.731 \cdot 10^{85}}{\mathrm{const}(\hat{\nu})} . \tag{3.49}$$

At the enormous age of the final state of the massive universe, we have assumed a complete decay of the N_{f} sterile neutrinos (see Sec. 3.1).

Consequently, in the massless universe, the beginning of the sterile neutrino decay, which must lead to the results (3.30) to (3.33), takes place in one photon and one sterile neutrino relic, so that instead of the volume V_{f} these 2 different decay products must yield the initial (i) volume V_{i} of the sterile neutrinos in the massless universe to

$$V_{\mathrm{i}} = \frac{2}{n(\hat{\nu})} = 11.24\,\mathrm{cm}^3 . \tag{3.50}$$

Thus, the initial number N_{i} of the sterile neutrinos of the massless universe must be defined by

$$N_{\mathrm{i}} = \frac{V_{\mathrm{i}}}{V(\hat{\nu})} = \frac{2}{\mathrm{const}(\hat{\nu})} . \tag{3.51}$$

Therefore, instead of Eq. (3.34), we can assume

$$N_{\mathrm{i}} = N_{\mathrm{f}}\, \mathrm{e}^{-t_{\mathrm{f}} / \tau_{\hat{\nu}}} , \tag{3.52}$$

so that via the Eqs. (3.48) to (3.52) we can form the expressions

$$N_{\mathrm{f}} = V(\hat{\nu})\, n_{\gamma}(R_{\mathrm{f}}) = \frac{n(\hat{\nu})\, V_{\mathrm{f}}}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\mathrm{f}}) \tag{3.53}$$

and

$$N_{\mathrm{i}} = V(\hat{\nu})\, n_\gamma(R_{\hat{\nu}}) = \frac{2}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_\gamma(R_{\hat{\nu}}) \,. \tag{3.54}$$

Then, the expressions (3.53) and (3.54) provide the conditions

$$\frac{n(\hat{\nu})\, V_{\mathrm{f}}}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_\gamma(R_{\mathrm{f}}) \tag{3.55}$$

and

$$\frac{2}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_\gamma(R_{\hat{\nu}}) \,. \tag{3.56}$$

Thus, the conditions (3.55) or (3.56) yield

$$\mathrm{const}(\hat{\nu}) = \left(\frac{n^2(\hat{\nu})\, V_{\mathrm{f}}}{n_\gamma(R_{\mathrm{f}})} \right)^{\frac{1}{2}} \cong 1.34 \cdot 10^{-99} \tag{3.57}$$

or

$$\mathrm{const}(\hat{\nu}) = \left(\frac{2\, n(\hat{\nu})}{n_\gamma(R_{\hat{\nu}})} \right)^{\frac{1}{2}} \cong 1.34 \cdot 10^{-99} \,, \tag{3.58}$$

so that Eq. (3.48) leads to the constant sterile neutrino volume

$$V(\hat{\nu}) = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})} \cong 7.528 \cdot 10^{-99}\ \mathrm{cm}^3 \,. \tag{3.59}$$

Taking Eqs. (3.20) and (3.33) as well as (3.53) to (3.59), we obtain the particle numbers (see above):

$$N_{\mathrm{f}} = V(\hat{\nu})\, n_\gamma(R_{\mathrm{f}}) \cong 2.04 \cdot 10^{184} \tag{3.60 a}$$

and

$$N_{\mathrm{i}} = V(\hat{\nu})\, n_\gamma(R_{\hat{\nu}}) \cong 1.50 \cdot 10^{99} \,. \tag{3.60 b}$$

Consequently, by Eqs. (3.20) and (3.33), considering the expressions (3.47) as well as (3.49 to (3.51)), the results (3.60 a) and (3.60 b) yield the following connections for the constant volume $V(\hat{\nu})$ of the sterile neutrinos

$$V(\hat{\nu}) = \frac{V_{\mathrm{f}}}{N_{\mathrm{f}}} = \frac{N_{\mathrm{f}}}{n_\gamma(R_{\mathrm{f}})} \cong 7.53 \cdot 10^{-99}\ \mathrm{cm}^3 \tag{3.61 a}$$

or

$$V(\hat{\nu}) = \frac{V_{\rm i}}{N_{\rm i}} = \frac{N_{\rm i}}{n_\gamma(R_{\hat{\nu}})} \cong 7.53 \cdot 10^{-99}\ {\rm cm}^3 \tag{3.61 b}$$

in accordance with Eq. (3.59).

Thus, the expressions (3.60 a) to (3.61 b) confirm the results (3.35) and (3.36) because of Eq. (3.34), i.e. we have again

$$\frac{t_{\rm f}}{\tau_{\hat{\nu}}} = \ln \frac{n_\gamma(R_{\rm f})}{n_\gamma(R_{\hat{\nu}})} = \ln \frac{N_{\rm f}}{N_{\rm i}} \cong 196.03\,. \tag{3.62}$$

With that, the assumption (3.34) is proved.

Using the data of Tables III to V, the results, derived in this Sec. 3.3 for the total universe, are also valid for the total anti-universe.

3.4 Heat accumulation in the final state (big bang) of the universe as reason of the very high energy density

In this chapter, we explain the high number densities (3.20) and (3.21) of the decay products of the sterile neutrinos as a result of the very high energy densities in the final state of the universe.

For this goal, we use the expression (3.29), which for positive pressure defines an energy decrease at the expansion of the massive universe. However, if for the final state of the universe the negative pressure $-P = \rho_{\rm vac}\, c^2$ is assumed, we obtain a gigantic energy increase, which by the heat accumulation leads to an overheat, so that the consequence is a heat death of this final state of the universe, since its heat death means simultaneously a transition in direction to the "massless" universe and the big bang, so that we get a relationship of the final state of the universe and the big bang. The reason is the increased concentration of the decay products of the sterile neutrinos by their deceleration in the final state of the universe, so that the very high temperature of the vacuum energy density $\rho_{\rm vac}\, c^2 = \rho_{\rm vac}(T)\, c^2$ of the universe (see Eq. (2.30)) can be attributed to the half dark energy $\frac{1}{2} E_{\rm d}$ (see Eqs. (3.3) and (3.4) as well as (3.11) and (3.12)). This assumption confirms the hypothesis of a joint origin of the dark matter and the dark energy by sterile neutrinos (see, e.g., Ref. [1]).

Because of the expressions (3.16) to (3.21), for the final state of the massive universe, the total energy density of the sterile neutrinos can be assumed to $\rho(\hat{\nu})\, c^2 = 2\rho_\gamma(R_{\rm eff})\, c^2 + 2\rho_{\frac{1}{2}\hat{\nu}}(R_{\rm eff})\, c^2$.

Then, using Eq. (2.30), in the final state of the massive universe, because of the condition $2\rho_\gamma(R_{\rm eff})\, c^2 = 2\rho_{\frac{1}{2}\hat{\nu}}(R_{\rm eff})\, c^2$, for the decay products of the sterile neutrinos, we can assume

$$2\frac{\pi^2}{15}\frac{(\frac{1}{2}E_{\rm d})^4}{(\hbar c)^3} = 6.787\cdot 10^{371}\ \text{eV cm}^{-3} =$$

$$= 2\rho_\gamma(R_{\rm eff})\, c^2 = 2\rho_{\frac{1}{2}\hat{\nu}}(R_{\rm eff})\, c^2 = \tfrac{1}{2}\rho(\hat{\nu})\, c^2 = 2\,\rho_{\rm vac}\, c^2 =$$

$$= 2\frac{1}{3}\frac{\pi^2}{15}\frac{(kT_1)^4}{(\hbar c)^3}\frac{1}{N(T)}, \tag{3.63}$$

so that for the final state of the massive universe we obtain the thermal energy

$$kT_1 = [3\,N(T)]^{\frac{1}{4}}\ \tfrac{1}{2}E_{\rm d}\,, \tag{3.64}$$

which leads to an overheat of the final state of the universe by the very high energy density (3.63), i.e. this universe meets one's heat death (see above).

Therefore, by the kinetic energy $E_{\rm K}(\gamma) = 2.70117 \times \frac{1}{2}E_{\rm d}$ (see Refs. [1, 2]), because of Eqs. (3.18) to (3.21), the energy densities (3.63) lead to the number density of the decay products of the sterile neutrinos

$$n(\hat{\nu}) = n_\gamma(R_{\rm f}) = 2n_\gamma(R_{\rm eff}) = 2.701178\,\frac{2\rho_\gamma(R_{\rm eff})\, c^2}{E_{\rm K}(\gamma)} =$$

$$= n_{\frac{1}{2}\hat{\nu}}(R_{\rm f}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\rm eff}) = 2.701178\,\frac{2\rho_{\frac{1}{2}\hat{\nu}}(R_{\rm eff})\, c^2}{E_{\rm K}(\gamma)} =$$

$$= 2\frac{\pi^2}{15}\left(\frac{\frac{1}{2}E_{\rm d}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\ \text{cm}^{-3}\,. \tag{3.65}$$

Similar to Eqs. (3.63) and (3.65), taking again Eqs. (3.18) to (3.21), we can express the energy densities $\frac{1}{2}\rho(\hat{\nu})\,c^2 = 2\rho_\gamma(R_{\rm eff})\,c^2 = 2\rho_{\frac{1}{2}\hat{\nu}}(R_{\rm f})\,c^2$ of the sterile neutrinos also as follows

$$\frac{\pi^2}{15}\frac{(kT_2)^4}{(\hbar c)^3} = \tfrac{1}{2}\rho(\hat{\nu})\,c^2 = 2\frac{\pi^2}{15}\frac{(\frac{1}{2}E_{\rm d})^4}{(\hbar c)^3} = 6.787\cdot 10^{371}\,\text{eV cm}^{-3}\,, \quad (3.66)$$

so that in the final state of the universe we get the thermal energy

$$kT_2 = 2^{1/4}\,\tfrac{1}{2}E_{\rm d}\,, \quad (3.67)$$

i.e. analogous to Eq. (3.65) we find

$$n(\hat{\nu}) = 2.701178\,\frac{\frac{1}{2}\rho(\hat{\nu})\,c^2}{E_{\rm K}(\gamma)} =$$

$$= 2\frac{\pi^2}{15}\left(\frac{\frac{1}{2}E_{\rm d}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\text{cm}^{-3}\,. \quad (3.68)$$

Indeed, by Eqs. (3.65) and (3.68), we can explain the high number densities (3.20) and (3.21) of the decay products of the sterile neutrinos. This result is confirmed by Eqs (3.64) and (3.67), since we can introduce the ratio

$$\frac{T_1}{T_2} = \left(\frac{3\,N(T)}{2}\right)^{1/4}, \quad (3.69)$$

so that for the final state of the universe (see Eqs. (3.37) to (3.41)) the entropy conservation (see, e.g., Refs. [6, 8]) yields

$$\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{3\,N(T)}{2}\right)^{3/4} = \frac{V_{\rm f2}}{V_{\rm f1}} = \left(\frac{d_{\rm f2}}{d_{\rm f1}}\right)^3. \quad (3.70)$$

Using the condition (3.41) in the result (3.70), we obtain

$$\left(\frac{3\,N(T)}{2}\right)^{3/4} \cong 1.052 \quad (3.71)$$

with the solution

$$N(T) \cong 0.713 \quad (3.72)$$

in accordance with

$$N(T) = \left(\frac{4}{11}\right)^{1/3} \cong 0.713 \quad \text{(see Eq. (2.7))}. \tag{3.73}$$

Then, by the conditions (3.41) as well as (3.70) to (3.73), the results (3.45) and (3.46) are confirmed because of

$$t_{f2} = t_{f1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 6883 \text{ Gyr} \tag{3.74 a}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 35.11 \text{ Gyr}. \tag{3.74 b}$$

With that, the conception of a heat death of the final state of the massive universe, which simultaneously means the transition in direction to the massless universe and the big bang, is confirmed via the corresponding vacuum energy density (see Eq. (2.30)) of the quantum gravity for the massive universe. Then, taking again the conditions (3.41) as well as (3.70) to (3.73), for the dark energy density (3,39), we can introduce

$$\rho_{f1}c^2 = \Omega_\Lambda \rho_{0C}c^2 \left(\frac{d_{\text{eff}}}{c\,t_{f2}}\right)^3 \left(\frac{T_1}{T_2}\right)^3 = \rho_{f2}c^2\left(\frac{T_1}{T_2}\right)^3 =$$

$$= \rho_{f2}c^2\left(\frac{3\,N(T)}{2}\right)^{3/4} \cong 4.56\cdot 10^{-4} \text{ eV cm}^{-3}, \tag{3.75 a}$$

whereas the reversal

$$\rho_{f2}c^2 = \Omega_\Lambda \rho_{0C}c^2 \left(\frac{d_{\text{eff}}}{c\,t_{f1}}\right)^3 \left(\frac{T_2}{T_1}\right)^3 = \rho_{f1}c^2\left(\frac{T_2}{T_1}\right)^3 =$$

$$= \rho_{f1}c^2\left(\frac{2}{3\,N(T)}\right)^{3/4} \cong 4.34\cdot 10^{-4} \text{ eV cm}^{-3} \tag{3.75 b}$$

is forbidden, i.e. the dark energy density $\rho_{f1}c^2 \equiv 4.56\cdot 10^{-4}\text{ eV cm}^{-3}$ of the final state of the universe is a function of t_{f2} because of $T_1 > T_2$.

Considering the data of Tables III to V, the results, derived in this Sec. 3.4 for the total universe, are also valid for the total anti-universe.

3.5 Mean (maximum) energies of "massless" and massive universe

In this chapter, we explain the mean (maximum) energies of the total ("massless" and massive) universe, applying the relationship between its final state and the big bang (see Sec. 3.11). In work [1], the interpretation of these mean (maximum) energies is incorrect. Therefore, in this work, we correct this interpretation. For this goal, for the "massless" and massive universe (see also Refs. [1, 2]). we consider their joint boundaries $R = R_{\text{Pl}}$ and $E = E_{\text{Pl}}$.

As starting point, we use the massive universe. Then, the boundary $E = E_{\text{Pl}}$ of the massive universe is the reason that we can assume the mean energy densities $E_{\text{Pl}}\, n_{\gamma}(R_{\text{f}})$ or $E_{\text{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}})$ for its final state, taking Eqs. (3.20) or (3.21). Thus, by the decay products of the sterile neutrinos, we have correspondingly the two identical mean energy densities

$$\bar{\rho}_1 c^2 = E_{\text{Pl}}\, n_{\gamma}(R_{\text{f}}) = 3.304\cdot 10^{310}\ \text{eV cm}^{-3} \qquad (3.76)$$

or

$$\bar{\rho}_1 c^2 = E_{\text{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}}) = 3.304\cdot 10^{310}\ \text{eV cm}^{-3}. \qquad (3.77)$$

Then, using the sterile neutrino volume $V(\hat{\nu}) = 7.528\cdot 10^{-99}\ \text{cm}^3$ (see Eq. (3.59)) for the final state of the massive universe, we can evaluate the mean (maximum) energy

$$\bar{E}_1 = \bar{\rho}_1 c^2 \times V(\hat{\nu}) \cong 2.49\cdot 10^{212}\ \text{eV}. \qquad (3.78)$$

By the sterile neutrino number $N_{\text{f}} = 2.04\cdot 10^{184}$ (see Eq. (3.60 a)), for the beginning of the massive universe, the greatest possible relativistic particle energy {Planck energy (see also Refs. [1, 2])} is again defined by

$$E_{\text{Pl}} = \frac{\bar{E}_1}{N_{\text{f}}} \cong 1.22\cdot 10^{28}\ \text{eV}. \qquad (3.79)$$

In the next step, we consider the mean (maximum) energy of the "massless" universe, using the transition direction from the final state of the universe to the "massless" universe and the big bang as a result of the complete decay of the sterile neutrinos (see Sec. 3.1). Taking the expressions (3.63) and (3.66) as well as (3.69) and (3.73), via the ratio $4\rho_{\text{vac}}/\rho(\hat{\nu})$, multiplied by $\rho(\hat{\nu})\, c^2 \times (T_2/T_1)^4$, because of $4\rho_{\text{vac}}/\rho(\hat{\nu})=1$, for the final state of the massive universe, we can derive the very high energy density

$$\rho_{\text{f}}(\hat{\nu})\,c^2=\rho(\hat{\nu})\,c^2\times\left(\frac{T_2}{T_1}\right)^4=4\rho_{\text{vac}}\,c^2\times\left(\frac{T_2}{T_1}\right)^4=$$

$$=4\frac{\pi^2}{15}\frac{(\tfrac{1}{2}E_{\text{d}})^4}{(\hbar c)^3}\frac{2}{3\,N(T)}\cong 1.268\cdot 10^{372}\,\text{eV cm}^{-3}\,. \qquad (3.80)$$

Similar to the expression (3.80), the fourfold vacuum energy density (3.25), multiplied by $(T_2/T_1)^4=2/3N(T)$, permits the introduction of the high energy density

$$\rho_{\text{BB}}(\hat{\nu})\,c^2=4\rho_{\text{vac}}(R_{\text{BB}})\,c^2\times\left(\frac{T_2}{T_1}\right)^4=\frac{8}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c\,R_{\text{BB}}^2}\left(\frac{T_2}{T_1}\right)^4=$$

$$=\frac{8}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c\,R_{\text{BB}}^2}\frac{2}{3\,N(T)}\cong 1.579\cdot 10^{248}\,\text{eV cm}^{-3}\,. \qquad (3.81)$$

Then, we can assume a mean energy density of the sterile neutrinos to

$$\bar{\rho}_2\,c^2=(\rho_{\text{f}}(\hat{\nu})\,c^2\times\rho_{\text{BB}}(\hat{\nu})\,c^2)^{1/2}\cong 1.415\cdot 10^{310}\,\text{eV cm}^{-3}\,, \qquad (3.82)$$

so that via the volume $V=\tfrac{4}{3}\pi R_{\text{Pl}}^3=1.768\cdot 10^{-98}\,\text{cm}^3$ we obtain the mean (maximum) energy of the "massless" universe to

$$\bar{E}_2=\bar{\rho}_2\,c^2\times V\cong 2.50\cdot 10^{212}\,\text{eV}\,, \qquad (3.83)$$

i.e. because of the sterile neutrino number $N_{\text{f}}=2.04\cdot 10^{184}$ (see Eq. (3.60 a)) we find again the Planck energy (see Eq. (3.79))

$$E_{\text{Pl}}=\frac{\bar{E}_2}{N_{\text{f}}}\cong 1.22\cdot 10^{28}\,\text{eV}\,. \qquad (3.84)$$

It is plausible that the mean energies (3.78) and (3.83) must be identical, since the ratio of the mean densities $\bar{\rho}_1$ (see Eqs. (3.76) and (3.77)) and $\bar{\rho}_2$ (see Eq. (3.82)) is defined by

$$\frac{\bar{\rho}_1}{\bar{\rho}_2}=\frac{V}{V(\hat{\nu})}\cong 2.34\,. \qquad (3.85)$$

We assume that we can interpret the mean (maximum) energy (3.83) as kinetic energy,, which is necessary for the complete expansion from the big bang of the "massless" universe to the final state of the massive universe. In contrast to energy (3.83), for the mean (maximum) energy (3.78), we assume its interpretation as potential energy, which is responsible for the transition from the final state of the massive universe in direction to the "massless" universe and the big bang. We prove these assumptions in the chapters 3.8 and 3.11, respectively.

Using the data of Tables III to V, the results, derived in this Sec. 3.5 for the total universe, are also valid for the total anti-universe.

3.6 The zero-point oscillations for the massless universe

In this chapter, we demonstrate that the existence of the "massless" universe ($0 \le R_{\rm BB} \le R \le R_{\rm Pl}$) can be attributed to the zero-point oscillations.

Therefore, at the distance $R_{\rm BB} = 2.069 \cdot 10^{-96}$ cm (see Eq. (3.23)), these zero-point oscillations must be formed via the greatest possible relativistic energy $E_{\rm BB} = 2\widetilde{E}_{\rm BB} = \hbar c / R_{\rm BB} = 9.537 \cdot 10^{90}\,{\rm eV}$ (see Eqs. (3.28 a) and (3.28 b)) because of the start of the universe at the distance $R_{\rm BB} = 2.069 \cdot 10^{-96}$ cm {see Eq. (3.23) and Sec. 3.10}.

