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ANALYTIC DISTORTED – WAVE APPROXIMATION FOR NUCLEONS QUASI – ELASTIC SCATTERING CALCULATIONS

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Abstract

On the basis of non-relativistic theory in the distorted-wave approximation in three dimensions, a theory of quasi-elastic knock-out of nucleons from nuclei by nucleons has been developed. On the basis of this theory the differential cross sections of quasi-elastic knock-out reactions of protons $1P$ and $1S$ in the with shells nucleus ^{16}O and ^{12}C have been calculated, which allows to determine the orbital moments of the nucleons in the nucleus before scattering by the angular distribution of emitted protons.

Keywords: Quasi-elastic nucleon scattering.

Excitation nucleus. Emitted protons

1 Introduction

The reactions of knocking out nucleons from nuclei by protons, sometimes called quasi-elastic scattering, in the region of intermediate initial energies (*150-1000MeV*), the experiments in which it was studied, confirmed the correctness of the model of direct knocking out of a particle from the nucleus and the shell structure of nuclei.

When using protons with an energy of $\sim 1GeV$ as test particles, the difficulties in interpreting experimental data are much less than when using particles of lower energies. Despite the qualitative progress in understanding quasi-elastic processes, unresolved problems still remain. First of all, there is no quantitative description of the experimental differential cross sections ([1], [2]).

It is convenient to begin a qualitative description of knockout reactions with an idealized single-particle approximation in which the inelastic interaction of nucleons with nuclei can be represented as a sequence of independent intranuclear nucleon-nucleon collisions. When calculating the transition matrix element, the wave function of the nucleus is usually selected from the shell or cluster models of the nucleus.

The experimental data obtained in the study of knockout reactions undoubtedly contain information about the structure of the nucleus; however, the complexity of the interpretation of the reaction mechanism makes it difficult to extract it [3].

The peculiarity of the reaction of knocking out protons and neutrons under the action of nuclei of intermediate energies is that this process can occur mainly with the most weakly bound nucleons that are on the "surface" of the nucleus. Therefore, such a process is called direct or surface interaction ([4],[5]).

In the theory of direct interaction of a falling nucleon with a nucleus, it is assumed that some of the nucleons interact with the nucleus in such a way that it transfers a significant part of its energy to one (or several) nucleons that are emitted before the energy has time to be distributed over other degrees of freedom associated with a change in the state of many nucleons.

2 The first part of the proposed theory

The aim of this work is to obtain a general rigorous expression for the amplitude of quasi-elastic scattering of intermediate-energy nucleons on nuclei from a single position based on three-dimensional quasi-classics in the framework of the high-energy approximation.

Let us consider the derivation of a formula that determines the angular distribution of protons in the reaction of the surface interaction of incident neutrons on the target nucleus.

We write the differential cross section of the reaction A(n, np)B in the following form [6]:

$$d\sigma_{n,np} = (2\pi)^4 \frac{m}{k} d\mathbf{p}_f d\mathbf{p}_p \delta(E_i - E_f - E_p - E_N - E_R) \frac{1}{2J_i+1} \sum_{\sigma_i \sigma_f} \sum_{M_i M_f} \left| T_{if} \right|^2 \quad (1)$$

Here, $\mathbf{p}_f$, $\mathbf{p}_p$ the three-dimensional moment of scattered neutrons and knocked-out protons, and E_i, E_f are the kinetic energies of incident and scattered neutrons. The energy of knocked-out protons is E_p, and the detachment energy of the least bound proton is - E_N. Finally, the recoil energy of the daughter nucleus is $E_R = \frac{P_R^2}{2M_{A-1}}$, which is determined using the recoil nucleus momentum ($\mathbf{P}_R$) and in turn, is associated with the missing mass (M_R) from the reaction based on the law of conservation of energy:

$$M_R = [(M_A - m_p + E_i - E_f - E_p)^2 - P_R^2]^{1/2} \quad (2)$$

For the missing mass, the connection between the nucleon separation energy (E_N) and the mass of the daughter nucleus (M_{A-1}), which is known from the experiment, is used.