Now, we prove the formation of these zero-point oscillations for the existence of the massless universe. For this goal, we use de Broglie waves. In the extremely relativistic case, between its momentum p of the particle as well as its relativistic energy E, we have the connection

$$p = \frac{E}{c}. \tag{3.86}$$

Then, the de Broglie wavelength λ of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$ and $\hat{\nu}_{\rm b}$, which describe the joint origin of the dark matter and dark energy (see Refs. [1, 2]), is defined by

$$\lambda = 2\pi \frac{\hbar}{p} = \frac{\hbar c.}{E/2\pi}. \tag{3.87}$$

For Eq. (3.87), we can write

$$\frac{\hbar c}{\lambda}=\frac{E}{2\pi}. \tag{3.88}$$

Because we have $N_{\hat{\nu}\uparrow}=3$ excited sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$ and $\hat{\nu}_{\rm b}$ as a result of the zero-point oscillation via Dirac's fermion-antifermion theory, we can introduce $E(\hat{\nu}_\Lambda)+E(\hat{\nu}_{\rm dm})+E(\hat{\nu}_{\rm b})\to E$ for $E(\hat{\nu}_\Lambda)=E(\hat{\nu}_{\rm dm})=E(\hat{\nu}_{\rm b})$ in Eqs. (3.87) and (3.88), so that analogous to Eq. (3.88) we get the condition $\hbar c/\lambda(\hat{\nu}_\Lambda)=\hbar c/\lambda(\hat{\nu}_{\rm dm})=\hbar c/\lambda(\hat{\nu}_{\rm b})$ with $\lambda(\hat{\nu}_\Lambda)=\lambda(\hat{\nu}_{\rm dm})=\lambda(\hat{\nu}_{\rm b})\to\lambda$ for the "massless" universe. The arrow $\uparrow$ describes their spin direction.

Analogously, the zero-point oscillations must excite simultaneously $N_{\hat{\hat{\nu}}\uparrow}=3$ sterile anti-neutrinos $\hat{\hat{\nu}}$ in the anti-universe, so that we obtain $N=3$ sterile neutrino-antineutrino pairs for these "massless" universes. If we consider correspondingly still $N_{\hat{\nu}\downarrow}=3$ sterile neutrinos and $N_{\hat{\hat{\nu}}\downarrow}=3$ anti neutrinos with opposite spin direction (Pauli exclusions principle), we have together $N=6$ excited sterile neutrino-antineutrino pairs.

Considering the simultaneous application of Dirac's fermion-antifermion theory for the universe and the anti-universe, we must introduce $N=12$ sterile neutrino-antineutrino pairs. This N doubling is extremely important at the calculation of the properties of the massive universe and anti-universe {see the application of Eq. (3.90 c) in Eq. (3.132) to (3.134)}.

For the massive universe (anti-universe), we assume that the excitation energy of the zero-point oscillations has an energy loss in form of the factor $(\Omega_{\rm dm}+\Omega_\Lambda)^{1/6}=0.9915$ by the induction of the decay of the 3 fundamental massive sterile neutrinos of the dark matter and dark energy into two massless decay products (see Sec. 4). In Ref. [1], we have neglected this influence of the dark matter and dark energy. In this work, we consider this effect.

Therefore, for the N excited sterile neutrino-antineutrino pairs, we must introduce the real excitation (exc) energy $E_{\rm exc}(N)=(\Omega_{\rm dm}+\Omega_\Lambda)^{1/6}E_{\rm BB}$, where $E_{\rm BB}\equiv 2\widetilde{E}_{\rm BB}=\hbar c/R_{\rm BB}=9.537\cdot 10^{90}$ eV {see Eqs. (3.28 b) and (3.130)}.

Consequently, at N excited sterile neutrino-antineutrino pairs, by the transition $E\to E_{\rm exc}(N)$, Eq. (3.88) must be transformed correspondingly to

$$\text{a)}\quad (\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6} N \frac{\hbar c}{\lambda} = \frac{E_{\rm exc}(N)}{2\pi},$$

$$\text{b)}\quad \lambda = \frac{(\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6}\hbar c}{E^{*}(N)},$$

$$\text{c)}\quad E^{*}(N) = \frac{E_{\rm exc}(N)}{N\,2\pi} = \frac{(\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6} E_{\rm BB}}{N\,2\pi}. \qquad (3.89)$$

Thus, for the cases $N = 3, 6, 12$, Eq. (3.89 c) yields

$$\text{a)}\quad E^{*}(3) = (\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6} E_{\rm BB}/6\pi = 5.017\cdot 10^{89}\ \text{eV},$$

$$\text{b)}\quad E^{*}(6) = (\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6} E_{\rm BB}/12\pi = 2.509\cdot 10^{89}\ \text{eV},$$

$$\text{c)}\quad E^{*}(12) = (\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6} E_{\rm BB}/24\pi = 1.254\cdot 10^{89}\ \text{eV}. \qquad (3.90)$$

Consequently, the cases $N = 3$ and $N = 6$ are described by the energies (3.90 a) and (3.90 b), which agree with dark energy $E_{\rm d} = 5.017\cdot 10^{89}$ eV (see Eq. (3.10)) and with the half dark energy $\frac{1}{2} E_{\rm d} = 2.509\cdot 10^{89}$ eV (see Eqs. (3.11) and (3.12)) in the massive universe, respectively. The case $N = 12$, which is defined by the doubling of the number $N = 6$ of the sterile neutrino-antineutrino pairs (see above), is taken into account by the energy (3.90 c). Therefore, the energy (3.90 c) must be identical with $\frac{1}{4} E_{\rm d} = 1.254\cdot 10^{89}$ eV. To this energy (3.90 c), we will return in Eqs. (3.112) and (3.113) as well as (3.132 to (3.134)).

By Eqs. (3.89 b) and (3.90), we find the corresponding wavelength λ to

$$\text{a)}\quad \lambda = \frac{(\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6}\hbar c}{E^{*}(3)} = \frac{\hbar c}{E_{\rm BB}/6\pi} = 3.90\cdot 10^{-95}\ \text{cm},$$

$$\text{b)}\quad \lambda = \frac{(\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6}\hbar c}{E^{*}(6)} = \frac{\hbar c}{E_{\rm BB}/12\pi} = 7.80\cdot 10^{-95}\ \text{cm},$$

$$\text{c)}\quad \lambda = \frac{(\Omega_{\rm dm}+\Omega_{\Lambda})^{1/6}\hbar c}{E^{*}(12)} = \frac{\hbar c}{E_{\rm BB}/24\pi} = 1.56\cdot 10^{-94}\ \text{cm}. \qquad (3.91)$$

Introducing the reduced wavelength $\lambda\!\!\!^{-} = \lambda/2\pi$, Eq. (3.91) provides

$$\text{a)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{\rm dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(3)} = \frac{\hbar c}{E_{\rm BB}/3} = 6.21\cdot 10^{-96}\ \text{cm}\,,$$

$$\text{b)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{\rm dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(6)} = \frac{\hbar c}{E_{\rm BB}/6} = 1.24\cdot 10^{-95}\ \text{cm}\,,$$

$$\text{c)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{\rm dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(12)} = \frac{\hbar c}{E_{\rm BB}/12} = 2.48\cdot 10^{-95}\ \text{cm}\,. \qquad (3.92)$$

By generalization of Eq. (3.92), we get $R_{\rm BB} = \lambda\!\!\!^{-}/N = \hbar c/E_{\rm BB}$, where N describes again the sterile neutrino-antineutrino pair number, i.e. we have $\lambda\!\!\!^{-} = N R_{\rm BB}$ because of N as the basis of the zero-point oscillations.

Therefore, we can assume that the reduced wavelengths (3.92) lie in the wavelength range $\lambda\!\!\!^{-}_{\rm BB} = R_{\rm BB} < \lambda\!\!\!^{-} < \lambda\!\!\!^{-}_{\hat\nu} = R_{\hat\nu}$ of the massless universe, where the distance $R_{\hat\nu} = 4.938\cdot 10^{-68}\text{cm}$ is defined by Eq. (3.32). Thus, for $\lambda\!\!\!^{-}_{\hat\nu}$, we have the condition $\lambda\!\!\!^{-}_{\hat\nu} = N R_{\rm BB} = R_{\hat\nu}$, whereat the number N of the particle-antiparticle pairs is determined by $N = R_{\hat\nu}/R_{\rm BB} = 2.387\cdot 10^{28}$. Then, the wavelength $\lambda\!\!\!^{-}_{\rm Pl} = N R_{\rm BB} = R_{\rm Pl}$ is also given by $N = R_{\rm Pl}/R_{\rm BB} = 7.812\cdot 10^{62}$.

Using the data of Tables III to V, the corresponding results, derived in this Sec. 3.6 for the massless universe, are also valid for the massless anti-universe.

With that, we have shown that the massless universe and anti-universe are connected with each other by the zero-point oscillations as a result of Dirac's fermion-antifermion theory.

We can thus assume that zero-point oscillations are also responsible for the massive and present universes, we will return to this problem in Sec. 3.7.

3.7 The particle horizon and the zero-point oscillations for the early and late massive universes (including the present universes)

In Ref. [1], the derivation of the particle horizon distances and of the sterile neutrino-antineutrino pair numbers are partly incorrect. Therefore, in this work, we correct now this inadmissibility.

According to Weinberg [8], the particle horizon distance $d_{\max}(t) = d_{\max}(z)$ of the known universe is defined by

$$d_{\max}(t) = d_{\max}(z) = R(t)\int_0^t \frac{c\,dt'}{R(t')}, \tag{3.93}$$

since the time t is a function of the redshift z (see Eq. (2.12)). This particle horizon limits the distance at which we can observe past events.

Using Eqs. (2.1) as well as (2.3) and (2.4), for the present (massive) universe ($z=0$), by its age $t=t_0$ (see Tables I and V), in accordance with Weinberg, the value $d_{\max}(t_0) = d_{\max}(0)$ is given by

$$d_{\max}(t_0) = d_{\max}(0) = \frac{c}{H_0}\int_0^1 \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_{\mathrm{m}} x^{-3} + \Omega_{\mathrm{r}} x^{-4})^{1/2}}, \tag{3.94}$$

where $x \equiv R/R_0 = 1/(1+z)$ (see Eq. (2.15)) and the corresponding quantities are explained by Tables I and V as well as by Eq. (2.5).

Then, for the known universe, we can introduce the general expression

$$d_{\max}(z) = \frac{R_0}{1+z}\int_0^{1/(1+z)} \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_{\mathrm{m}} x^{-3} + \Omega_{\mathrm{r}} x^{-4})^{1/2}}. \tag{3.95}$$

However, for $x = 1/(1+z) \to 0$, the integrals (3.94) and (3.95) would be infinite. Consequently, we must determine a new lower integration limit. For this goal, via the big bang, we can assume as the lower integration limit $x = 1/(1+z_{\mathrm{BB}})$, if we select $1+z_{\mathrm{BB}} = R_0/R_{\mathrm{BB}} = 6.646\cdot 10^{123}$, so that because of $1+z_{\mathrm{M}} = 2.916\cdot 10^{30}$ {see the new inflation model (2.14) to (2.18) as well as Refs. [1-3]} we have $1+z_{\mathrm{BB}} >> 1+z_{\mathrm{M}}$, i.e., for example, Eq. (3.95) can be converted into its smallest possible value

$$d_{\max}(z_{\mathrm{M}}) = \frac{R_0}{1+z_{\mathrm{M}}}\int_{1/(1+z_{\mathrm{BB}})}^{1/(1+z_{\mathrm{M}})} \frac{dx}{\Omega_{\mathrm{r}}^{1/2}}. \tag{3.96}$$

Then, at the limit $R \to R_{\mathrm{Pl}}$, because of this new inflation model, we must apply (see Eq. (2.5)) and $N(T) = 1/2\,\Omega_\gamma$ (see Refs. [1-3]), so that Eq. (3.96) yields

$$d_{\max}(z_{\mathrm{M}}) = \frac{R_0}{(1+z_{\mathrm{M}})}\left[\frac{2}{1+z_{\mathrm{M}}} - \frac{2}{1+z_{\mathrm{BB}}}\right] = 2R_{\mathrm{Pl}} = c2t_{\mathrm{Pl}}\,. \tag{3.97}$$

For the quantum mechanical range $1+z_{\mathrm{BB}} \geq 1+z \geq 1+z_{\mathrm{M}}$ of the massless universe ($R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$ or $t_{\mathrm{BB}} \leq t \leq t_{\mathrm{Pl}}$) [1, 2], we must apply the result $R = ct$ (see Eq. (2.43)) of the quantum gravity [1, 2] in Eq. (3.93) with the lower integration limit $t' = t_{\mathrm{BB}}$, so that the particle horizon (3.93) has the limiting expression

$$d(t) = c(t - t_{\mathrm{BB}})\,, \tag{3.98}$$

which disappears for $t = t_{\mathrm{BB}} = 6.901 \cdot 10^{-107}$ s (see Eq. (3.24)).

Consequently, because the results (3.97) and (3.98), we obtain the plausible condition

$$d(t) = c\,(t_{\mathrm{Pl}} - t_{\mathrm{BB}}) < d_{\max}(z_{\mathrm{M}}) = c2t_{\mathrm{Pl}}\,. \tag{3.99}$$

Thus, for the total (massless and massive) universe, Eqs. (3.94) to (3.99) yield the connection

$$0 \leq d(t \leq t_{\mathrm{Pl}}) < 2R_{\mathrm{Pl}} \leq d_{\max}(z \geq z_{\mathrm{M}}) \leq d_{\max}(0)\,. \tag{3.100}$$

According to the end of Sec. 3.6, the value $1+z_{\mathrm{BB}} = R_0/R_{\mathrm{BB}} = 6.646 \cdot 10^{123}$ is identical with the sterile neutrino-antineutrino pair number

$$N = N_0 = \frac{R_0}{R_{\mathrm{BB}}} \tag{3.101}$$

for the universe and anti-universe, i.e. we must assume that the sterile neutrino (antineutrino) number $\frac{1}{2}N_0$ is alone responsible for the universe (anti-universe).

Now, we prove this assumption by the new inflation model (see Eqs. (2.14) to (2.18) or Refs. [1, 2]) via the early ($R_{\mathrm{Pl}} \leq \widetilde{R} \leq \widetilde{R}_0$) for the late ($\widetilde{R}_0 \leq R \leq R_0$) massive universe [1, 2]. For this goal, by Eqs. (2.1) as well as (2.13) and (2.14), we can use $\dot{\widetilde{R}}/\widetilde{R} = (\frac{1}{2}N(T)\,\Omega_\gamma)^{\frac{1}{2}} H_0 (1+z)^2$, so that because of $N(T) = 1/2\,\Omega_\gamma$ at $t = t_{\mathrm{Pl}}$ we can write

$$\frac{\dot{\widetilde{R}}(t_{\mathrm{Pl}})}{\widetilde{R}(t_{\mathrm{Pl}})} = H_{\mathrm{Pl}} = \frac{1}{2\,t_{\mathrm{Pl}}} = \tfrac{1}{2} H_0 (1+z_{\mathrm{M}})^2 \tag{3.102}$$

or

$$\frac{H_{\mathrm{Pl}}}{H_0} = \frac{1}{2H_0\,t_{\mathrm{Pl}}} = \frac{\frac{1}{2}R_0}{R_{\mathrm{Pl}}} = \frac{\frac{1}{2}N_0}{N_{\mathrm{Pl}}}\,. \tag{3.103}$$

Indeed, via the result (3.101), Eq. (3.103) yields the sterile neutrino (antineutrino) number alone for the universe (anti-universe) to

$$\tfrac{1}{2} N_0 = \tfrac{1}{2} R_0 N_{\mathrm{Pl}} / R_{\mathrm{Pl}} = \tfrac{1}{2} R_0 / R_{\mathrm{BB}} = 3.323 \cdot 10^{123}, \quad (3.104)$$

whereat

$$N_{\mathrm{Pl}} = R_{\mathrm{Pl}} / R_{\mathrm{BB}} = 7.812 \cdot 10^{62} \quad \text{(see Sec. 3.6)}. \quad (3.105)$$

By Eq. (3.104), we have supported the assumption to the end of Sec. 3.6 that the massive universe can be attributed also to zero-point oscillations, which are based on the sterile neutrino-antineutrino pairs.

Using the data of Tables III to V, the results, derived in this Sec. 3.7 for the massive universe, are also valid for the massive anti-universe.

Then, by Eq. (3.105), the Planck energy of the early massive universe or anti-universe is determined by

$$E_{\mathrm{Pl}} = N_{\mathrm{Pl}} \times kT_{\mathrm{BB}} \cong 1.221 \cdot 10^{28}\ \mathrm{eV}. \quad (3.106)$$

The Hubble (H) energy E_{H} of the present late massive universe or anti-universe can be evaluated by the present critical energy density given by $\rho_{0\mathrm{C}}\, c^2 = 4.77 \cdot 10^3\ \mathrm{eV\,cm^{-3}}$ (see Tables I and V), and the Hubble length $R_0 = 1.375 \cdot 10^{28}\,\mathrm{cm}$ (see Tables I and V as well as Eq. (2.17)) to

$$E_{\mathrm{H}} \cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3} \pi R_0^3 \cong 5.194 \cdot 10^{88}\ \mathrm{eV}. \quad (3.107)$$

This energy (3.107) can be determined also by the smallest thermal (particle) energy $kT_{\mathrm{BB}} = 1.536 \cdot 10^{-35}\,\mathrm{eV}$ (see Eq. (3.27)), multiplied by the sterile neutrino (antineutrino) number (3.104), so that the Hubble energy of the present universe (anti-universe) is also defined by

$$E_{\mathrm{H}} = \tfrac{1}{2} N_0 \times kT_{\mathrm{BB}} \cong 5.194 \cdot 10^{88}\ \mathrm{eV}. \quad (3.108)$$

The results (3.107) and (3.108) support again the assumption that the present late massive universe (anti-universe) can be described also by zero-point oscillations (see the end of Sec. 3.6).

Similarly, by Eqs. (3.20) and (3.27), we get

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\, c^2 = n_\gamma(R_{\mathrm{f}}) \times kT_{\mathrm{BB}} \cong 4.227 \cdot 10^{247}\ \mathrm{eV\,cm^{-3}} \quad (3.109)$$

in excellent agreement with the result (3.25).

3.8 The greatest possible gravitational energy and the hypothetical superforce of the particle interactions

Using in the massless universe the Planck energy $E_{\mathrm{Pl}} = (\hbar c^5/G_N)^{1/2}$ for the big bang at $R = R_{\mathrm{BB}}$ (see Eq. (3.23)), we obtain the normal gravitational energy $\overline{E}_{\mathrm{BB}}$ at the big bang for the massless universe and anti-universe by the gravitational potential energy to

$$\overline{E}_{\mathrm{BB}} = \frac{G_N}{c^4}\frac{E_{\mathrm{Pl}} \times E_{\mathrm{Pl}}}{R_{\mathrm{BB}}} = \frac{\hbar c}{R_{\mathrm{BB}}} = 9.537 \cdot 10^{90}\ \mathrm{eV}. \qquad (3.110)$$

The result (3.110) is equivalent to the energy (3.28 b). This energy (3.28 b) was interpreted as the energy $E_{\mathrm{BB}} = \hbar c/R_{\mathrm{BB}}$ via the potential of a new attractive force, which is now assumed as the attractive, hypothetical superforce of the particle interactions.

Therefore, we can assume that the normal gravitational energy (3.110) of the big bang corresponds to the highest value of the potential energy $E = \hbar c/R$ of the attractive, hypothetical superforce of the particle interactions.

At $R = R_{\mathrm{Pl}} = (\hbar G_N/c^3)^{1/2}$, the normal potential energy of this hypothetical superforce has its lowest value E_{Pl}, where the unification of the strong and electroweak interaction (grand unification) begins, since the gravitational force and this hypothetical superforce are here again equivalent because of

$$E_{\mathrm{Pl}} = \frac{G_N}{c^4}\frac{E_{\mathrm{Pl}} \times E_{\mathrm{Pl}}}{R_{\mathrm{Pl}}} = \frac{\hbar c}{R_{\mathrm{Pl}}} = 1.220932 \cdot 10^{28}\ \mathrm{eV}. \qquad (3.111)$$

Because of Eq. (3.110), the normal gravitational energy $\overline{E}_{\mathrm{BB}} = \hbar c/R_{\mathrm{BB}}$ at the big bang and the highest energy value $E_{\mathrm{BB}} = \hbar c/R_{\mathrm{BB}}$ (see Eq. (3.110)) of this hypothetical superforce are conserved quantities with the same values.

Consequently, the greatest possible value $2\overline{E}_{\max}$ of the gravitational energy at the big bang (origin) for the universe and anti-universe {distance $2R_{\mathrm{BB}}$ (see Eqs. (3.28 a), (3.128) and (3.129))} is precisely and uniquely determined by the energy $E^*(12) = (\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6}\, E_{\mathrm{BB}}/24\pi = 1.254 \cdot 10^{89}$ eV (see Eq. (3.90 c)) to

$$2\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{E^*(12)\}\times\{E^*(12)\}}{2R_{\mathrm{BB}}} \cong 5.03\cdot 10^{212}\,\mathrm{eV}\,, \qquad (3.112)$$

so that for the universe (anti-universe) alone we find the greatest possible gravitational energy $\overline{E}_{\max}$ to

$$\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{E^*(12)\}\times\{E^*(12)\}}{2\times 2R_{\mathrm{BB}}} \cong 2.51\cdot 10^{212}\,\mathrm{eV}\,. \qquad (3.113)$$

This energy (3.113), which is interpreted as the kinetic energy for the complete expansion of the total ("massless" and massive) universe (anti-universe), is so identical with the mean (maximum) energies (3.83) {see also the end of Sec. 3.5}.

More generally, for the potential $V(R)$ of the hypothetical superforce of the particle interactions, in the massless universe, analogous to the gravitational potential, we can assume

$$V(R) = -\frac{\hbar c}{R}\,, \qquad (3.114)$$

so that we find the law of this hypothetical superforce via the potential (3.114) to

$$\vec{K}(\vec{R}) = -\frac{\partial V(R)}{\partial \vec{R}} = -\frac{\vec{R}}{R}\frac{dV(R)}{dR} =$$

$$= -\frac{\hbar c}{R^2}\frac{\vec{R}}{R} \quad \text{for} \quad R_{\mathrm{BB}} \le R \le R_{\mathrm{Pl}}\,. \qquad (3.115)$$

Using the data of Tables III to V, the results, derived in this Sec. 3.8 for the "massless" and massive universe, are also valid for the "massless" and massive anti-universe.

3.9 The very high temperature of the big bang

Using Eqs. (2.40) and (2.41), we can introduce the vacuum energy density $\widehat{\rho}_{\mathrm{vac}}\,c^2$ of the quantum gravity of the massless universe as a function of the temperature, i.e. we obtain

$$\rho_{\text{vac}}(R)\,c^2 \to \widehat{\rho}_{\text{vac}}\,c^2 = \widehat{\rho}_{\text{vac}}(T)\,c^2 = \frac{2}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,(kT)^2}\,. \tag{3.116}$$

Because of the relationship of the final state of the massive universe and the big bang, at $t = t_{\text{BB}}$, in the massless universe, for the big bang, according to Eq. (3.129), the relativistic energy uncertainty

$$\widetilde{E}_{\text{BB}} = k\widetilde{T}_{\text{BB}} = \frac{\hbar}{2t_{\text{BB}}} = \frac{\hbar c}{2R_{\text{BB}}} = 4.769\cdot 10^{90}\ \text{eV} \tag{3.117}$$

must be connected with the dark energy density of the final state of the massive universe (see Eq. (3.75 a))

$$\rho_{\text{f}\,1}c^2 = \Omega_\Lambda \rho_{0\text{C}}c^2\left(\frac{d_{\text{eff}}}{c\,t_{\text{f}\,2}}\right)^3\left(\frac{T_1}{T_2}\right)^3 = \Omega_\Lambda \rho_{0\text{C}}c^2\left(\frac{d_{\text{eff}}}{c\,t_{\text{f}\,2}}\right)^3\left(\frac{3\,N(T)}{2}\right)^{3/4} =$$

$$= \rho_{\text{f}\,2}c^2\left(\frac{3\,N(T)}{2}\right)^{3/4} = 4.56\cdot 10^{-4}\ \text{eV cm}^{-3}\,. \tag{3.118}$$

Thus, taking the result (3.118), because of the relationship of the final state of the massive universe and the big bang, we can so assume the connection $\widehat{\rho}_{\text{vac}} = \rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ with $N(T) = (4/11)^{1/3}$, so that we obtain

$$\frac{8}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,(k\widetilde{T}_{\text{BB}})^2} = \rho_{\text{f}1}\,c^2 \tag{3.119}$$

with the solution

$$k\widetilde{T}_{\text{BB}} = \widetilde{E}_{\text{BB}} = \left[\frac{8}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,\rho_{\text{f}1}\,c^2}\right]^{1/2} \cong 4.76\cdot 10^{90}\ \text{eV}\,, \tag{3.120}$$

i.e. the result (3.28 a) is correct by $\widehat{\rho}_{\text{vac}} = \rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ (see above), since the energy $\widetilde{E}_{\text{BB}} \cong 4.76\cdot 10^{90}\ \text{eV}$ realizes also the state of $\rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ {see also Sec. 3.11}. Because of the result (3.120), the very high temperature of the start of the universe (see also Sec. 3.10 or 3.11) is given by

$$\widetilde{T}_{\text{BB}} = \widetilde{E}_{\text{BB}}/k \cong 5.52\cdot 10^{94}\ \text{K}\,. \tag{3.121}$$

Using the data of Tables III to V, these considerations, derived in this Sec. 3.9 for the universe, are also valid for the anti-universe (see Sec. 3.10).

3.10 The time reversal solution for universe and anti-universe

At the 4-vectors and the tensors, the time reversal leads to a sign change of their time component. Consequently, for the universe and the time-reversal anti-universe, we must apply the results of Tables IV and V. However, because the parameters of Table I and V agree excellently, we use predominantly the parameters of Table I in this work. The universe is treated by the known way, for example, using the Friedmann equation (2.1). The time-reversal anti-universe is also determined, for example, by the Friedmann equation (2.1), whereat we must however consider the minus sign of the root as well as the results of Tables IV and V, i.e. we must take into account the sign "$-$" of the root together with $-\dot{R}$. Thus, we obtain the "same" solution for the scale factor of the anti-universe as at the universe. However, in contrast to the universe, at the anti-universe, we have a negative velocity and a negative time.

Then, using the result (2.43) of the quantum gravity [1, 2] for the "massless" ($R_{\rm Pl} \geq R \geq R_{\rm BB}$ and $R_{\rm BB} \leq R \leq R_{\rm Pl}$) anti-universe and universe as well as the general solutions [1-3, 5] of Eqs. (2.1) and (2.13) via the results (2.14) to (2.18) for the early ($\widetilde{R}_0 \geq \widetilde{R} \geq R_{\rm Pl}$ and $R_{\rm Pl} \leq \widetilde{R} \leq \widetilde{R}_0$) and, for example, the late ($R \geq \widetilde{R}_0$ and $\widetilde{R}_0 \leq R$) radiation-dominated massive anti-universe and universe, we have the positive distances and scale factors

$$R = c\,t \tag{3.122}$$

as well as

$$\widetilde{R} = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\rm Pl}\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\,(t_{\rm pl}\,t)^{1/2} \tag{3.123}$$

and

$$R = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t}{H_0}\right)^{1/2}, \tag{3.124}$$

respectively. Then, the corresponding velocities are given by

$$\dot{R} = \frac{dR}{dt} = \pm c \tag{3.125}$$

as well as

$$\dot{\tilde{R}} = \frac{d\tilde{R}}{dt} = \pm(2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c/t)^{1/2} =$$

$$= \pm c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t_{\mathrm{Pl}}}{t}\right)^{1/2} \tag{3.126}$$

and

$$\dot{R} = \frac{dR}{dt} = \pm(2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c/t)^{1/2} =$$

$$= \pm c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{1}{H_0 t}\right)^{1/2}, \tag{3.127}$$

where according to Table IV the sign "+" and the sign "−" are valid for the universe and the anti-universe, respectively. Therefore, we have the positive time direction (from the origin (big bang) to the future), which is valid for all fundamental physical processes in the universe, so that we have here an expanding universe with scale factors greater than zero as well as positive velocities $c \geq v > 0$. Correspondingly, for the anti-universe, we get a negative time direction (i.e. from the origin (big bang) to the past as a result of the time reversal), which is valid for all fundamental physical processes in the anti-universe, i.e. we have here an expanding anti-universe with scale factors greater than zero as well as negative velocities $-c \leq -v < 0$ and the transitions $H_0 \to -H_0$, $t_{\mathrm{Pl}} \to -t_{\mathrm{Pl}}$ and $t \to -t$. The velocities (3.125) to (3.127) of the anti-universe possess a negative sign because of the transition $t \to -t$ in the differential quotients of the velocities, so that we get here the negative velocities in contrast to the universe. Thus, we have simply explained the long-sought problem of the separation between matter (universe) and antimatter (anti-universe.

It is now clear that the results of the universe, derived in this work and the papers [1-5], are completely transferable to the anti-universe, using the data of Tables III to V.

Because of this time reversal solution, the beginning (big bang) of the anti-universe and the universe are a result of two equivalent energy uncertainties of the vacuum (origin), which arise from quantum fluctuations of the time or the distance according to the uncertainty relation. Then, according to Table IV, via the result (3.28 a), for the antimatter (anti-universe) and the matter (universe), these two equivalent energy uncertainties are given by

$$\widetilde{E}_{\mathrm{BB}} = \frac{-\hbar}{2(-t_{\mathrm{BB}})} = \frac{(-\hbar)(-\mathrm{c})}{2R_{\mathrm{BB}}} \cong 4.769 \cdot 10^{90}\ \mathrm{eV} \tag{3.128}$$

and

$$\widetilde{E}_{\mathrm{BB}} = \frac{\hbar}{2t_{\mathrm{BB}}} = \frac{\hbar \mathrm{c}}{2R_{\mathrm{BB}}} \cong 4.769 \cdot 10^{90}\ \mathrm{eV}\ , \tag{3.129}$$

respectively. Thus, for a transition from the anti-universe to the universe, for the zero-point oscillations (see Sec. 3.6 or 3.7), we need the total energy (see also Eq. (3.28 b))

$$E_{\mathrm{BB}} = 2\widetilde{E}_{\mathrm{BB}} = \frac{\hbar}{t_{\mathrm{BB}}} = \frac{\hbar \mathrm{c}}{R_{\mathrm{BB}}} = 9.537 \cdot 10^{90}\ \mathrm{eV}\ . \tag{3.130}$$

This total energy $E_{\mathrm{BB}} = 9.537 \cdot 10^{90}$ eV, released by the vacuum (origin), is applied as the excitation energy for the zero-point oscillation which act again on the particles and antiparticles in the total {massless (see Sec. 3.6) and massive (see Sec. 3,7)} universe and anti-universe, respectively.

Therefore, at $t = 0$ (origin), the antimatter (3.128) and the matter (3.129) disappear by annihilation, so that the anti-universe (antimatter) and the universe (matter) expand in opposite time directions. These universes are based on the zero-point oscillations formed by the sterile neutrino-antineutrino pairs according to the chapters 3.6 and 3.7.

Now, we can explain the new inflation model {see Eqs. (2.14) to (2.18) or Refs. [1-5]} via a gigantic resonance as a result of the excitation of the magnetic monopoles [1-5] by the zero-point oscillations. This resonance, which is possible because of the binding of these monopoles to the early massive universe, ends at the rest energy of the X and Y gauge bosons [1-5], so that during a short inflationary phase [1-5] an enormous energy is released. This released energy is so large that it is enough for the inflation of the total early massive universe ($R_{\mathrm{Pl}} \leq \widetilde{R} \leq \widetilde{R}_0$), i.e. it leads to an inflation, in which the scale factors of the early universe are enlarged by the enormous factor $1 + z_{\mathrm{M}} = 2.916 \cdot 10^{30}$ (see Eq. (2.18)). However, for the excitation of this resonance, the expended energy is very small in comparison to the excitation energy $E_{BB} = 9.537 \cdot 10^{90}$ eV of the zero-point oscillations, so that the zero-point oscillation are not influenced.

Thus, similarly, at the beginning of the present accelerated expansion (see Eqs. (2.27) and (2.35)), we can assume the formation of a new very large resonance by excitation of the here dominating sterile neutrino-antineutrino pairs of the dark energy (see Eq. 3.132)) via the zero-point oscillations. Consequently, the very large energy of this new resonance could be responsible for the present accelerated expansion of the universe (see also Sec. 7).

The kinetic energy, which is necessary for the complete expansion of the total ("massless" and massive) universe, is given by the mean (maximum) energy (3.83), which is identical with the greatest possible gravitational energy $\overline{E}_{max} = 2.51 \cdot 10^{212}$ eV (see Eq. (3.113)).

In this Sec. 3.10, the results of the last four paragraphs are also valid for the anti-universe.

3.11 The eternal cyclic evolution for universe and anti-universe

For the final state of the universe, because of $\widehat{\rho}_{vac} = \rho_{f1} = \rho_{f2} \times (T_1/T_2)^3$, the result $k\widetilde{T}_{BB} = \widetilde{E}_{BB}$ of Eq. (3.120) gives again the big bang distance

$$R_{BB} = \frac{\hbar c}{2k\widetilde{T}_{BB}} = \frac{\hbar c}{2\widetilde{E}_{BB}} \cong 2.069 \cdot 10^{-96} \text{ cm} \quad \text{(see Eq. (3.23))}. \qquad (3.131)$$

Thus, this expression (3.131) proves uniquely the relationship between the final state of the universe and the big bang.

Via the dark energy $E_d = 5.017 \cdot 10^{89}$ eV (see Eq. (3.10)), the case $N = 12$, which is based on the simultaneous application of the Dirac theory for the universe and the anti-universe (see Sec. 3.6), leads to the connection

$$\tfrac{1}{4} E_d = E^*(12) = (\Omega_{dm} + \Omega_\Lambda)^{1/6} E_{BB}/24\pi = 1.254 \cdot 10^{89} \text{ eV}, \qquad (3.132)$$

considering the results of the zero-point oscillations (see Eq. (3.90 c)).

Consequently, via Eq. (3.132), we obtain the greatest possible gravitational energy for the universe and the anti-universe to

$$2\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{\frac{1}{4}E_d\}\times\{\frac{1}{4}E_d\}}{2R_{BB}} \cong 5.03\cdot 10^{212}\,\text{eV}\,, \tag{3.133}$$

so that for the universe alone we find the greatest possible gravitational energy $\overline{E}_{\max}$ to

$$\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{\frac{1}{4}E_d\}\times\{\frac{1}{4}E_d\}}{2\times 2R_{BB}} \cong 2.51\cdot 10^{212}\,\text{eV}\,. \tag{3.134}$$

Then, this enormous energy (3.134), which can be interpreted in the extreme case as the potential energy for a gravitational collapse at the transition of the total ("massless" and massive) universe from its final state to the big bang, is so identical with the mean (maximum) energy (3.78) {see also the end of Sec. 3.5}. In this extreme case, the potential energy $\overline{E}_{\max} \cong 2.51\cdot 10^{212}\,\text{eV}$ leads to a very fast transition via the reduction of the dimension of the final state of the universe (see Eq. (3.37)) to the distance of the big bang (see Eq. (3.23)) in the time $t_{BB} = 6.901\cdot 10^{-117}\,\text{s}$ (see Eq. (3.24)), i.e. the start of an eternal cycle of evolution is found. However, by a deeper analysis of the result (3.131), in connection with the zero-point oscillations (see also Eqs. (3.128) to (3.130)), we can exclude this extreme case of a gravitational collapse, so that the gigantic potential energy $\overline{E}_{\max} \cong 2.51\cdot 10^{212}\,\text{eV}$ must be responsible for a very slow transition from the final state of the universe to the big bang in the time $t_{f2} = 6883\,\text{Gyr}$ (see Eqs. (3.36) and (3.45)). i.e. we obtain the start of an eternal cyclic evolution of the universe by the corresponding reversal of its expansion. Consequently, we have proved finally the relationship between the final state of the universe and the big bang.

Because we have used the assumption $\widehat{\rho}_{\text{vac}} = \rho_{f1} = \rho_{f2}\times(T_1/T_2)^3$ for the result (3.131), we must still explain the application of the dark mass density ρ_{f1} as $\rho_{f2}\times(T_1/T_2)^3$ (see Eq. (3.75 a)), i.e. we have here the application of $d_{f2} = 6.512\cdot 10^{30}\,\text{cm}$ (see Eq. (3.38)) instead of $d_{f1} = 6.404\cdot 10^{30}\,\text{cm}$ (see Eq. (3.37)). Then, according to uncertainty relation, via the quantum fluctuation

$d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29}$ cm of the distance d_{f1}, we obtain the energy uncertainty

$$E_{12} = 6\,kT_{BB} = \frac{\hbar c}{2d_{f21}} \cong 9.14 \cdot 10^{-35}\ \text{eV} \tag{3.135}$$

for the decay products of the 3 sterile neutrino types of the dark energy (see Sec. 3.1). Therefore, in the final state d_{f2} of the (massive) universe, for $E_0^{\hat{\nu}}(\gamma) = E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$, the energy uncertainty $6\,kT_{BB} = 3 \times E_0^{\hat{\nu}}(\gamma) + 3 \times E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$ of these decay products is correspondingly identical with the triple sum of the rest energies of the photons $E_0(\gamma) = kT_{BB} \cong 1.563 \cdot 10^{-35}$ eV (see Eq. (6.1 b)) and of the gravitons $E_0(G) = kT_{BB} \cong 1.563 \cdot 10^{-35}$ eV (see Eq. (6.1 a))}. These two new assumptions are based on the existence of the sterile neutrino-antineutrino pairs in the final state of the universe because of the Dirac theory, since one sterile neutrino-antineutrino pair determines these decay properties of the sterile neutrinos. Thus, the final state d_{f2} of the universe is a result of these photons and gravitons in an unstable equilibrium, which is destroyed by the gigantic potential energy (3.134), so that it gives the transition from the final state of the universe in the direction to the big bang. Then, the proper distance d_{f2} is determined by the quantum fluctuation $d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29}$ cm, i.e. the assumption $\hat{\rho}_{vac} = \rho_{f1} = \rho_{f2} \times (T_1/T_2)^3$ is a function of $d_{f2} = c\,t_{f2}$ (see Eqs. (3.75 a) or (3.118)).

Using the data of Tables III to V, all these events are also valid for the transition from the final state of the anti-universe in direction to the big bang.

With that, we have precisely and uniquely proved the eternal cyclic evolution of the anti-universe and the universe.

4 The explanation of the present dark matter and dark energy

Because of several incorrect interpretations in Ref. [1], we perform here once more the explanation of the present dark matter and dark energy, whereat these results are used for the derivation of new cosmological parameters instead of that in Table V.

According to Table I, the present, (ionisable, visible or baryonic) matter is $\Omega_b = 0.0499$, whereas the present, dark matter Ω_{dm} and dark energy Ω_Λ are given by $\Omega_{dm} = 0.265$ and $\Omega_\Lambda = 0.685$.

They possess the total relation $\Omega_b + \Omega_{dm} + \Omega_\Lambda = 1$, which was generally derived in Refs. [2-4].

The fluxes $\Phi(\hat{\nu}_\Lambda)$, $\Phi(\hat{\nu}_{dm})$ and $\Phi(\hat{\nu}_b)$, calculated correctly in Ref. [1], show uniquely that the joint origin of the dark matter and the dark energy is based on the three sterile neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{dm}$ and $\hat{\nu}_b$ [1-4] without the consideration of the Dirac theory. The Dirac theory leads to the corresponding sterile neutrino-anti-neutrino pairs (see Sec. 3.6 or 3.7). Therefore, we assume the decay of the corresponding sterile neutrinos via the gravitation in one massless photon (γ) with the energy of their half rest energy and one sterile neutrino relic ($\frac{1}{2}\hat{\nu}$) with the energy of their half rest energy (see also Eq. (3.135)). Consequently, we must together consider the present dark matter and dark energy, i.e. the sum $\Omega_{dm} + \Omega_\Lambda = 0.265 + 0.685 = 0.950$.

The decay of these sterile neutrinos is again determined by the universal decay law $\overline{N}(t) = \overline{N}_0\, e^{-t/\tau_{\hat{\nu}}}$, where $\tau_{\hat{\nu}}$ is the lifetime "$\tau_{\hat{\nu}} = 35.11$ Gyr" of the sterile neutrinos (see Eq. (3.15)). Assuming $\overline{N}_0 = \Omega_{dm} + \Omega_\Lambda$, the remaining dark matter and dark energy $\overline{N}(t)$ is given by $\overline{N}(t) = (\Omega_{dm} + \Omega_\Lambda)\, e^{-t/\tau_{\hat{\nu}}}$. Using the Hubble (H) time $\tau_H = 1/H_0 = 14.53\,\text{Gyr}$ (see Table I), which is valid for all galaxies (which, for example, expand by the velocity $v = H_0 R$ at very small redshifts, i.e. for $v << c$), we can assume the time $t = \tau_{\hat{\nu}} - \tau_H = 20.58\,\text{Gyr}$, so that we obtain $e^{-(\tau_{\hat{\nu}} - \tau_H)/\tau_{\hat{\nu}}} = 0.5565$.