$$E_N = M_R - M_{A-1} \quad (3)$$

Consider the derivation of the formula that determines the angular distribution of knocked-out protons in the reaction of the surface interaction of the incident neutron on the nucleus - A. For this case, we write the wave function of the initial state of the system $n + A$ in the following form:

$$\psi_{A-1}(\mathbf{r})\psi_{nl}(r_p)Y_{lm}(\theta_p, \varphi_p)\psi_{\mathbf{k}_i}(\mathbf{r}'), \quad (4)$$

where r_p, θ_p, φ_p - are the polar coordinates of the proton in the nucleus - A, $\mathbf{r}$ -, are the coordinates of all other nucleons in the nucleus; $\mathbf{r}'$ and $\mathbf{k}_i$ - respectively, the coordinate and the wave vector of the incident neutron. After the proton emits a nucleus, the final state corresponds to a nucleus $A-1$, whose wave function with a large distance of nucleons from the nucleus has the form:

$$\psi_{A-1}(\mathbf{r})\psi_{\mathbf{k}_f}(\mathbf{r}')\psi_{\mathbf{k}_p}(\mathbf{r}_p), \qquad (5)$$

where $\mathbf{k}_f$ and $\mathbf{r}'$ - are the wave vector and coordinate of the scattered neutron, respectively, and $\mathbf{k}_p$ - is the wave vector of the knocked-out proton.

The potential responsible for the direct interaction $v(|r' - r_p|)$ – represents the interaction of the incident neutron with the proton of the nucleus - A, a $U(|\mathbf{r}_p - \mathbf{r}|)$ is the potential of the interaction of the emitted proton with the nucleus of the residue, which is nonzero only on the "surface" of the nucleus, that is, under the condition $r_n = r_p = R$.

The matrix element of the transition of the nucleus is represented as ([8], [11]).

$$T_{if} = < f \left| \int d\mathbf{r}' d\mathbf{r}_p \Pi(r') \psi^{(+)}_{\mathbf{k}_i}(\mathbf{r}') M(r_p) \right| i >, \ (6)$$

here

$$\Pi(\boldsymbol{r}') = \psi^{(-)*}_{k_f}(\boldsymbol{r}')\psi^{*}_{k_p}(r_p) v(|\boldsymbol{r}' - r_p|)$$

and

$$M(\boldsymbol{r_p}) = \psi_{nl}(\boldsymbol{r_p}) Y_{lm} W(\boldsymbol{r_p}) \delta(|\,\boldsymbol{r_p} - \boldsymbol{R}|)$$

where

$$W(\mathbf{r}_p) = \int \psi^{*}_{A-1}(\mathbf{r}) U(|\mathbf{r}_p - \mathbf{r}|) \psi_{A-1}(\mathbf{r}) d\mathbf{r} \qquad (7)$$

The wave functions of scattered nucleons obtained in [7] from the solution of the non-relativistic Schrödinger equation are written in the following form

$$\psi^{(-)*}_{\mathbf{k}_f}(\mathbf{r}')\psi^{(+)}_{\mathbf{k}_i}(\mathbf{r}') = \ell^{i[\mathbf{q}\mathbf{r}' + \phi(\mathbf{r}', \mathbf{q})]}, \qquad (8)$$

here $\phi(\mathbf{r}', \mathbf{q})$ it is a distorting function of scattered nucleons and depends on the distribution of nucleon density in the nucleus.

Expressing now in (6) the potential of two-nucleon interaction through the amplitudes of nucleon-nucleon scattering -

$$f_{NN}(\mathbf{q}') = -\frac{m_N}{4\pi\hbar^2} \int \ell^{i\mathbf{q}'\mathbf{r}'} \upsilon\ (|\mathbf{r}' - \mathbf{r}_\mathbf{N}|) d\mathbf{r}', \ (9)$$

using the Fourier transform, for the matrix element of the transition of the nucleus we get

$$T_{if} = \frac{-\hbar^2}{2\pi^2 m_p} \langle f | \int H_1 f_{NN}(q')\, M(\boldsymbol{r_p}) d\boldsymbol{r}' d\boldsymbol{q}' d\boldsymbol{r_p} | i \rangle \qquad (10)$$

here

$$H_1 = e^{i(qr'+\Phi)}e^{-ik_p r_p}e^{-iq(r'-r_p)}.$$

In order to simplify the calculation, the wave function of the knocked-out proton is taken in the form of a plane wave.