This assumption is supported by the fact that according to Eq. (2.17) this Hubble time (see Table I) defines also the scale factor $R_0 = c\tau_H = c/H_0$ of the present (massive) universe for $z = 0$, using Eq. (2.15).

Then, the decay products of the dark matter and dark energy (see above) are defined via the slow decay process

$$(\Omega_{\text{dm}} + \Omega_{\Lambda})(1 - e^{-(\tau_{\hat{\nu}} - \tau_{\text{H}})/\tau_{\hat{\nu}}}) = 0.4213 \tag{4.1}$$

to

$$\{\Omega_{\text{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\text{decay}} = 0.2107 \tag{4.2}$$

and

$$\{\Omega_{\text{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\text{decay}} = 0.2107\,. \tag{4.3}$$

For these decay products, the dark radiation energy (4.2) lies in an invisible range of the radiation spectrum, whereas the neutral dark sterile neutrino relics (4.3) are thus also invisible.

Because of Eqs. (4.1) to (4.3), the remaining dark matter and dark energy (see above) is determined to

$$(\Omega_{\text{dm}} + \Omega_{\Lambda})\, e^{-(\tau_{\hat{\nu}} - \tau_{\text{H}})/\tau_{\hat{\nu}}} = 0.5287\,. \tag{4.4}$$

Now, we assume that the repulsive (rep) force of the dark energy leads to a breakup of the remaining dark matter and dark energy (4.4) into two equivalent invisible parts of the massive sterile breakup neutrinos

$$\{\Omega_{\text{dm}} + \Omega_{\Lambda}\}_{\text{rep}} = 0.2644 \tag{4.5}$$

and of the sum of the two equal massless sterile breakup neutrinos products

$$\{\Omega_{\text{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\text{rep}} + \{\Omega_{\text{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\text{rep}} = 0.2644\,, \tag{4.6}$$

so that results (4.5) and (4.6) are in equilibrium.

For Eq. (4.6), on the left hand side, the first term $\{\Omega_{\text{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\text{rep}}$ gives the fraction of the breakup radiation, which lies in the invisible range of the radiation spectrum, whereas the second term $\{\Omega_{\text{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\text{rep}}$ gives the fraction of the invisible massless sterile breakup neutrino relics.

These two fractions $\{\Omega_{\text{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\text{rep}}$ and $\{\Omega_{\text{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\text{rep}}$ exist independently on the corresponding decay products (4.2) and (4.3).

In this connection, by the slow decay process (4.1) to (4.3), we must consider that the massless photons (4.2) move with the velocity $v = c$, whereas the sterile massless neutrino relics (4.3) possess also the velocity $v = c$ via Eq. (3.135), so that the results (4.2) and (4.3) cannot be connected with the (ionisable or visible) massive matter Ω_{b} because of $v = c$.

A completely other situation exists for the breakup process (4.4) to (4.6), where we have the sterile massive breakup neutrino packet (4.5) with the velocity $v \approx c$ and the two sterile massless breakup neutrino products (4.6) with the velocity $v = c$, so that only the massive ($v \approx c$) sterile breakup neutrino packet (4.5) can exist as the massive dark matter Ω_{dm} together with the massive (ionisable or visible) matter Ω_{b}, whereas the massless ($v = c$) sterile breakup neutrino products (4.6) cannot be connected with the massive (ionisable or visible) matter Ω_{b}.

Therefore, for the total matter $\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}} + \{\Omega_{\mathrm{dm}} + \Omega_{\Lambda}\}_{\mathrm{rep}} = 0.3143$, we obtain the ratio

$$\frac{\text{total matter}}{\text{matter}} = \frac{\Omega_{\mathrm{m}}}{\Omega_{\mathrm{b}}} = 6.30\,, \qquad (4.7)$$

which agrees excellently with the experimental value $(\Omega_{\mathrm{b}} + \Omega_{\mathrm{dm}})/\Omega_{\mathrm{b}} = 6.31$ (see above).

This agreement confirms our above-mentioned assumptions, i.e. the results (4.1) to (4.6) can be summarized correspondingly in the gigantic present universe after the process (4.4) to (4.6) because of the particle conservation to the massive dark matter

$$\Omega_{\mathrm{dm}} = \{\Omega_{\mathrm{dm}} + \Omega_{\Lambda}\}_{\mathrm{rep}} = 0.2644 \qquad (4.8)$$

and to the massless dark energy

$$\Omega_{\Lambda} = \{\Omega_{\mathrm{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\mathrm{rep}} + \{\Omega_{\mathrm{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\mathrm{rep}} + \\ + \{\Omega_{\mathrm{dm}}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{\mathrm{decay}} + \{\Omega_{\mathrm{dm}}(\tfrac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\tfrac{1}{2}\hat{\nu})\}_{\mathrm{decay}} = 0.686 \qquad (4.9)$$

via the sum of the invisible massless sterile breakup neutrino products (see Eq. (4.6)) as well as the invisible massless decay products (see Eqs. (4.2) and (4.3)) of the sterile neutrinos.

With that, we have found a simple model for the explanation of the present dark matter and dark energy, i.e. it was proved that the invisible sterile breakup neutrinos $\hat{\nu}_{\Lambda}$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ as well as their invisible decay and breakup products form the joint origin of the dark matter and the dark energy.

Because of the two decay products of the 3 sterile neutrinos $\hat{\nu}_{\Lambda}$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$, we have already introduced the new factor $(\Omega_{\mathrm{dm}} + \Omega_{\Lambda})^{1/6} = 0.9915$ for the excitation energy $E_{\mathrm{exc}}(N) = (\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} E_{\mathrm{BB}}$ at the zero-point oscillations in Sec. 3.6. Consequently, this factor describes the energy loss of the excitation energy for the zero-point oscillations, since they excite the decay of the 3 fundamental sterile neutrinos into the two decay products.

Then, via Eqs. (4.1) and (4.4), at the massless dark energy, because of the equivalence of the results (4.5) and (4.6), by the two massless decay (see Eqs. (4.2) and (4.3)) and the two massless breakup (see Eqs. (4.6)) products, i.e. because of 4 events, we can assume semi-empirically

$$\Omega_{\mathrm{dm}} + \Omega_\Lambda \cong \left[\frac{1 - \mathrm{e}^{-(\tau_{\hat{\nu}} - \tau_{\mathrm{H}})/\tau_{\hat{\nu}}}}{\mathrm{e}^{-(\tau_{\hat{\nu}} - \tau_{\mathrm{H}})/\tau_{\hat{\nu}}}} \right]^{1/4} \cong 0.945 . \tag{4.10}$$

Indeed, the result (4.10) agrees well with the sum $\Omega_{\mathrm{dm}} + \Omega_\Lambda = 0.950$ of the present dark matter and dark energy (see above).

However, according to Refs. [1-4], we have 4 heavy neutrinos $\tilde{\nu}_\Lambda$, $\tilde{\nu}_{\mathrm{dm}}$, $\tilde{\nu}_{\mathrm{b}}$ and $\tilde{\nu}_{\mathrm{CMB}}$, which are derived by aid of the light neutrinos [1-4]. These 4 heavy neutrinos are again coupled correspondingly with the 4 sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$, $\hat{\nu}_{\mathrm{b}}$ and $\hat{\nu}_{\mathrm{CMB}}$ [1-4]. In Refs. [1-4], we have also assumed that the fourth sterile neutrino $\tilde{\nu}_{\mathrm{CMB}}$ could be responsible for the photon decoupling. In Refs. [1, 2], we have in detail discussed the properties of these heavy and sterile neutrinos. However, in Refs. [1, 2], the value 629.2 of the semi-empirical explanation of the sterile neutrino calculation, estimated by the rest energies of the light and heavy neutrinos, for example, via formula (5.30) in Ref. [1], is useless, i.e. it is invalid more generally.

In Eqs. (4.1) to (4.10), we have shown that only the 3 sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ describe the dark matter and the dark energy. Therefore, instead of the particle-defined present cosmological parameters of Table V, on the same way, we must introduce several new particle-defined present cosmological parameters of the universe.

The old particle-defined density parameters of Table V (see, e.g., Ref. [2]) are defined by

$$\Omega_\nu(\nu_{\mathrm{e}}) \cong (0.683^{+0.034}_{-0.042})\,\Omega_\gamma = (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} h^{-2} , \tag{4.11}$$

$$\Omega_{\tilde{\nu}}(\tilde{\nu}_{\rm CMB}) = \Omega_\nu(\nu_{\rm e})\,\frac{E_{\rm Pl}}{2\sqrt{2}\,E_0({\rm X})} =$$

$$= (110{,}2^{+5.5}_{-6.8})\,\Omega_\gamma = (2.72^{+0.14}_{-0.17})\cdot 10^{-3} h^{-2}, \tag{4.12}$$

$$\Omega_{\rm b} = \Omega_\nu(\nu_\mu)\,\frac{E_{\rm Pl}}{2E_0({\rm X})} + \Omega_\nu =$$

$$= (894^{+35}_{-36})\,\Omega_\gamma = (0.02211^{+0.00089}_{-0.00091})\,h^{-2}, \tag{4.13}$$

$$\Omega_{\rm dm} = \Omega_\nu(\nu_\tau)\,\frac{E_{\rm Pl}}{2E_0({\rm X})} + \Omega_\nu =$$

$$= (4862^{+234}_{-206})\,\Omega_\gamma = (0.1202^{+0.0059}_{-0.0052})\,h^{-2}, \tag{4.14}$$

$$\Omega_{\rm m} = \Omega_{\rm dm} + \Omega_{\rm b} = (5756^{+234}_{-206})\,\Omega_\gamma = (0.1423^{+0.0059}_{-0.0052})\,h^{-2} \tag{4.15}$$

and

$$\Omega_\Lambda = \Omega_\nu(\nu_\mu)\,\frac{E_{\rm Pl}}{2E_0({\rm Y})} + \Omega_\nu\,\frac{E_{\rm Pl}}{E_0({\rm Y})} =$$

$$= (12589^{+559}_{-505})\,\Omega_\gamma = (0.311^{+0.015}_{-0.013})\,h^{-2}, \tag{4.16}$$

i.e. via $\Omega_{\rm tot} = 1 = \Omega_\Lambda + \Omega_{\rm m} + 1.681322\,\Omega_\gamma = (18347^{+560}_{-506})\,\Omega_\gamma$ we obtain the radiation density parameter to

$$\Omega_\gamma = (5.45^{+0.15}_{-0.17})\cdot 10^{-5}, \tag{4.17}$$

so that because of $\Omega_\gamma = (2.4728 \pm 0.0025)\cdot 10^{-5} h^{-2}$ [1-5] we can derive

$$h = 0.6736^{+0.0105}_{-0.0096}\,. \tag{4.18}$$

The applied neutrino density parameters of the universe are defined by Eqs. (2.51) to (2.56). The Planck energy has the value $E_{\rm Pl} = 1.220932\cdot 10^{28}$. For the X and Y gauge bosons, the rest energies $E_0({\rm X})$ and $E_0({\rm Y})$ are defined by Eq. (2.57). The corresponding values (4.13) to (4.18) are tabulated in Table V.

Then, in this work, instead of these old density parameters of the universe, we assume their new corresponding values to

$$\Omega^*_\nu(\nu_{\rm e}) \cong (0.683^{+0.034}_{-0.042})\,\Omega^*_\gamma = (1.69^{+0.09}_{-0.11})\cdot 10^{-5} h_*^{-2}, \tag{4.19}$$

$$\Omega^*_{\tilde{\nu}}(\tilde{\nu}_{\rm CMB}) = \Omega^*_\nu(\nu_{\rm e})\,\frac{E_{\rm Pl}}{2\sqrt{2}\,E_0({\rm X})} =$$

$$= (110{,}2^{+8.1}_{-9.2})\,\Omega^*_\gamma = (2.72^{+0.20}_{-0.23})\cdot 10^{-3}\,h_*^{-2}, \tag{4.20}$$

$$\Omega^*_{\rm b} = \Omega^*_\nu(\nu_\mu)\,\frac{E_{\rm Pl}}{2E_0({\rm X})} + \Omega^*_\nu =$$

$$= (894^{+35}_{-36})\,\Omega^*_\gamma = (0.02211^{+0.00089}_{-0.00091})\,h_*^{-2}, \tag{4.21}$$

$$\Omega^*_{\rm dm} = \Omega^*_\nu(\nu_\tau)\,\frac{E_{\rm Pl}}{2E_0({\rm X})} + \Omega^*_\nu =$$

$$= (4862^{+234}_{-206})\,\Omega^*_\gamma = (0.1202^{+0.0059}_{-0.0052})\,h_*^{-2}, \tag{4.22}$$

$$\Omega^*_{\rm m} = \Omega^*_{\rm dm} + \Omega^*_{\rm b} = (5756^{+234}_{-206})\,\Omega^*_\gamma = (0.1423^{+0.0059}_{-0.0052})\,h_*^{-2}, \tag{4.23}$$

$$\Omega^{**}_\Lambda = \Omega^*_\Lambda + \Omega^*_\nu = \Omega^*_\nu(\nu_\mu)\,\frac{E_{\rm Pl}}{2E_0({\rm Y})} + \Omega^*_\nu\,\frac{E_{\rm Pl}}{E_0({\rm Y})} + \Omega^*_\nu =$$

$$= (12615^{+560}_{-506})\,\Omega^*_\gamma = (0.312^{+0.014}_{-0.013})\,h_*^{-2} \tag{4.24}$$

and

$$\Omega^{***}_\Lambda = \Omega^{**}_\Lambda + \Omega^*_{\tilde{\nu}}(\tilde{\nu}_{\rm CMB}) =$$

$$= (12725^{+565}_{-514})\,\Omega^*_\gamma = (0.315^{+0.014}_{-0.013})\,h_*^{-2}, \tag{4.25}$$

i.e. via $\Omega^*_{\rm tot} = 1 = \Omega^{**}_\Lambda + \Omega^*_{\rm m} + 1.681322\,\Omega^*_\gamma = (18483^{+566}_{-515})\,\Omega^*_\gamma$ we have found the radiation density parameter to

$$\Omega^*_\gamma = (5.41^{+0.15}_{-0.17})\cdot 10^{-5}, \tag{4.26}$$

so that because of $\Omega^*_\gamma = (2.4728 \pm 0.0025)\cdot 10^{-5}\,h_*^{-2}$ we can determine

$$h_* = 0.676^{+0.011}_{-0.010}\,. \tag{4.27}$$

In Eqs. (4.24) and (4.25), instead of the old dark energy Ω_Λ (see Eq. (4.16)), the two new values $\Omega^{**}_\Lambda = \Omega^*_\Lambda + \Omega^*_\nu$ and $\Omega^{***}_\Lambda = \Omega^{**}_\Lambda + \Omega^*_{\tilde{\nu}}(\tilde{\nu}_{\rm CMB})$ were defined. The values (4.20) to (4.27) are tabulated in Table VI. The first value Ω^{**}_Λ, which describes alone the normal dark energy of the 3 sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$ and $\hat{\nu}_{\rm b}$, is improved by the term Ω^*_ν in analogy to the expressions (4.13) and (4.14), i.e. $(\Omega_\Lambda + \Omega_\nu)^* \to \Omega_\Lambda^{**}$. At the second value Ω_Λ^{***}, which yields the total dark energy of the 4 sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$, $\hat{\nu}_{\rm b}$ and $\hat{\nu}_{\rm CMB}$, the term $\Omega_{\tilde{\nu}}^*(\tilde{\nu}_{\rm CMB})$ could be responsible again for the decoupling of the photons [1, 2].

Then, by Refs. [1, 2, 6], the age of the present massive universe ($z = 0$), which now depends on the dark matter $\Omega_{\rm m}^*$ and the total dark energy Ω_Λ^{***}, is defined by

$$t_0^* = \frac{2}{3\, H_0^*\, \Omega_\Lambda^{***\frac{1}{2}}} \ln \frac{\sqrt{\Omega_\Lambda^{***}} + \sqrt{\Omega_\Lambda^{***} + \Omega_{\rm m}^*}}{\sqrt{\Omega_{\rm m}^*}} . \tag{4.28}$$

The values (4.20) to (4.28) are tabulated in Table VI. These values of Table VI agree well with the most recent data of Ref. [16], which are given in Table VII.

According to Table VI, the new sum $\Omega_{\rm dm}^* + \Omega_\Lambda^{**} = 0.946$ confirms the value (4.10). Because of the heavy neutrino number density $n(\tilde{\nu}_{\rm CMB}) = 112\,{\rm cm}^{-3}$ (see Refs. [1-4]), the above-mentioned assumption, where the fourth heavy neutrino $\tilde{\nu}_{\rm CMB}$ could be responsible for the decuopling of the photons, is supported by its rest energy $E_0(\tilde{\nu}_{\rm CMB})$ via the connection [1-4]

$$E_0(\tilde{\nu}_{\rm CMB}) = \frac{\Omega_{\tilde{\nu}}^*(\tilde{\nu}_{\rm CMB})\; \rho_{0{\rm C}}^*\, c^2}{n(\tilde{\nu})} \cong 0.256_{-0.016}^{+0.013}\,{\rm eV} , \tag{4.29}$$

since by Eq. (4.29) we obtain the redshift of the photon decoupling to

$$z_{\rm dec}^* = \frac{E_0(\tilde{\nu}_{\rm CMB})/k}{T_0} - 1 \cong 1089_{-68}^{+56} \tag{4.30}$$

in excellent agreement with the corresponding value $z_* = 1089.9 \pm 0.4$ of the redshift (at which optical depth equals unity) in Table VII. Within the error limits, the cosmological parameter values of Tables I, II, V, VI and VII agree excellently. Thus, in this work, we do not correct generally all results derived by the data of Tables I and V, since the data of Table VII yield the same results for all considerations of the works [1-5] and the present paper within the error limits, i.e. we apply always the data of Table I (see also the paragraph before last in Sec. 2).

Table VI. The estimated (present-day) values of the new particle-defined cosmological parameters for the universe. [a] see Eq. (2.56).

Symbol, equation	**Value**
h_*	$0.676^{+0.011}_{-0.010}$
H_0^*	$100\,h\ \text{km s}^{-1}\text{Mpc}^{-1} = 67.6^{+1.1}_{-1.0}\ \text{km s}^{-1}\text{Mpc}^{-1} =$ $= h_* \times (9.777752\ \text{Gyr})^{-1} = (2.191^{+0.034}_{-0.032})\cdot 10^{-18}\ \text{s}^{-1}$
t_0^*	$(4.356^{+0.017}_{-0.015})\cdot 10^{17}\ \text{s} = 13.80 \pm 0.05\ \text{Gyr}$
$R_0^* = c/H_0^*$	$(1.368^{+0.020}_{-0.021})\cdot 10^{26}\ \text{m}$
$\rho_{0C}^* = 3H_0^{*2}/8\pi G_N$	$(4.82^{+0.16}_{-0.14})\cdot 10^3\,(\text{eV}/c^2)\ \text{cm}^{-3}$
Ω_ν^* [a]	$(6.35^{+0.16}_{-0.14})\cdot 10^{-4} h_*^{-2} = (1.390^{+0.035}_{-0.031})\cdot 10^{-3}$
$\Omega_{\bar{\nu}}^*(\tilde{\nu}_{\text{CMB}})$	$(2.72^{+0.14}_{-0.17})\cdot 10^{-3} h_*^{-2} = (5.95^{+0.31}_{-0.37})\cdot 10^{-3}$
$\Omega_b^* = \rho_b^*/\rho_{0C}^*$	$0.02211^{+0.00089}_{-0.00091}\,h_*^{-2} = 0.0484^{+0.0019}_{-0.0020}$
$\Omega_{\text{dm}}^* = \rho_{\text{dm}}^*/\rho_{0C}^*$	$0.1202^{+0.0059}_{-0.0052}\,h_*^{-2} = 0.263^{+0.013}_{-0.011}$
$\Omega_m^* = \Omega_{\text{dm}}^* + \Omega_b^*$	$0.1423^{+0.0059}_{-0.0052}\,h_*^{-2} = 0.311^{+0.013}_{-0.011}$
$\Omega_\Lambda^{**} = \rho_\Lambda^{**}/\rho_{0C}^*$	$0.311^{+0.014}_{-0.013}\,h_*^{-2} = 0.683^{+0.031}_{-0.028}$
$\Omega_\Lambda^{***} = \rho_\Lambda^{***}/\rho_{0C}^*$	$0.315^{+0.014}_{-0.013}\,h_*^{-2} = 0.689^{+0.031}_{-0.028}$
$\Omega_\gamma^* = \rho_\gamma^*/\rho_{0C}^*$	$(2.4728 \pm 0.0025)\cdot 10^{-5} h_*^{-2} = (5.41^{+0.15}_{-0.17})\cdot 10^{-5}$
Ω_{tot}	1

Table VII. The most recent (present-day) values of the cosmological parameters for the universe according to Ref. [16]

Symbol, equation	**Value**
T_0	2.7255(6) K
h	0.678 ± 0.009
H_0	$100\,h$ km s^{-1}Mpc^{-1} $= 67.8 \pm 0.9$ km s^{-1}Mpc^{-1} = $= h \times (9.777752\ \text{Gyr})^{-1} = (2.197 \pm 0.029) \cdot 10^{-18}\ \text{s}^{-1}$
t_0	13.80 ± 0.04 Gyr
$R_0 = c/H_0$	$0.9250629 \cdot 10^{26} h^{-1}\ \text{m} = (1.364 \pm 0.018) \cdot 10^{26}$ m
$\rho_{0C} = 3H_0^2/8\pi G_N$	$1.05371 \cdot 10^4 h^2 (\text{eV}/c^2)\ \text{cm}^{-3}$ = $= (4.84 \pm 0.13) \cdot 10^3 (\text{eV}/c^2)\ \text{cm}^{-3}$
Ω_ν	< 0.016 (Planck CMB); ≥ 0.0012 (mixing)
$\Omega_b = \rho_b/\rho_{0C}$	$0.02226 \pm 0.00023\, h^{-2} = 0.0484 \pm 0.0010$
$\Omega_{dm} = \rho_{dm}/\rho_{0C}$	$0.1186 \pm 0.0020\, h^{-2} = 0.258 \pm 0.011$
$\Omega_m = \rho_m/\rho_{0C}$	0.308 ± 0.012
$\Omega_\Lambda = \rho_\Lambda/\rho_{0C}$	0.692 ± 0.012
$\Omega_\gamma = \rho_\gamma/\rho_{0C}$	$(2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2} = (5.38 \pm 0.15) \cdot 10^{-5}$
z_*	1089.9 ± 0.4

Exceptionally, because of the new introduction of the normal ($\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$, $\hat{\nu}_{\rm b}$) and the total ($\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$, $\hat{\nu}_{\rm b}$, $\hat{\nu}_{\rm CMB}$) dark energy (see above), we use the improved data of Table VI in Eqs. (7.19) to (7.29) for the important confirmation of the far-reaching conclusion in Sec. 7.

Consequently, using the data of Tables III to V including VI, the results, derived in this Sec. 4 for the universe, are also valid for the anti-universe.

5 The new thermal equilibrium and the light neutrinos

Via the results (2.47) to (2.50), we can directly support the introduction of the new thermal equilibrium by the assumptions

$$kT = kT_0(\nu_{\rm e}) = E_0(\nu_{\rm e}) \cong (1.589^{+0.078}_{-0.098})\cdot 10^{-3}\,{\rm eV}\,, \tag{5.1}$$

$$kT = kT_0(\nu_\mu) = E_0(\nu_\mu) \cong (8.85^{+0.14}_{-0.16})\cdot 10^{-3}\,{\rm eV}\,, \tag{5.2}$$

$$kT = kT_0(\nu_\tau) = E_0(\nu_\tau) \cong (4.93^{+0.12}_{-0.10})\cdot 10^{-2}\,{\rm eV} \tag{5.3}$$

$$kT = kT_0(\textstyle\sum_i \nu_i) = \sum_i E_0(\nu_i) \cong (5.97^{+0.14}_{-0.13})\cdot 10^{-2}\,{\rm eV}\,. \tag{5.4}$$

Then, the assumptions (5.1) to (5.4) yield the redshift conditions

$$1 + z_0(\nu_{\rm e}) = \frac{E_0(\nu_{\rm e})}{kT_0} = 6.766\,, \tag{5.5}$$

$$1 + z_0(\nu_\mu) = \frac{E_0(\nu_\mu)}{kT_0} = 37.68\,, \tag{5.6}$$

$$1 + z_0(\nu_\tau) = \frac{E_0(\nu_\tau)}{kT_0} = 209.9 \tag{5.7}$$

and

$$1 + z_0(\textstyle\sum_i \nu_i) = \frac{\sum_i E_0(\nu_i)}{kT_0} = 254.2\,. \tag{5.8}$$

For $N(T) = 3.362644$ (see Eq. (2.6)), the expression (2.5) yields

$$\Omega_{\rm r} = \frac{1}{2} N(T)\,\Omega_\gamma = 9.18\cdot 10^{-5}\,. \tag{5.9}$$

Then, taking Eqs. (2.61) to (2.67) as well as (5.5) to (5.9), we get semi-empirically the identical connections

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\sum_i \nu_i)}{1+z_{\mathrm{reion}}(\sum_i \nu_i)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{\sum_i E_0(\nu_i)}{kT(\sum_i \nu_i)}\right]^{1/2} = 0.0391\,, \qquad (5.10)$$

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_\tau)}{1+z_{\mathrm{reion}}(\nu_\tau)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_\tau)}{kT(\nu_\tau)}\right]^{1/2} = 0.0391\,, \qquad (5.11)$$

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_\mu)}{1+z_{\mathrm{reion}}(\nu_\mu)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_\mu)}{kT(\nu_\mu)}\right]^{1/2} = 0.0391 \qquad (5.12)$$

and

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_e)}{1+z(\nu_e)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_e)}{kT(\nu_e)}\right]^{1/2} = 0.0391 \qquad (5.13)$$

in excellent agreement with Eq. (2.59).

The results (5.10) to (5.13) can be considered also as a reasonable argument for the introduction of the new thermal equilibrium in Refs. [1-5].

Using the data of Tables III to V, these considerations, derived in this Sec. 5 for the universe, are also valid for the anti-universe.

6 Rest energy of photons and gravitons including conclusions

To this day, it was assumed that the rest energies of the photons and the gravitons are zero. Therefore, we expect for their rest energies greater than zero also the same values. Then, the results (3.108) and (3.109) permit unique conclusions about the rest energy of the gravitons (G) and the photons (γ). Consequently, by Eq. (3.108), we have the first condition

$$E_0(\mathrm{G}) = kT_{\mathrm{BB}} = \frac{E_{\mathrm{H}}}{\frac{1}{2}N_0} = \frac{\rho_{0\mathrm{C}}\, c^2\, \frac{4}{3}\pi R_0^3}{\frac{1}{2}N_0} \cong 1.563\cdot 10^{-35}\ \mathrm{eV}\,, \qquad (6.1\mathrm{a})$$

whereas Eq. (3.109) yields the second condition

$$E_0(\gamma) = kT_{\text{BB}} = \frac{\rho_{\text{vac}}(R_{\text{BB}})\,c^2}{n_\gamma(R_{\text{f}})} \cong 1.563\cdot 10^{-35}\ \text{eV}\,. \qquad (6.1\,\text{b})$$

Thus, we can assume that the results (6.1 a) and (6.1 b) describe the smallest possible thermal (particle) energies in the total (massless and massive) universe. Therefore, we suggest that because of the new thermal equilibrium these smallest possible thermal (particle) energies (6.1 a) and (6.1 b) of the big bang should be responsible for the rest energies $E_0(\text{G})$ and $E_0(\gamma)$ of the gravitons and photons, respectively, i.e. we assume also

$$E_0(\gamma) = E_0(\text{G}) = kT_{\text{BB}} \cong 1.563\cdot 10^{-35}\text{eV}\,. \qquad (6.2)$$

Thus, they are practically massless. The suggestion (6.2) is supported by the gravitational potential energy (see Refs. [1, 2])

$$E_0(\gamma) = \frac{G_N}{c^4}\,\frac{E_0(\gamma)\times E_0(\gamma)}{R_{\text{BB}}} =$$

$$= E_0(\text{G}) = \frac{G_N}{c^4}\,\frac{E_0(\text{G})\times E_0(\text{G})}{R_{\text{BB}}} \cong 1.563\cdot 10^{-35}\text{eV}\,, \qquad (6.3)$$

where R_{BB} is given by Eq. (3.23). These results (6.1 a) to (6.3) agree with the observed limits $E_0(\text{G}) < 9.0\cdot 10^{-34}$ eV and $E_0(\gamma) < 3\cdot 10^{-27}$ eV (see Ref. [16]).

Now, the considerations are restricted to photons. Then, assuming the present relativistic thermal energy $E(\gamma) = kT_0 + E_0(\gamma)$ (see Table I for T_0), the redshift condition for photons yields

$$1 + z_0(\gamma) = \frac{E(\gamma)}{kT_0} = 1 + \frac{E_0(\gamma)}{kT_0}\,, \qquad (6.4)$$

i.e. we find the redshift of the photons to

$$z_0(\gamma) = \frac{E_0(\gamma)}{kT_0} \cong 6.655\cdot 10^{-32}\,. \qquad (6.5)$$

Assuming also for the big bang at $R_{\text{BB}} = c\,t_{\text{BB}} = 2.069\cdot 10^{-96}$ cm (see Eqs. (3.23) and (3.24)) because of the Robertson-Walker metric the validity of the Hubble relation $v = cz = \overline{H}R$ as a result of the Doppler effect for $v << c$ (see, e.g., Ref. [9]), the quantum mechanical zero-point velocity v_{BB} is given by

$$v_{\mathrm{BB}} = \dot{R}_{\mathrm{BB}} = c\, z_0(\gamma) = \overline{H} R_{\mathrm{BB}} = \overline{H}_{\mathrm{BB}} R_{\mathrm{BB}} \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.6}$$

so that for the big bang we obtain

$$\overline{H}_{\mathrm{BB}} = \frac{\dot{R}_{\mathrm{BB}}}{R_{\mathrm{BB}}} = \frac{z_0(\gamma)}{t_{\mathrm{BB}}} \cong 9.643 \cdot 10^{74}\,\mathrm{s^{-1}}. \tag{6.7}$$

Introducing the limiting values

$$v_{\mathrm{BB}} = \dot{R}_{\mathrm{BB}} = c\, z_0(\gamma) = \overline{H}_{\mathrm{Pl}} R_{\mathrm{Pl}} \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.8}$$

we find

$$\overline{H}_{\mathrm{Pl}} = \frac{\dot{R}_{\mathrm{BB}}}{R_{\mathrm{Pl}}} = \frac{z_0(\gamma)}{t_{\mathrm{Pl}}} \cong 1.234 \cdot 10^{12}\,\mathrm{s^{-1}}. \tag{6.9}$$

Using the result $H_{\mathrm{Pl}} = 1/2t_{\mathrm{Pl}}$ of Eq. (3.102), the expression (6.9) yields the connections

$$\overline{H}_{\mathrm{Pl}} = \frac{z_0(\gamma)}{t_{\mathrm{Pl}}} = 2\, z_0(\gamma)\, H_{\mathrm{Pl}} = z_0(\gamma)\, H_0\, (1 + z_{\mathrm{M}})^2. \tag{6.10}$$

By the limiting values of Eqs. (6.6) to (6.9), we can generally introduce

$$v_{\mathrm{BB}} = \dot{R}_{\mathrm{BB}} = c\, z_0(\gamma) = \overline{H} R \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.11}$$

i.e. because of Eq. (2.43) for $t_{\mathrm{BB}} \le t \le t_{\mathrm{Pl}}$ we get

$$\overline{H} = \frac{\dot{R}_{\mathrm{BB}}}{R} = \frac{z_0(\gamma)}{t}. \tag{6.12}$$

Because of $kT_0 >> E_0(\gamma)$, we have $E(\gamma) = kT_0$, so that via Eq. (3.104) we can estimate the present total energy $E_{\mathrm{tot}}(\gamma)$ of the photons to

$$E_{\mathrm{tot}}(\gamma) = E(\gamma) \times \tfrac{1}{2} N_0 = kT_0 \times \tfrac{1}{2} N_0 \cong 7.805 \cdot 10^{119}\,\mathrm{eV}. \tag{6.13}$$

The present total photon energy (6.13), multiplied by $z_0(\gamma) = 6.655 \cdot 10^{-32}$ (see Eq. (6.5)), yields again the Hubble energy E_{H} (see Eqs (3.107) and (3.108)) of the present universe as follows

$$E_{\mathrm{H}} = z_0(\gamma) \times E_{\mathrm{tot}}(\gamma) \cong 5.194 \cdot 10^{88}\,\mathrm{eV}. \tag{6.14}$$

Similar results can be found for all remaining extreme massive particles in the universe. For example, we consider the X and Y gauge bosons as well as the protons and the electron neutrino.

According to Eq. (2.57) or Refs. [1-3, 5], for the X and Y gauge bosons, if we introduce $E_0(\mathrm{X}) = E_0(\mathrm{X, Y})$ and $E_0(\mathrm{Y}) = E_0(\mathrm{X, Y})$, we find

$$E_0(\mathrm{X, Y}) = 2.675 \cdot 10^{25}\ \mathrm{eV} \quad \text{(see Eq.(2.57))}, \qquad (6.15)$$

$$\overline{R}_{\mathrm{X, Y}} = \frac{G_N}{c^4} E_0(\mathrm{X, Y}) = 3.541 \cdot 10^{-36}\ \mathrm{cm} \quad \text{(see Eq. (2.40))}, \qquad (6.16)$$

$$E_0(\mathrm{X, Y}) = \frac{G_N}{c^4} \frac{E_0(\mathrm{X, Y}) \times E_0(\mathrm{X, Y})}{\overline{R}_{\mathrm{X, Y}}} = 2.675 \cdot 10^{25}\ \mathrm{eV}, \qquad (6.17)$$

$$R_{\mathrm{BB}} = \frac{E_0(\gamma)}{E_0(\mathrm{X, Y})} \overline{R}_{\mathrm{X, Y}} = 2.069 \cdot 10^{-96}\ \mathrm{cm}, \qquad (6.18)$$

$$1 + z_{\mathrm{X, Y}} = \frac{E_0(\mathrm{X, Y})}{kT_0} = 1.139 \cdot 10^{29}, \qquad (6.19)$$

$$R_{\mathrm{X, Y}} = \frac{R_0}{1 + z_{\mathrm{X, Y}}} = 1.207 \cdot 10^{-1}\ \mathrm{cm} \quad \text{(see Eqs. (2.15) and (2.17))}, \qquad (6.20)$$

$$\tfrac{1}{2} N_{\mathrm{X, Y}} = \frac{\tfrac{1}{2} R_{\mathrm{X, Y}}}{R_{\mathrm{BB}}} = 2.917 \cdot 10^{94}, \qquad (6.21)$$

$$E_{\mathrm{tot}}(\mathrm{X, Y}) = E_0(\mathrm{X, Y}) \times \tfrac{1}{2} N_{\mathrm{X, Y}} = 7.804 \cdot 10^{119}\ \mathrm{eV} \qquad (6.22)$$

and

$$E_{\mathrm{H}} = z_0(\gamma) \times E_{\mathrm{tot}}(\mathrm{X, Y}) = 5.194 \cdot 10^{88}\ \mathrm{eV}. \qquad (6.23)$$

For the Higgs boson (see Ref. [16]), we have

$$E_0(\mathrm{H}^0) \cong 1.26 \cdot 10^{11}\ \mathrm{eV}, \qquad (6.24)$$

$$\overline{R}_{\mathrm{H}^0} = \frac{G_N}{c^4} E_0(\mathrm{H}^0) = 1.668 \cdot 10^{-50}\ \mathrm{cm}, \qquad (6.25)$$

$$E_0(\mathrm{H}^0) = \frac{G_N}{c^4} \frac{E_0(\mathrm{H}^0) \times E_0(\mathrm{H}^0)}{\overline{R}_{\mathrm{H}^0}} = 1.260 \cdot 10^{11}\ \mathrm{eV}, \qquad (6.26)$$

$$R_{\mathrm{BB}} = \frac{E_0(\gamma)}{E_0(\mathrm{H}^0)} \overline{R}_{\mathrm{H}^0} = 2.069 \cdot 10^{-96}\ \mathrm{cm}, \qquad (6.27)$$

$$1 + z_{\mathrm{H}^0} = \frac{E_0(\mathrm{H}^0)}{kT_0} = 5.365 \cdot 10^{14}, \qquad (6.28)$$

$$R_{H^0} = \frac{R_0}{1+z_{H^0}} = 2.563 \cdot 10^{13}\ \text{cm}, \tag{6.29}$$

$$½ N_{H^0} = \frac{½ R_{H^0}}{R_{BB}} = 6.194 \cdot 10^{108}, \tag{6.30}$$

$$E_{tot}(H^0) = E_0(H^0) \times ½ N_{H^0} = 7.804 \cdot 10^{119}\ \text{eV} \tag{6.31}$$

and

$$E_H = z_0(\gamma) \times E_{tot}(H^0) = 5.194 \cdot 10^{88}\ \text{eV}. \tag{6.32}$$

For the electron [10], we obtain

$$E_0(e) = 5.109989 \cdot 10^5\ \text{eV}, \tag{6.33}$$

$$\overline{R}_e = \frac{G_N}{c^4} E_0(e) = 6.764 \cdot 10^{-56}\ \text{cm}, \tag{6.34}$$

$$E_0(e) = \frac{G_N}{c^4} \frac{E_0(e) \times E_0(e)}{\overline{R}_e} = 5.110 \cdot 10^5\ \text{eV}, \tag{6.35}$$

$$R_{BB} = \frac{E_0(\gamma)}{E_0(e)} \overline{R}_e = 2.069 \cdot 10^{-96}\ \text{cm}, \tag{6.36}$$

$$1+z_e = \frac{E_0(e)}{kT_0} = 2.176 \cdot 10^9, \tag{6.37}$$

$$R_e = \frac{R_0}{1+z_e} = 6.319 \cdot 10^{18}\ \text{cm}, \tag{6.38}$$

$$½ N_e = \frac{½ R_e}{R_{BB}} = 1.527 \cdot 10^{114}, \tag{6.39}$$

$$E_{tot}(e) = E_0(e) \times ½ N_e = 7.804 \cdot 10^{119}\ \text{eV} \tag{6.40}$$

and

$$E_H = z_0(\gamma) \times E_{tot}(e) = 5.194 \cdot 10^{88}\ \text{eV}. \tag{6.41}$$

Consequently, for the extremely massive particles, the total energies (see, Eqs. (6.22, (6.31) and (6.40)) exist also during the evolution of the massive universe at the different times $t = t(z)$, defined by Eq. (2.12) via the corresponding redshifts (see Eqs. (6.19), (6.28) and (6.37)) and values $N(T)$ [1-3, 10]. These total energies, multiplied by the redshift $z_0(\gamma) = 6.655 \cdot 10^{-32}$

(see Eq. (6.5)), yield again the Hubble energy E_{H} (see Eq. (3.108)) of the present universe according to Eqs. (6.23), (6.32) and (6.41). Therefore, this fact means that the zero-point oscillations describe also completely the massive universe. Using Eqs. (3.102) to (3.109), this result is confirmed by the ratios

$$\tfrac{1}{2}(1+z_{\mathrm{M}})^2 = \frac{1}{2\,H_0\,t_{\mathrm{Pl}}} = \frac{\tfrac{1}{2}R_0}{R_{\mathrm{Pl}}} = \frac{\tfrac{1}{2}N_0}{N_{\mathrm{Pl}}} = \frac{E_{\mathrm{H}}}{E_{\mathrm{Pl}}} = 4.254\cdot 10^{60}, \qquad (6.42)$$

since the basis of the mean (maximum) energies (3.78) and (3.83) of the universe and the anti-universe is the Planck energy E_{Pl} (see Eqs. (3.79) and (3.84)) as the greatest possible relativistic particle energy. Thus, new results are possible by coupling of all corresponding expressions. For example, in this chapter, the coupling of Eqs. (6.10) and (6.42) leads to

$$(1+z_{\mathrm{M}})^2 = \frac{\overline{H}_{\mathrm{Pl}}}{H_0\,z_0(\gamma)} = 2\,\frac{E_{\mathrm{H}}}{E_{\mathrm{Pl}}}\,. \qquad (6.43)$$

Using the data of Tables III to V, the results, derived by (6.1 a) to (6.43) for the total (massless and massive) universe, are also valid for the corresponding anti-universe.

In Refs. [1, 2], for particles (P) and antiparticles ($\overline{\mathrm{P}}$), we have introduced the Schwarzschild (S) radius $R_{\mathrm{S}} = R'$, where R' is defined by

$$R' = \frac{G_N}{c^4}\left[E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}})\right], \qquad (6.44)$$

so that because of $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$ we obtain as Schwarzschild radius [9, 11]

$$R_{\mathrm{S}} = 2\,\frac{G_N}{c^4}\,E_0(\mathrm{P}) \qquad (6.45)$$

as distance between particle-antiparticle pairs in the quantum vacuum [1, 2], which penetrates completely the space-time continuum of the total (massless and massive) universe and anti-universe (see Eqs. (6.46) to (6.48)).

For example, at Higgs boson-antiboson pairs, we have the Schwarzschild radius

$$R_{\mathrm{S}} = R_{\mathrm{S}}(\mathrm{H}^0) = 2\overline{R}_{\mathrm{H}^0} = 3.336\cdot 10^{-50}\ \mathrm{cm}\,, \qquad (6.46)$$

where $\overline{R}_{\mathrm{H}^0}$ is given by Eq. (6.25). In the case of photons, for the corresponding photon-photon pairs, we obtain as Schwarzschild radius

$$R_S = R_S(\gamma) = 2R_{BB} = 4.138 \cdot 10^{-96}\ \text{cm}, \tag{6.47}$$

where the big bang distance $R_{BB} = 2.069 \cdot 10^{-96}$ cm (see Eq. (3.23)) is defined according to Eq. (2.40) by $R_{BB} = (G_N/c^4)\, E_0(\gamma)$ via Eqs. (6.1 b). For gravitons, we can assume an "analogous" situation.

According to Eqs. (3.108), for the Hubble energy E_H of the present ($z = 0$) massive universe and anti-universe, we find plausibly for their Schwarzschild radius

$$R_S = 2\frac{G_N}{c^4} E_H = R_0 = 2 \times \tfrac{1}{2} R_0 = 1.375 \cdot 10^{28}\ \text{cm}. \tag{6.48}$$

Then, we find the escape (esc) velocity v_{esc} (see, e.g., Refs. [9, 11]) for the present massive universe or anti-universe to

$$v_{esc} = \left(\frac{2\, G_N\, (E_H/c^2)}{R_0} \right)^{1/2} = c, \tag{6.49}$$

i.e. they are black holes because the surfaces of the present massive universe or anti-universe are limited by their Schwarzschild radius $R_S = R_0$, so that even the light cannot escape from the present massive universe or anti-universe.

Because of $R_{BB} < R_S$ (see Eq. (6.47)), an analogous situation has the big bang, so that we get no information about the big bang, since from a mass with the surface within the range of the Schwarzschild radius no events can be observed.

A completely other situation is observed at the extremely massive particles and antiparticles, which form the corresponding universe and anti-universe. For example, at the Higgs bosons and Higgs antibosons, their scale factors $R_{H^0} = 2.563 \cdot 10^{13}$ cm (see Eq. (6.29)) and analogously $R_{\overline{H}^0} = 2.563 \cdot 10^{13}$ cm do not lie within the range of the Schwarzschild radius (6.46).

Now, we try to calculate the rest energy of the photons via the general galactic magnetic field, assuming a coupling with the magnetic neutrino moment.

In Sec. 4, we have shown that the joint origin of the dark matter and the dark energy is a result of the invisible decay and breakup products of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$ and $\hat{\nu}_{\rm b}$. Therefore, we can assume that their magnetic (mag) energy density $\hat{\varepsilon}_{\rm mag}$ must be equivalent to

$$\hat{\varepsilon}_{\rm mag} = (\Omega_{\rm dm} + \Omega_\Lambda)\,\Omega_\gamma\,\rho_{0\rm C}\,c^2 = (\Omega_{\rm dm} + \Omega_\Lambda)\,\rho_0(\gamma)\,c^2 \qquad (6.50)$$

because of the definition (see, e.g., Refs. [7, 8])

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_{0\rm C}} = \frac{\rho_0(\gamma)}{\rho_{0\rm C}} = 5.46\cdot 10^{-5} \quad \text{(see Table I)}, \qquad (6.51)$$

where $\rho_\gamma = \rho_0(\gamma)$ is the electromagnetic mass density of the photons of the present cosmic microwave background (CMB) explained, for example, in Ref. [8]. Taking

$$\rho_0(\gamma)\,c^2 = \frac{\pi^2}{15}\,\frac{(kT_0)^4}{(\hbar c)^3} = 0.26057\,\text{eV cm}^{-3} \qquad (6.52)$$

and

$$\rho_{0\rm C}\,c^2 = 4.77\cdot 10^3\ \text{eV cm}^{-3}, \qquad (6.53)$$

we obtain the magnetic energy density $\hat{\varepsilon}_{\rm mag}$ to

$$\hat{\varepsilon}_{\rm mag} = (\Omega_{\rm dm} + \Omega_\Lambda)\,\rho_0(\gamma)\,c^2 = 0.2475\,\text{eV cm}^{-3}. \qquad (6.54)$$

The galactic magnetic (gmag) energy density $\varepsilon_{\rm gmag}$ is given by

$$\varepsilon_{\rm gmag} = \frac{1}{2\mu_0}\,B^2 = \frac{1}{8\pi\cdot 1.602176565\cdot 10^{-20}}\left(\frac{B}{\rm T}\right)^2 \text{eV/cm}^3, \qquad (6.55)$$

where μ_0 describes the permeability of free space and B represents the general galactic magnetic field. Then, by the condition $\varepsilon_{\rm gmag} = \hat{\varepsilon}_{\rm mag}$, we find the present general galactic magnetic field B to

$$B = \left[(\Omega_{\rm dm} + \Omega_\Lambda)\,\rho_0(\gamma)c^2 \times 8\pi \times 1.6022\cdot 10^{-20}\ \text{cm}^3/\text{eV}\right]^{1/2}\text{T} =$$
$$= 3.157\cdot 10^{-10}\,\text{T} \qquad (6.56)$$

in excellent agreement with the observations [17], which lie in the small range $(1.4 \pm 0.2)\cdot 10^{-10}\,\text{T} \le B \le (4.4 \pm 0.9)\cdot 10^{-10}\,\text{T}$.

Consequently, the remaining energy density $\varepsilon_{\rm b} = \Omega_{\rm b}\,\rho_0(\gamma)\,c^2$ must provide the present luminosity density of all galaxies to

$$\Lambda_0 = \Omega_b H_\Lambda \rho_0(\gamma) c^2 = 4.544 \cdot 10^{-33} \text{ W/m}^3 =$$
$$= 2.886 \cdot 10^8 L_{SOL}/\text{Mpc}^3, \quad (6.57)$$

where the Hubble parameter $H_\Lambda = \Omega_\Lambda^{1/2} H_0 = 1.805 \cdot 10^{-18} \text{ s}^{-1}$, which characterizes the present day situation of the massive universe by the exponential expansion (2.27) for $t = t_0$, and the solar (SOL) luminosity $L_{SOL} = 3.828 \cdot 10^{26}$ W [10] were used. The result (6.57) agrees with the limiting value of the total luminosity $\Lambda_{tot} \lesssim 3 \cdot 10^8 L_{SOL}/\text{Mpc}^3$ of all galaxies in Ref. [9]. This agreement supports the upper assumptions.

Because of the estimations (6.54) to (6.56), we can assume that the gravitons yield no contribution to this general galactic magnetic field.

Therefore, we assume that a connection must exist between the magnetic field (6.56) as well as the rest energy of the photons via the magnetic moments of the light neutrinos [1-3, 5, 10, 18] if we consider the sum of ratios of the corresponding rest energies [1-3] of the end products of the 3 transformation types [2, 3] (in form of light neutrinos ν into heavy neutrinos $\tilde{\nu}$, heavy neutrinos $\tilde{\nu}$ into sterile neutrinos $\hat{\nu}$ and light neutrinos ν into sterile neutrinos $\hat{\nu}$) as well as the neutrino number densities $n(\nu) = n(\tilde{\nu}) = 112 \text{ cm}^{-3}$ and $n(\hat{\nu}) = 0.178 \text{ cm}^{-3}$ [1-3]. This sum provides $3\Omega_{dm}$, since it contains once $\Omega_{dm} = E_0(\tilde{\nu}_{dm})/[E_0(\tilde{\nu}_\Lambda) + E_0(\tilde{\nu}_{dm}) + E_0(\tilde{\nu}_b)] = 0.265$ and twice $\Omega_{dm} = E_0(\hat{\nu}_{dm})/[E_0(\hat{\nu}_\Lambda) + E_0(\hat{\nu}_{dm}) + E_0(\hat{\nu}_b)] = 0.265$. These rest energies possess the values $E_0(\tilde{\nu}_\Lambda) = 29.3 \text{ eV}$, $E_0(\tilde{\nu}_{dm}) = 11.31 \text{ eV}$, $E_0(\tilde{\nu}_b) = 2.08 \text{ eV}$, $E_0(\hat{\nu}_\Lambda) = 18436 \text{ eV}$, $E_0(\hat{\nu}_{dm}) = 7120 \text{ eV}$ and $E_0(\hat{\nu}_b) = 1309 \text{ eV}$ [1-4]. Thus, we find semi-empirically the constant of this process to $3\Omega_{dm}\, n(\tilde{\nu})/n(\hat{\nu})$, which must be distributed symmetrically among the two decay products of the sterile neutrinos, i.e. this process must be characterized presently by the constant factor

$$[3\,\Omega_{dm}\, n(\tilde{\nu})/n(\hat{\nu})]^{1/2} \cong 22.37. \quad (6.58)$$

Using the geometric mean from the product of the neutrino number density ratio $n(\tilde{\nu})/n(\hat{\nu}) = 629.2$ (see above) and the ratio of the results (4.1) and (4.4), for the present universe, a similar factor is plausibly found to

$$\Omega_{\text{dm}}\left[\frac{n(\tilde{\nu})}{n(\hat{\nu})}\,\frac{(1-\mathrm{e}^{-(\tau_{\hat{\nu}}-\tau_{\text{H}})/\tau_{\hat{\nu}}})}{\mathrm{e}^{-(\tau_{\hat{\nu}}-\tau_{\text{H}})/\tau_{\hat{\nu}}}}\right]^{1/2} \cong 22.39\,. \qquad (6.59)$$

According to Refs. [1-3, 5, 10, 18], the sum of the magnetic moment of all light massive neutrinos can be derived to

$$\mu_{\nu}(\sum_i \nu_i) = \frac{3}{8\pi^2\sqrt{2}}\,e\,[G_F/(\hbar c)^3]\,\hbar c^2 \sum_i E_0(\nu_i) =$$

$$= 3.203\cdot 10^{-19}(\sum_i E_0(\nu_i))/\text{eV})\,\mu_B \qquad (6.60)$$

with the Bohr magneton [10]

$$\mu_B = \frac{e\,\hbar\,c^2}{2\,E_0(\text{e})} = (5.7883818066(38)\cdot 10^{-11}\,\text{MeV/T}\;, \qquad (6.61)$$

where the new quantities e, $[G_F/(\hbar c)^3]$, $\sum_i E_0(\nu_i) = 5.97\cdot 10^{-2}\text{eV}$ and $E_0(\text{e})$ describes the elementary charge, the Fermi coupling constant [10], the sum of the rest energies of the light neutrinos [1-3, 5] and the rest energy of the electron [10], respectively. The subscript $i = \text{e}, \mu, \tau$ characterizes the light e, μ and τ neutrino. Thus, for the magnetic moment (6.60), we can also write

$$\mu_{\nu}(\sum_i \nu_i) = 1.107\cdot 10^{-24}\,\text{eV}\,\text{T}^{-1}, \qquad (6.62)$$

so that, for example, via the factor (6.58), we find semi-empirically the connection between the magnetic field (6.56) as well as the rest energy of the photons (see Eq. (6.1 b)) to

$$B = [3\,\Omega_{\text{dm}}\,n(\tilde{\nu})/n(\hat{\nu})]^{1/2}\,\frac{E_0(\gamma)}{\mu_{\nu}(\sum_i \nu_i)} \qquad (6.63)$$

Indeed, if we use $B = 3.157\cdot 10^{-10}\,\text{T}$, the expression (6.63) yields

$$E_0(\gamma) = \frac{B}{[3\,\Omega_{\text{dm}}\,n(\tilde{\nu})/n(\hat{\nu})]^{1/2}}\,\mu_{\nu}(\sum_i \nu_i) = 1.563\cdot 10^{-35}\,\text{eV}\,. \qquad (6.64)$$

With that, we have confirmed the existence of the photon rest energy by the experimental value of the general galactic magnetic field (see, e.g., Ref. [17]), i.e. by a convincing direct experimental observation.

Because of the assumption $E_0(\gamma) = E_0(\mathrm{G})$ [see Eq. (6.2)], the results (3.107) and (3.108) mean that for the universe we must assume the proper energy

$$2E_{\mathrm{H}} \cong 2\rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi R_0^3 \cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi (R_0\, 2^{1/3})^3 \cong$$
$$\cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi (R_0\, r)^3 \cong 1.039\cdot 10^{89}\ \mathrm{eV}\,, \tag{6.65}$$

where $r = 2^{1/3} \cong 1.26$ is again the dimensionless time-independent comoving coordinate of the proper distance $d = R_0 r$.

Using the data of Tables III to V, the results, derived from Eqs. (6.44) to (6.65) for the massive universe, are also valid for the massive anti-universe.

7 Hubble "constants" as a function of cosmic evolution epochs

The Hubble "constants" determine how fast the universe expands over the time. Because of Eq. (6.10), they must possess discontinuities in the cosmic evolution.

By Eqs. (6.6) to (6.9), we have derived the values of the Hubble parameters $\overline{H}_{\mathrm{BB}} = z_0(\gamma)/t_{\mathrm{BB}} = 9.643\cdot 10^{74}\ \mathrm{s}^{-1} = 2.976\cdot 10^{94}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Eq. (6.7)) and $\overline{H}_{\mathrm{Pl}} = z_0(\gamma)/t_{\mathrm{Pl}} = 1.234\cdot 10^{12}\ \mathrm{s}^{-1} = 3.808\cdot 10^{31}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Eq. (6.9)) for the massless universe ($R_{\mathrm{BB}} \le R \le R_{\mathrm{Pl}}$). They are the limiting values of the continuous function (6.12). However, between the Hubble parameters $\overline{H}_{\mathrm{Pl}} = z_0(\gamma)/t_{\mathrm{Pl}} = 1.234\cdot 10^{12}\ \mathrm{s}^{-1} = 3.808\cdot 10^{31}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see above) and $H_{\mathrm{Pl}} = 1/2t_{\mathrm{Pl}} = 9.275\cdot 10^{42}\ \mathrm{s}^{-1} = 2.862\cdot 10^{62}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [lower limiting value of the early massive universe ($R_{\mathrm{Pl}} \le R \le \widetilde{R}_0$) according to the new inflation model (see Eqs. (2.14) to (2.18))], we have a discontinuity (see Eq. (6.10)). These Hubble parameters are father than that of the Planck observations 2013 [12], which yield $H_0 = 2.181\cdot 10^{-18}\ \mathrm{s}^{-1} = 67.3\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Table I). The connection between all these values is given by Eq. (6.10). Consequently, instead of Eqs. (6.7) and (6.10), we can also write

$$\overline{H}_{\mathrm{BB}} = \frac{z_0(\gamma)}{t_{\mathrm{BB}} t_{\mathrm{Pl}} (1+z_{\mathrm{M}})^2 H_0} = 2.976 \cdot 10^{94}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{7.1}$$

$$\overline{H}_{\mathrm{Pl}} = z_0(\gamma)\left[1+z_{\mathrm{M}}\right]^2 H_0 = 3.808 \cdot 10^{31}\ \mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{7.2}$$

and

$$H_{\mathrm{Pl}} = \tfrac{1}{2}(1+z_{\mathrm{M}})^2 H_0 = 2.862 \cdot 10^{62}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{7.3}$$

i.e. Eqs. (7.1) and (7.2) show clearly a continuous connection by $\overline{H} = z_0(\gamma)/t$ for $t_{\mathrm{BB}} \le t \le t_{\mathrm{Pl}}$, whereas between Eqs. (7.2) and (7.3) we see again a large discontinuity.

According to Ref. [1], in the radiation-dominated early ($R_{\mathrm{Pl}} \le R \le \widetilde{R}_0$) and late ($\widetilde{R}_0 \le R \le R_0$) massive universe {$z \ge 10^5$ for the new inflation model [see Eqs. (2.14) to (2.18)]}, because of $t = 1/(2N(t)\Omega_\gamma)^{1/2}(1+z)^2 H_0$ as well as $\widetilde{R} \propto t^{1/2}$ and $R \propto t^{1/2}$ (see, e.g., Ref. [1]), we have the continuous connection

$$H = \dot{R}/R = 1/2t = \tfrac{1}{2}(2N(t)\Omega_\gamma)^{1/2}(1+z)^2 H_0 . \tag{7.4}$$

For $t = t_{\mathrm{Pl}}$, because of $N(T) = 1/2\Omega_\gamma$ and $z = z_{\mathrm{M}}$ [1, 2], Eq. (7.4) gives again the expression (7.3).

Because Eqs. (7.2) and (7.3) yield a discontinuity at the Hubble parameters $\overline{H}_{\mathrm{Pl}}$ and H_{Pl}, we expect also a similar discontinuity between H_0 of the Planck observations (see above) and H_{acc} (see below) of the accelerated expansion (2.27). This assumption is clear because H_0 was derived from measurements of the CMB, which was formed $3.72 \cdot 10^5$ years after the big bang [7], whereas the accelerated expansion began $7{,}70 \cdot 10^9$ years after the big bang (see Eq. (2.35) and also below), i.e. at two very different epochs of the evolution of the universe. Therefore, the Hubble expansion rate $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, which in Ref. [7] was assumed as the present Hubble "constant" of the universe, is interpreted as the present "Hubble constant" of the CMB, so that this new Hubble "constant" $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ can be used further as basis for all hitherto existing considerations for the evolution of the universe, i.e. it must also determine all Hubble "constants" of the universe. Thus, we expect that this present CMB value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ must also yield a new Hubble "constant" for the beginning of the "present" accelerated expansion of the universe, so that this "constant" has a larger value $H_{\mathrm{acc}} > H_0$ (see Eq. (7.6)).

Then, the accelerated expansion can be described uniquely and continuously via Eq. (2.38). Then, from the beginning (2.35) of the accelerated expansion, we can write

$$H_{\mathrm{acc}} = \frac{(c^2\widetilde{\Lambda}/3)}{\Omega_\Lambda^{1/2}} = \frac{t_0 - t}{t_0 - \widetilde{t}} H_0 . \tag{7.5}$$

Then, taking the data at the expression (2.38), we can apply $t_0 - t = 2.72\cdot 10^{17}\,\mathrm{s}$ and $t_0 - \widetilde{t} = 1.928\cdot 10^{17}\,\mathrm{s}$ ($t_0 = (4.358 \pm 0.016)\cdot 10^{17}\mathrm{s}$ see Table I), so that Eq. (7.5) gives

$$H_{\mathrm{acc}} = \frac{t_0 - t}{t_0 - \widetilde{t}} H_0 = 3.08\cdot 10^{-18}\,\mathrm{s^{-1}} = 95.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}} , \tag{7.6}$$

i.e. the epoch of Eq. (7.6) begins at $\widetilde{t} = 2.43\cdot 10^{17}\mathrm{s} = 7.70\,\mathrm{Gyr}$.

The result (7.6) means an extremely rapid expansion in contrast to the value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. To this problem, we will return below.

In Eq. (7.6), for $1 + z_{\mathrm{acc}} = 1.632$ (beginning of the accelerated expansion [1, 2]), by the result (2.15), via the accelerated expansion (2.27), we have determined the time $t = 1.638\cdot 10^{17}\mathrm{s}$, whereas for $z_{\mathrm{eq}} >> z \geq 1$ (z_{eq} see Table I) according to Refs. [1, 2, 6] the time $\widetilde{t} = 2.43\cdot 10^{17}\mathrm{s}$ is defined by

$$\widetilde{t} = t(z) = \frac{2}{3\,H_0\,\Omega_\Lambda^{1/2}} \ln \frac{\sqrt{\Omega_\Lambda(1+z)^{-3}} + \sqrt{\Omega_\Lambda(1+z)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}} , \tag{7.7}$$

whereat the corresponding scale factor [1, 2, 6] is given by

$$\frac{R}{R_0} = \frac{1}{1+z} = \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3} \left[\sinh(3/2\,\Omega_\Lambda^{1/2} H_0\,\widetilde{t})\right]^{2/3} =$$

$$= \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3} \left[\frac{e^{3/2\,\Omega_\Lambda^{1/2} H_0 \widetilde{t}} - e^{-3/2\,\Omega_\Lambda^{1/2} H_0 \widetilde{t}}}{2}\right]^{2/3} . \tag{7.8}$$

Analogously, for $1+z=1.05$, we have estimated $t=4.088\cdot 10^{17}\text{s}$ and $\widetilde{t}=4.139\cdot 10^{17}\text{s}$, so that instead of Eq. (7.6) we find now

$$H_{\text{acc}}=\frac{t_0-t}{t_0-\widetilde{t}}H_0=2.69\cdot 10^{-18}\,\text{s}^{-1}=83.0\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}, \tag{7.9}$$

whereat the value $1+z=1.05$ represents a lower limit because of the accuracy of the data. Consequently, the result (7.9) means a decrease of the accelerated expansion if it is compared with the value (7.6).

Then, for $z=0$, because of $t=t_0$ and $\widetilde{t}=t_0$, Eqs. (7.6) and (7.9) yield the indefinable value $H_{\text{acc}}=(0/0)H_0$. Instead of this value, we apply the results (3.42) according to Refs. [1, 2] i.e. we can now estimate the present Hubble parameter by the semi-empirical expression

$$H_{\text{acc},0}=\frac{(c^2\widetilde{\Lambda}/3)}{\Omega_{\Lambda}^{1/2}}=\frac{t_{\text{eff}}-t_0}{(\tau_{\bar{\nu}2}\ln 2)-t_0}H_0\,. \tag{7.10}$$

Consequently, taking the data, applied in the result (3.42), we have $t_{\text{eff}}-t_0=3.676\cdot 10^{17}\text{s}$ and $(\tau_{\bar{\nu}2}\ln 2)-t_0=3.322\cdot 10^{17}\text{s}$, so that Eq. (7.10) yields (via the cosmological parameters of Table I) as present Hubble parameter

$$H_{\text{acc},0}=\frac{t_{\text{eff}}-t_0}{(\tau_{\bar{\nu}2}\ln 2)-t_0}H_0=2.41\cdot 10^{-18}\text{s}=74.4\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}, \tag{7.11}$$

i.e. the epoch of Eq. (7.11) begins at $t_0=4.358\cdot 10^{17}\text{s}=13.81\,\text{Gyr}$ (see Table I) after the big bang. Thus, we need no new physical assumptions.

Then, for $t\geq t_0$, at $t_{\text{eff}}=8.034\cdot 10^{17}\text{s}$ and $\tau_{\bar{\nu}2}\ln 2=7.680\cdot 10^{17}\text{s}$ [1, 2], we assume semi-empirically

$$H_{\text{acc},}(t)=\left(\frac{t_{\text{eff}}-t_0}{(\tau_{\bar{\nu}2}\ln 2)-t_0}\right)^2\frac{(\tau_{\bar{\nu}2}\ln 2)-t}{t_{\text{eff}}-t}H_0\,. \tag{7.12}$$

Thus, for $H_{\text{acc}}(t)=H_0$, we obtain

$$t=\frac{\left([t_{\text{eff}}-t_0]/[(\tau_{\bar{\nu}2}\ln 2)-t_0]\right)^2\tau_{\bar{\nu}2}\ln 2-t_{\text{eff}}}{\left([t_{\text{eff}}-t_0]/[(\tau_{\bar{\nu}2}\ln 2)-t_0]\right)^2-1}=$$

$$=6.103\cdot 10^{17}\text{s}=19.34\,\text{Gyr}\,. \tag{7.13}$$

Therefore, in future, from $t = 19.34\,\mathrm{Gyr}$, for $H_{\mathrm{acc}}(t) < H_0$, the expansion of the universe is still slower, i.e. the accelerated expansion is decelerated.

Consequently, the results (7.6) as well as (7.9) and (7.11) mean a father expansion than the value $H_0 = 2.181 \cdot 10^{-18}\,\mathrm{s}^{-1} = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see above). However, the Hubble parameters (7.9) and (7.11) yield a slower expansion than the value (7.6). Because of the result (7.13), the accelerated expansion of the universe is extremely reduced at $t = 19.34\,\mathrm{Gyr}$. This result agrees with the derivation of a slow linear expansion [1, 2], which is characterized by the expression [1, 2]

$$\frac{1}{\mathrm{AU}}\frac{d}{dt}\mathrm{AU} = 2.733 \cdot 10^{-20}\,\mathrm{s}^{-1}, \qquad (7.14)$$

where $\mathrm{AU} = 1.49597870700(3) \cdot 10^{11}\,\mathrm{m}$ describes the astronomical unit [10], whereas $d\mathrm{AU}/dt$ [1, 2] represents the astronomical unit changing [1, 2]

$$\frac{d}{dt}\mathrm{AU} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} =$$

$$= (12.9^{+1.2}_{-1.3})\ \mathrm{cm\,yr}^{-1} \qquad (7.15)$$

in accordance with the observation [19]

$$\frac{d}{dt}\mathrm{AU} = (15 \pm 4)\ \mathrm{cm\,yr}^{-1}. \qquad (7.16)$$

The values $R_{\mathrm{earth}} = 6.378137 \cdot 10^6\,\mathrm{m}$ and $R_{\mathrm{sun}} = (6.9551 \pm 0.0004) \cdot 10^8\,\mathrm{m}$ are the equatorial radii of the earth and the sun [10].

Then, by Eqs. (7.14) and (7.15), because of $R_0 = c/H_0$ (see Table I), for the slow linear (lin) expansion, we can assume its Hubble parameter to

$$H_{\mathrm{lin}} = \frac{1}{\mathrm{AU}}\frac{d}{dt}\mathrm{AU} =$$

$$= \left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{\mathrm{AU}} H_0 =$$

$$= 0.843\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \qquad (7.17)$$

Consequently, in future, the result (7.17) means a slow linear expansion of the late massive universe (see above) in comparison with the Hubble parameters $95.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \geq H_{\mathrm{acc}} > 0.843\mathrm{km\,s^{-1}\,Mpc^{-1}}$ {see Eqs. (7.6), (7.9), (7.11) and (7.17)}.

Thus, at $H_{\mathrm{acc}}(t) = H_{\mathrm{lin}}$, we get

$$t = \frac{(H_0/H_{\mathrm{lin}})\left([t_{\mathrm{eff}} - t_0]/[(\tau_{\bar{\nu}2}\ln 2) - t_0]\right)^2 \tau_{\bar{\nu}2}\ln 2 - t_{\mathrm{eff}}}{(H_0/H_{\mathrm{lin}})\left([t_{\mathrm{eff}} - t_0]/[(\tau_{\bar{\nu}2}\ln 2) - t_0]\right)^2 - 1} =$$

$$= 7.676 \cdot 10^{17}\,\mathrm{s} = 24.32\,\mathrm{Gyr}\,. \tag{7.18}$$

Therefore, in future, at $H_{\mathrm{acc}}(t) = H_{\mathrm{lin}}$, i.e. from $t = 24.32\,\mathrm{Gyr}$, the slow linear expansion of the universe dominates up to the final state of the massive universe.

Indeed, the present Hubble parameter $H_{\mathrm{acc},0} = 74.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Eq. (7.11)) is confirmed by the observations of Riess et al. [20], which yield a value for the present Hubble "constant" of $74.03\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The deviation between these two expansion rates is about 0.5%. Thus, we cannot more denote the Hubble parameter $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ as present expansion rate of the universe, since it was interpreted as the present Hubble "constant" of the CMB, so that we have assumed that we can express all Hubble parameters as a function of the present CMB value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ of the Planck observations 2013, since the present CMB value is slower in comparison with the present Hubble "constant" $H_{\mathrm{acc},0} = 74.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Thus. we have used the present CMB value H_0 as basis for the description of the evolution of the universe, since it was reasonably derived via data [12] of the CMB formed at $3.72 \cdot 10^5$ years after the big bang. Because of these assumptions, in this work, all considerations are correct.

Consequently, our result of a slower expansion of the massive universe is in contrast to the interpretation of Riess et al. [20], in which the universe expands always father, i.e. this interpretation contradicts the known physics and is only understandable in the framework of a new physics [20], whereas our far-reaching result agrees with the known physics [Λ cold dark matter model ($\Lambda\mathrm{CDM}$) confirmed experimentally].

Because of a better explanation of the dark energy in the second half of Sec. 4, for the confirmation of this far-reaching conclusion, we use the new particle-defined cosmological parameter values of Table VI, since the accelerated expansion is determined by the normal dark energy $\Omega_\Lambda^{**} = 0.683^{+0.038}_{-0.028}$ (introduced by Eq. (4.24) and Table VI) via the vacuum energy density $\rho_{\text{vac}}^* c^2 = \Omega_\Lambda^{**} \rho_{0C}^* c^2 = (3.29^{+0.15}_{-0.14}) \cdot 10^3 \text{ eV cm}^{-3}$, which is again defined by the 3 sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\text{dm}}$ and $\hat{\nu}_{\text{b}}$ (see Sec. 4), i.e. we expect that we can estimate a still better present Hubble "constant" $H_{\text{acc,0}}^*$ than $H_{\text{acc,0}} = 74.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, using again the result (7.10). Therefore, we apply following procedure in 3 steps.

Firstly, via this vacuum energy density $\rho_{\text{vac}}^* c^2 = (3.29^{+0.15}_{-0.14}) \cdot 10^3 \text{ eV cm}^{-3}$, according to Eq. (3.8), we determine the distance

$$d_{\text{eff}}^* = \left(\frac{E_{\text{Pl}}^2}{\hbar c \, \tfrac{1}{2} \Omega_\Lambda^{**} \rho_{0C}^* c^2 \, \tfrac{4}{3} \pi} \right)^{1/2} = (3.311^{+0.070}_{-0.075}) \cdot 10^{26} \text{ m} , \qquad (7.19)$$

so that because of Eqs. (3.15) and (3.42) including (7.10) the lifetime $\tau_{\hat{\nu}}^* \rightarrow \tau_{\hat{\nu}2}^*$ of the sterile neutrinos is found to

$$\tau_{\hat{\nu}2}^* = \frac{d_{\text{eff}}^*}{c} = (1.104^{+0.023}_{-0.025}) \cdot 10^{18} \text{ s} = 34.98^{-0.73}_{-0.79} \text{ Gyr} . \qquad (7.20)$$

Thus, the corresponding expression $(\tau_{\hat{\nu}2}^* \ln 2) - t_0^*$ yields

$$(\tau_{\hat{\nu}2}^* \ln 2) - t_0^* = (3.30^{+0.14}_{-0.15}) \cdot 10^{17} \text{ s} . \qquad (7.21)$$

Secondly, using Eq. (2.30) with $N(T) = 3.362644$, we can form the condition

$$\rho_{\text{vac}}(T^*) c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT^*)^4}{(\hbar c)^3} \frac{1}{N(T)} = \rho_{\text{vac}}^* c^2 =$$

$$= \Omega_\Lambda^{**} \rho_{0C}^* c^2 = (3.29^{+0.15}_{-0.14}) \cdot 10^3 \text{ eV cm}^{-3} , \qquad (7.22)$$

i.e. the temperature T^* can be determined to

$$T^* = 51.49^{+0.59}_{-0.55} \text{ K} , \qquad (7.23)$$

so that we obtain the redshift condition

$$1+z_{\Lambda}^{*}=\frac{T^{*}}{T_{0}}=18.89_{-0.20}^{+0.22}, \qquad (7.24)$$

where now the influence of the normal dark matter Ω_{Λ}^{**} begins (see Eq. (2.20)). Then, taking the blueshift condition $1+z(\nu_{e})=0.406_{-0.025}^{+0.020}$ of the electron neutrino (see Eq. (2.67)), analogous to Eq. (2.22), we estimate now the mean redshift condition $1+z_{MNA}^{*}$ to

$$1+z_{MNA}^{*}=\left([1+z_{\Lambda}^{*}][1+z(\nu_{e})]\right)^{1/2}=2.769_{-0.100}^{+0.084}, \qquad (7.25)$$

where it defines the mean negative acceleration $\ddot{R}_{MNA}^{*}$ via Eqs. (2.23) and (2.24), i.e. we obtain this mean negative acceleration to

$$\ddot{R}_{MNA}^{*}=-\frac{1}{2}c\,H_{0}^{*}\left[\Omega_{m}^{*}(1+z_{MNA}^{*})^{2}+\Omega_{\Lambda}^{**}/(1+z_{MNA}^{*})\right]=$$

$$=(-8.64_{-0.69}^{+0.60})\cdot10^{-8}\,\mathrm{cm\,s^{-2}}, \qquad (7.26)$$

if the transformations $H_{0}\rightarrow H_{0}^{*}=2.191\cdot10^{-18}\mathrm{s}^{-1}$, $\Omega_{m}\rightarrow\Omega_{m}^{*}=0.311$ and $\Omega_{\Lambda}\rightarrow\Omega_{\Lambda}^{*}=0.683$ are used according to Tables V and VI. Consequently, via Eq. (2.26), the effective scale factor R_{eff}^{*} is given by

$$R_{eff}^{*}=\frac{-\ddot{R}_{MNA}^{*}}{\Omega_{\Lambda}^{**}H_{0}^{*2}}=(2.63_{-0.21}^{+0.18})\cdot10^{26}\,\mathrm{m}, \qquad (7.27)$$

so that via Eqs. (2.27) or (2.28) by the corresponding values of Table VI we can determine the time difference

$$t_{eff}^{*}-t_{0}^{*}=\frac{1}{\Omega_{\Lambda}^{**1/2}H_{0}^{*}}\ln\frac{R_{eff}^{*}}{R_{0}^{*}}=(3.61_{-0.23}^{+0.19})\cdot10^{17}\,\mathrm{s}=11.44_{-0.73}^{+0.60}\,\mathrm{Gyr}. \qquad (7.28)$$

Thirdly, using the results (7.21) and (7.28), via the corresponding expression (7.10), we find more exactly the present Hubble "constant" $H_{acc,0}^{*}$ to

$$H_{acc,0}^{*}=\frac{t_{eff}^{*}-t_{0}^{*}}{(\tau_{\nu2}^{*}\ln2)-t_{0}^{*}}H_{0}^{*}=74.0_{-2.6}^{+2.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \qquad (7.29)$$

i.e. the deviation between the observed value $74.03\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see above) and the result (7.29) is very small.

This excellent agreement confirms our hypothesis that the joint origin of the dark matter and dark energy is based on the sterile neutrinos [1-4] as well as their breakup and decay products (see above and Ref. [1]). This result supports also the introduction of the normal (Ω_Λ^{**}) and the total (Ω_Λ^{***}) dark energy in the present work (see the second half of Sec. 4).

Using the data of Tables III to VI, the corresponding results, derived in Sec. 7 for the massive universe, are also valid for the massive anti-universe.

8. The time dependence of the cosmological "constant"

The cosmological "constant" problem has a complex history [21]. In this work, for the total (massless and massive) universe, the vacuum energy densities or cosmological "constants", introduced already in Refs. [1, 2] as variable quantities, are compared with the prediction of the quantum field theory. For this goal, they are written in their time-dependent form, whereat the considerations are initially restricted to the results of Sec. 3.1. Firstly, we take into account only the 3 limiting cases (Hubble time ($\tau_{\rm H}$), Planck time ($t_{\rm Pl}$) and big bang ($t_{\rm BB}$)). Secondly, we treat generally the case $t \le t_{\rm Pl}$. Thirdly, we consider the case $t \ge t_{\rm Pl}$.

Now, we describe firstly the 3 limiting cases (Hubble time, Planck time and big bang). For the Hubble time $\tau_{\rm H} = 1/H_0 = 4.585 \cdot 10^{17}\,{\rm s}$, we have

$$\rho_{{\rm vac},\Lambda}\, c^2 = \Omega_\Lambda\, \rho_{0{\rm C}}\, c^2 = \Omega_\Lambda \frac{3\, H_0^2\, c^2}{8\pi\, G_N} =$$

$$= \Omega_\Lambda \frac{3\, c^2}{8\pi\, G_N\, \tau_{\rm H}^2} = 3.27 \cdot 10^3\ {\rm eV\, cm^{-3}} \tag{8.1}$$

or

$$\Lambda = \Lambda_\Lambda = \frac{3\,\Omega_\Lambda}{R_0^2} = \frac{3\,\Omega_\Lambda}{c^2/H_0^2} =$$

$$= \frac{3\,\Omega_\Lambda}{c^2\,\tau_{\rm H}^2} = 1.087 \cdot 10^{-52}\ {\rm m^{-2}}. \tag{8.2}$$

For the Planck units as the limits between massless and massive universe, according to Eqs. (2.30) and (2.31), because of $N(T) = 1/2\Omega_\gamma$ [1, 2], we obtain

$$\rho_{vac}(T_{Pl})\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT_{Pl})^4}{(\hbar c)^3}\frac{1}{N(T)} = \rho_{vac}(t_{Pl})\,c^2 =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{\hbar}{c^3 t_{Pl}^4}\frac{1}{N(T)} = 6.93\cdot 10^{121}\,\mathrm{eV\,cm^{-3}} \qquad (8.3)$$

or

$$\Lambda = \Lambda_{Pl} = \frac{8\pi^3}{45}\frac{1}{R_{Pl}^2\,N(T)} =$$

$$= \frac{8\pi^3}{45}\frac{1}{c^2 t_{Pl}^2\,N(T)} = 2.30\cdot 10^{66}\,\mathrm{m^{-2}}. \qquad (8.4)$$

At the big bang (see Eqs. (3.23) to (3.26)), we get

$$\rho_{vac}(R_{BB})\,c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{Pl}^2}{\hbar c\,R_{BB}^2} = \rho_{vac}(t_{BB})\,c^2 =$$

$$= \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{Pl}^2}{\hbar c^3 t_{BB}^2} = 4.227\cdot 10^{247}\,\mathrm{eV\,cm^{-3}} \qquad (8.5)$$

or

$$\Lambda = \Lambda_{BB} = \frac{16\pi^3}{45}\frac{\Omega_\gamma}{R_{BB}^2} = \frac{16\pi^3}{45}\frac{\Omega_\gamma}{c^2 t_{BB}^2} = 1.406\cdot 10^{192}\,\mathrm{m^{-2}}. \qquad (8.6)$$

Using Eqs. (8.1) to (8.4) at $t = t_{Pl}$ and $t = \tau_H$ as well as Eqs. (8.3) to (8.6) at $t = t_{BB}$ and $t = t_{Pl}$, we can introduce

$$\frac{\rho_{vac}(t_{Pl})}{\rho_{vac,\Lambda}} = \frac{\Lambda_{Pl}}{\Lambda_\Lambda} = \frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\frac{\tau_H^2}{t_{Pl}^2} \cong 2.119\cdot 10^{118} \qquad (8.7)$$

as well as

$$\frac{\rho_{vac}(t_{BB})}{\rho_{vac}(t_{Pl})} = \frac{\Lambda_{BB}}{\Lambda_{Pl}} \cong 6.10\cdot 10^{125}, \qquad (8.8)$$

respectively. We summarize correspondingly these calculated data (8.1) to (8.6) in the Tables VIII and IX.

Table VIII. The calculated time-dependent vacuum energy densities or cosmological "constants" for $t_{\mathrm{BB}} \leq t \leq t_{\mathrm{Pl}}$.

State	kT (eV)	Time (s)	$\rho_{\mathrm{vac}}c^2$ ($\mathrm{eV\,cm^{-3}}$)	Λ ($\mathrm{m^{-2}}$)
Big bang	$1.563\cdot10^{-35}$	$6.901\cdot10^{-107}$	$4.23\cdot10^{247}$	$1.41\cdot10^{192}$
$E_0(\mathrm{e})$	$5.110\cdot10^{5}$	$2.256\cdot10^{-66}$	$3.95\cdot10^{166}$	$1.32\cdot10^{111}$
$E_0(\mathrm{H}^0)$	$1.260\cdot10^{11}$	$5.564\cdot10^{-61}$	$1.49\cdot10^{156}$	$2.16\cdot10^{100}$
$E_0(\mathrm{X,Y})$	$2.675\cdot10^{25}$	$1.181\cdot10^{-46}$	$1.44\cdot10^{127}$	$4.80\cdot10^{71}$
Planck	$1.221\cdot10^{28}$	$5.391\cdot10^{-44}$	$6.93\cdot10^{121}$	$2.30\cdot10^{66}$

Because of the new inflation model [1, 2], we have a discontinuity in the evolution of the universe, so that Eqs. (8.7) and (8.8) do not agree with the result of the quantum field theory, which predicts a value of about 10^{122} for the result (8.7) at a continuous evolution of the universe [11]. However, we can simulate a continuous evolution for the total (massless and massive) universe by $(\Lambda_{\mathrm{BB}}\Lambda_{\Lambda})^{1/2}/\Lambda_{\Lambda} = (\Lambda_{\mathrm{BB}}/\Lambda_{\Lambda})^{1/2} = 1.137\cdot10^{122}$, so that we obtain an excellent agreement with the quantum field theory. In other words, the expression $(\Lambda_{\mathrm{BB}}/\Lambda_{\Lambda})^{1/2} = 1.137\cdot10^{122}$ corresponds to the geometric mean of the results (8.7) and (8.8).

Secondly, we treat now generally the case $t_{\mathrm{BB}} \leq t \leq t_{\mathrm{Pl}}$, which is also tabulated in Table VIII. Here, as examples, we estimate still the corresponding data for the X and Y gauge bosons as well as Higgs boson and the electron, given also in Table VIII.

For $t_{\mathrm{BB}} \le t \le t_{\mathrm{Pl}}$, according to Eqs. (2.41) and (2.42), for $R = \overline{R}$ (see, e.g., Eq. (6.16)), because of $R = ct$ [see Eq. (2.43)), we have the variable (time-dependent) vacuum energy density or cosmological "constant:

$$\rho_{\mathrm{vac}}(R)\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c\, R^2} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c^3 t^2} \tag{8.9}$$

or

$$\Lambda = \frac{16}{45}\pi^3\, \Omega_\gamma \frac{1}{R^2} = \frac{16}{45}\pi^3\, \Omega_\gamma \frac{1}{c^2 t^2}\,. \tag{8.10}$$

For example, at the X and Y gauge bosons, we have $\overline{R}_{\mathrm{X,\,Y}} = 3.541 \cdot 10^{-36}\ \mathrm{cm}$ (see Eq. (6.16)), i.e. we obtain $t = \overline{R}_{\mathrm{X,\,Y}} / c = 1.181 \cdot 10^{-46}\ \mathrm{s}$, so that we get

$$\rho_{\mathrm{vac}}(\mathrm{X,\,Y})\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c\, \overline{R}_{\mathrm{X,\,Y}}^2} =$$

$$= \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c^3 t^2} = 1.443 \cdot 10^{127}\ \mathrm{eV\,cm^{-3}} \tag{8.11}$$

or

$$\Lambda_{\mathrm{X,\,Y}} = \frac{16}{45}\pi^3\, \Omega_\gamma \frac{1}{\overline{R}_{\mathrm{X,\,Y}}^2} =$$

$$= \frac{16}{45}\pi^3\, \Omega_\gamma \frac{1}{c^2 t^2} = 4.80 \cdot 10^{71}\ \mathrm{m}^{-2}\,. \tag{8.12}$$

The data (8.11) and (8.12) are tabulated in Table VIII.

By the results (8.9) and (8.10), we have estimated also the corresponding data for the Higgs boson and the electron, given also in Table VIII.

Thirdly, we describe now generally the case $t \ge t_{\mathrm{Pl}}$ (tabulated in Table IX), whereat the considerations are here restricted particularly to the radiation-dominated universe ($z \ge 10^5$, $T \ge 3 \cdot 10^5\ \mathrm{K}$ or $t \le 3 \cdot 10^9\ \mathrm{s}$). Here, according to Eqs. (2.30) and (2.31), via the results (2.10) and (2.12), we have generally the variable vacuum energy density or cosmological "constant

$$\rho_{\rm vac}(T)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \tag{8.13}$$

or

$$\Lambda = \frac{8\pi G_N}{c^2}\rho_{\rm vac}(T) = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\rm Pl}^2(\hbar c)^2}\frac{1}{N(T)}\,. \tag{8.14}$$

i.e. explicitly for the radiation-dominated universe their time dependence is considered according to Refs. [1-5] by the connection

$$t = \frac{1}{2}\left(\frac{90\,\hbar^3 c^5}{8\,\pi^3 G_N}\right)^{1/2}\frac{1}{\sqrt{N(T)}\,(kT)^2} =$$

$$= \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\rm MeV}{kT}\right)^2 \rm s\,. \tag{8.15}$$

Table IX. The calculated time-dependent vacuum energy densities or cosmological "constants" for $t \geq t_{\rm Pl}$.

State	$N(T)$	kT (eV)	Time (s)	$\rho_{\rm vac}c^2$ (eV cm^{-3})	Λ (m^{-2})
Planck	$1/2\Omega_\gamma$	$1.221\cdot10^{28}$	$5.39\cdot10^{-44}$	$6.93\cdot10^{121}$	$2.30\cdot10^{66}$
$E_0(\rm X, Y)$	160.75	$2.675\cdot10^{25}$	$2.67\cdot10^{-40}$	$9.09\cdot10^{112}$	$3.02\cdot10^{57}$
$E_0(\rm H^0)$	385/4	$1.260\cdot10^{11}$	$1.55\cdot10^{-11}$	$7.48\cdot10^{55}$	2.49
$E_0(\rm e)$	43/4	$5.110\cdot10^{5}$	2.83	$1.81\cdot10^{35}$	$6.02\cdot10^{-25}$
$\Omega_\Lambda\rho_{0C}\,c^2$	3.362644	$4.430\cdot10^{-3}$	$6.65\cdot10^{15}$	$3.27\cdot10^{3}$	$1.09\cdot10^{-52}$
final	$(4/11)^{1/3}$	$5.811\cdot10^{-5}$	$2.17\cdot10^{20}$	$4.57\cdot10^{-4}$	$1.52\cdot10^{-59}$

Using the results (8.13) to (8.15), we get as their time dependence

$$\rho_{\text{vac}}(t)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{5.85809\,\text{MeV}^4}{(\hbar c)^3[N(T)]^2}\frac{\text{s}^2}{t^2} \tag{8.16}$$

or

$$\Lambda = \frac{8}{45}\pi^3\frac{5.85809\,\text{MeV}^4}{(\hbar c)^2 E_{\text{Pl}}^2[N(T)]^2}\frac{\text{s}^2}{t^2}\,. \tag{8.17}$$

Now, by the results (8.13) to (8.17), we estimate also the corresponding data for the X and Y gauge bosons as well as the Higgs boson and the electron, given also in Table IX.

For example, at the X and Y gauge bosons, we have $N(T) = 160.75$ (see Refs. [1-3, 5, 10]) and $kT = 2.675\cdot 10^{25}\,\text{eV}$ (see Eq. (6.15)), i.e. we can determine $t = 2.67\cdot 10^{-40}\,\text{s}$ via Eq. (8.15), so that we get the results

$$\rho_{\text{vac}}(\text{X, Y})\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{5.85809\,\text{MeV}^4}{(\hbar c)^3[N(T)]^2}\frac{\text{s}^2}{t^2} = 9.09\cdot 10^{112}\,\text{eV cm}^{-3} \tag{8.18}$$

or

$$\Lambda_{\text{X, Y}} = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\text{Pl}}^2(\hbar c)^2}\frac{1}{N(T)} =$$

$$= \frac{8}{45}\pi^3\frac{5.85809\,\text{MeV}^4}{(\hbar c)^2 E_{\text{Pl}}^2[N(T)]^2}\frac{\text{s}^2}{t^2} = 3.02\cdot 10^{57}\,\text{m}^{-2}\,, \tag{8.19}$$

tabulated in Table IX.

Analogous to the X and Y gauge bosons (see Eqs. (8.18) and (8.19)), we have calculated still the corresponding data for the Higgs boson H^0 ($N(T) = 385/4$) and the electron ($N(T) = 43/4$), for which the rest energy is given in Sec. 7 and their value $N(T)$ is defined in Refs. [1-3, 5, 10]. They are also tabulated in Table IX, where we have also summarized all calculated values for the time-dependent vacuum energy densities or cosmological "constants" for $t \geq t_{\mathrm{Pl}}$.

At a temperature of $T \approx 5 \cdot 10^9\ \mathrm{K}$, the neutrinos decouple from the photons [6], i.e. from this point we have $N(T) = 3.362644$ [6, 8] in formula (8.15), which is valid for $T \geq 3 \cdot 10^5\ \mathrm{K}$. However, from $T \approx 5 \cdot 10^9\ \mathrm{K}$, we have $T_\nu = (4/11)^{1/3} T$ (see also Sec. 2), where T is again the photon temperature, i.e. the photons and neutrinos dominate now the universe. Then, by $N(T) = 3.362644$, for $T \leq 3 \cdot 10^5\ \mathrm{K}$, via the formulae (8.13) and (8.14), the corresponding time of the variable vacuum energy densities or cosmological "constants" can be determined only by the redshift $1 + z = T/T_0$ (see Eq. (2.10)), using the result (2.12) for the time calculation.

Then, for Table IX, the corresponding time of the limiting values (2.32) and (2.33) is again evaluated by $1 + z = T/T_0 = 18.86$ (see Eq. (2.34)) via the expression (see Eq. (7.7))

$$t = t(z) = \frac{2}{3 H_0 \Omega_\Lambda^{1/2}} \ln \frac{\sqrt{\Omega_\Lambda (1+z)^{-3}} + \sqrt{\Omega_\Lambda (1+z)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}} . \tag{8.20}$$

Using Eqs. (3.8) and (3.38) as well as interpreting because of Eq. (8.1) the result (3.75 a) as vacuum energy density of the final state of the massive universe, by Eqs. (3.10) and (8.13), for $N(T) = (4/11)^{1/3}$ (see Eq. (3.73)), we can write

$$\rho_{\mathrm{vac}}(T)\, c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3} \frac{1}{N(T)} = \rho_{\mathrm{f1}} c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \left(\frac{d_{\mathrm{eff}}}{c t_{\mathrm{f2}}} \right)^3 \left(\frac{T_1}{T_2} \right)^3 =$$

$$= \frac{E_{\mathrm{d}}}{\frac{4}{3} \pi (c t_{\mathrm{f2}})^3} \left(\frac{3\, N(T)}{2} \right)^{3/4} \cong 4.56 \cdot 10^{-4}\ \mathrm{eV\, cm^{-3}} , \tag{8.21}$$

so that via the middle term of Eq. (8.14) we can estimate the corresponding cosmological "constant" to

$$\Lambda_\mathrm{f} = \frac{8\pi\, G_N}{c^2}\,\rho_\mathrm{f1} = 8\pi\,\frac{\hbar c}{E_\mathrm{Pl}^2}\,\frac{E_\mathrm{d}\{3\,N(T)/2\}^{3/4}}{\frac{4}{3}\pi\,(ct_\mathrm{f2})^3} = 1.519\cdot 10^{-59}\,\mathrm{m}^{-2}\,, \qquad (8.22)$$

where the time $t_\mathrm{f2} = 2.172\cdot 10^{20}\,\mathrm{s}$ is defined by Eq. (3.36).

Then, using again $N(T) = (4/11)^{1/3}$ and the connection (8.21), we can estimate the energy kT of the state $\rho_\mathrm{f1}\,c^2$ of the massive universe to

$$kT = \left[\frac{45}{\pi^2}\,(\hbar c)^3\,\rho_\mathrm{f1}\,c^2\,N(T)\right]^{1/4} = 5.811\cdot 10^{-5}\,\mathrm{eV}\,. \qquad (8.23)$$

The data (8.21) to (8.23) are also tabulated in Table IX.

Via the geometric means $(\Lambda_\mathrm{BB}\Lambda_\mathrm{f})^{1/2}$ and $(\Lambda_\Lambda\Lambda_\mathrm{f})^{1/2}$, we can once more simulate a continuous evolution of the total (massless and massive) universe by $(\Lambda_\mathrm{BB}\Lambda_\mathrm{f})^{1/2}/(\Lambda_\Lambda\Lambda_\mathrm{f})^{1/2} = (\Lambda_\mathrm{BB}/\Lambda_\Lambda)^{1/2} = 1.137\cdot 10^{122}$ as a consequence of the geometric mean $\left\{\left[(\Lambda_\mathrm{BB}\Lambda_\mathrm{f})/\Lambda_\mathrm{Pl}^2\right]\times\left[\Lambda_\mathrm{Pl}^2/(\Lambda_\Lambda\,\Lambda_\mathrm{f})\right]\right\}^{1/2} = (\Lambda_\mathrm{BB}/\Lambda_\Lambda)^{1/2} \cong 10^{122}$, so that we obtain again an excellent agreement with the quantum field theory (see above).

We mention still that because of the result (2.59) for the magnetic monopoles their rest energy $E_0(\mathrm{M}) = 6.849\cdot 10^{17}\,\mathrm{GeV}$ (see Eq. (2.58)) must be correct. However, their assumed statistical quantity $N(T) = 1/2\Omega_\gamma$ [1-3] is only correct for the estimation of their relativistic maximum energy E_Pl. Its real value must lie in the range $1/2\Omega_\gamma > N(T) > 160.75$. Because this "unknown" value $N(T)$ is necessary for corresponding calculations, we have not considered the magnetic monopoles for Tables VIII and IX.

With that, we have solved the problem of the time-dependent vacuum energy densities or cosmological "constants" on the basis of our quantum gravity [1, 2].

Thus, we have shown that the energy density (8.1) of the dark energy and the cosmological "constant" (8.2), which describe the present accelerated expansion of the universe, can be identified with the vacuum energy density or cosmological "constant" of the quantum field theory, so that we have confirmed the derivation of the quantum gravity [1, 2] by the gravitation via the particle-antiparticle pairs of the quantum vacuum [1, 2].

Besides, this solution corroborates the correctness of the estimation of the parameters of the big bang [1, 2], the new inflation model [1-5], the (light [1-5], heavy [1-4] and sterile [1-4]) neutrinos as well as the SUSY GUT particles (X and Y gauge bosons [1-5] as well as magnetic monopoles [1-5]). Consequently, the conception of the SUSY GUT is now well established within the theories of the early universe.

Using the data of Tables III to V, the results, derived in this Sec. 8 for the massless and massive universe, are also valid for the massless and massive anti-universe.

9 Summary

We have derived the transition from the final state of the universe and anti-universe to the big bang (origin) via the complete sterile (anti)neutrino decay and the quantum gravity. With that, we have solved precisely and uniquely this fundamental problem, whereat we have confirmed the explanation that the present dark matter and dark energy can be attributed to the invisible decay and breakup products of the sterile neutrinos. We have proved that the massless universe and anti-universe exist by zero-point oscillations. Finally, we have also shown that the massive universe and anti-universe can be explained reasonably by zero-point oscillations.

By aid of the time-dependent vacuum energy densities or cosmological "constants", we confirm the predictions of the quantum field theory.

The final age of the universe and the anti-universe was confirmed to $t_{f2} = 6883\,\mathrm{Gyr}$, derived in Refs. [1, 2]. The lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos [1, 2] was also confirmed to $\tau_{\hat{\nu}} = 35.11\,\mathrm{Gyr}$ [1, 2]. The rest energy of the photons was estimated to $E_0(\gamma) \cong 1.563 \cdot 10^{-35}\,\mathrm{eV}$. This estimation is confirmed by the measured general galactic magnetic field. It was assumed that the rest energy of the gravitons has probably the same value as at the photons.

In the framework of the ΛCDM model, we determine various values of the Hubble "constant" in the evolution of the universes as a function of the present CMB Hubble constant $H_0 = 67.3 \pm 1.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ so that we can determine the present Hubble constant of the accelerated expansion to "$H_{\mathrm{acc},0}$"= $74.0^{+2.0}_{-2.6}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ in excellent agreement with its most recent observed value $74.03\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ by Riess et al. in 2019, i.e. we need no new physical assumptions. However, the accelerated expansion began with a large Hubble constant $H_{\mathrm{acc}} = 95.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, so that the accelerated expansion is decelerated. In future, this result is in accordance with a slow linear expansion, which has the small Hubble constant $H_{\mathrm{lin}} = 0.843\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

For the big bang, we have confirmed the distance $R_{\mathrm{BB}} = 2.069 \cdot 10^{-98}\ \mathrm{m}$ and the time $t_{\mathrm{BB}} = 6.901 \cdot 10^{-107}\ \mathrm{s}$ of Refs. [1, 2]. The vacuum energy density or the cosmological "constant" of the big bang were also confirmed to $\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = 4.227 \cdot 10^{247}\ \mathrm{eV\,cm^{-3}}$ and $\Lambda = 1.406 \cdot 10^{192}\ \mathrm{m^{-2}}$ (see Refs. [1, 2]). The very high temperature $\widetilde{T}_{\mathrm{BB}} = \widetilde{E}_{\mathrm{BB}}/k \cong 5.52 \cdot 10^{94}\ \mathrm{K}$ was estimated for the hot big bang. Therefore, for the direct investigation of the big bang, ultrahigh-energy accelerator experiments, which under terrestrial conditions also in the near future have not the necessary energies, are utopian. Thus, for example, they should be stopped in favour of the neutrino physics.

Using the Friedmann equation as well as the known properties of particles and antiparticles, we have found a time reversal solution for the anti-universe $(-t_{\mathrm{f2}} \leftarrow -t \leftarrow -t_{\mathrm{BB}} \leftarrow 0)$ and the universe $(0 \rightarrow t_{\mathrm{BB}} \rightarrow t \rightarrow t_{\mathrm{f2}}.)$, since they expand in the opposite time direction by scale factors greater than zero and at velocities with opposite sign (see Table IV). The beginning of the anti-universe (antimatter) and the universe (matter) is a result of two equivalent energy uncertainties by one quantum fluctuation of the vacuum according to the uncertainty relation in form of $\widetilde{E}_{\mathrm{BB}} = -\hbar/2(-t_{\mathrm{BB}})$ and $\widetilde{E}_{\mathrm{BB}} = \hbar/2t_{\mathrm{BB}}$, respectively. The total energy $E_{\mathrm{BB}} = 2\widetilde{E}_{\mathrm{BB}} = \hbar/t_{\mathrm{BB}}$, which must yield the vacuum, is used for the excitation of the zero-point oscillations. At $t = 0$ (origin), these two energy uncertainties disappear by annihilation.

. With that, we have simply solved the fundamental problem of the separation of antimatter and matter because the unknown anti-universe must be existent in the past, whereas the known universe exists in the future. Besides, we have shown that the existence of the anti-universe and the universe is determined by an eternal cyclic cosmic evolution, for which is responsible the transition from their final state in the direction to the big bang.

. If we go in direction to the end of the universes, we have shown that we have an enormous increase of the energy density and the temperature because of the relationship "final state of the universe and big bang". To this day, in these processes, at the dark energy, the complete sterile neutrino decay was not considered, so that this work yields a strongly improved picture of the universe and the corresponding anti-universe in contrast to the hitherto existing big bang models.

In these hitherto existing big bang models [6-12], where the physics of the early universe ($t \leq 10^{-6}\mathrm{s}^{-1}$) is still very uncertain, their theories are not well established [6]. They predict, e.g., for the elusive dark matter, the existence of many exotic particles [6, 8, 11], for which however there is no experimental evidence [6].

In contrast to these theories, by aid of time-dependent vacuum energy densities or cosmological "constants" [1, 2], this work yields new contributions for the picture of the universe and the corresponding anti-universe by the derivation of the parameters of the big bang (as a result of the complete sterile (anti)neutrino decay) via the results of the quantum gravity [1, 2], which was derived by the gravitation via the particle-antiparticle pairs of the quantum vacuum, using the new thermal equilibrium between photons and particles [1-3] as well as the new inflation model [1-3].

By these new contributions, the SUSY GUT transition is now well established at the theories of the early universe. These new arguments are based on (convincing) direct experimental observations as the 3-neutrino oscillation parameters (see, e.g., Ref. [10]), the Planck 2013 results (see, e.g., Refs. [7, 12]), "the sterile neutrino decay" (see, e.g., Refs. [13, 22-26]), "the Pioneer anomaly" (see, e.g., Refs. [14, 15]), the general galactic magnetic field [9, 17],"the astronomical unit changing" (see Ref. [19]) and the present Hubble "constant" [20] as well as "well" established theories as the neutrino statistics (see, e.g., Refs. [6-11]), "the magnetic neutrino moments of the standard

$SU(2)\times U(1)$ model" (see, e.g., Refs. [10, 18]), the hot big bang model (see, e.g., Refs. [6-12]) and "the SUSY GUT transition" (see, e.g., Ref. [7]), used in Refs. [1-5], which are the fundamental basis of this work.

The "dark energy" was not introduced a priori in the ΛCDM model. This dark energy is based on the convincing experimental observation of the expansion of the universe by Hubble because by this discovery Einstein' cosmological constant Λ, introduced by him to get a static universe, has obtained a Renaissance via the quantum field theory as vacuum energy density (negative pressure), which in the works [1, 2] was thus connected with the cosmological "constant" and the dark energy as variable quantities. This work supports once more this hypothesis [1, 2] by the transition to a time-dependent vacuum energy density or cosmological "constant" in the eternal cyclic evolution of the total (massless and massive) universe and anti-universe. This assumption is confirmed by discovery of the present accelerated cosmic expansion [7], defined as the corresponding "present dark energy" [1, 2].

10 Acknowledgment

We acknowledge Peter Ressel, who provided several useful comments in the discussion.

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