To calculate the matrix element (10), replacing the variables in the phase $\mathbf{u} = \mathbf{r}' - \mathbf{r}_p$ after integration we obtain:

$$T_{if} = \frac{-4\pi\hbar^2}{m_p}\langle f|\int H_2 W(\boldsymbol{r_p})\delta(|\boldsymbol{r_p} - \boldsymbol{R}|)d\boldsymbol{r_p}|i\rangle \quad (11)$$

here

$$H_2 = e^{i[qr_p+\Phi(r_p)-ik_p r_p]}\psi_{nl}(\boldsymbol{r_p})Y_{lm}f_{NN}(\boldsymbol{q_{eff}}).$$

Now, we choose the coordinate system in which $Oz \uparrow\uparrow \mathbf{q}$. We denote $\cos(\hat{\mathbf{q}}\,\hat{\mathbf{r}}) = \mu$ where $\mathbf{r} = \{r\mu\varphi\}$. This allows taking into account the energy loss, the momentum transmitted by the incident nucleon to the target nucleus can be written as:

$$|\mathbf{q}| = |\mathbf{k}_i - \mathbf{k}_f| = \sqrt{k_i^2 + k_f^2 - 2k_i k_f \cos\vartheta} =$$
$$= \left(\frac{2m}{\hbar^2}\right)^{1/2}\sqrt{E_i + E_f - 2E_i^{1/2}E_f^{1/2}\cos\vartheta} \quad (12)$$

The scattering angle $\vartheta = \vartheta_1 + \vartheta_2$ and the deflection angles of the incident (ϑ_1) and scattered particles relative to the axis OX and $\vartheta_3 = (\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j)$ the scattering angle of the knocked out particle in a three-dimensional coordinate system are related as follows :

$$\cos(\hat{\mathbf{r}\mathbf{k}}_i) = \mu\sin\vartheta_1 + \cos\vartheta_1\sqrt{1-\mu^2}\cos\varphi$$
$$\cos(\hat{\mathbf{r}\mathbf{k}}_f) = -\mu\sin\vartheta_2 + \cos\vartheta_2\sqrt{1-\mu^2}\cos\varphi \quad (13)$$
$$\cos(\hat{\mathbf{r}\mathbf{k}}_j) = -\mu\sin(\vartheta_3 - \vartheta_1) + \cos(\vartheta_3 - \vartheta_1)\sqrt{1-\mu^2}\cos\varphi$$
$$tg\vartheta_1 = \frac{E_i^{1/2}}{E_f^{1/2}}\frac{1}{\sin\vartheta} - ctg\vartheta$$

We immediately note that the angles ϑ and ϑ_3 are determined from the experiment.

Now, to calculate the integral expression (10), we apply the recurrence formula obtained in [7]. In this case, the amplitude of the free nucleon – nucleon interaction is selected in the following parameterized form [9]:

$$f_{n,p}(q') = \frac{ik\sigma_{p,n}}{4\pi}(1 - i\varepsilon_{p,n})\ell^{-\frac{\beta_0^2 q'^2}{2}} \quad (14)$$

Immediately, we note that in the calculations for the parameters of free - interaction, the following values are taken in expression (14).

$$\sigma_p = 4.75\ \text{фм}^2,\quad \varepsilon_p = -0.05,\quad \beta_0^2 = 0.21\ \text{фм}^2;$$
$$\sigma_n = 4.04\ \text{фм}^2,\quad \varepsilon_n = -0.5; \quad (15)$$

After that, the amplitude of the process, simplifying, takes the following form:

$$T_{if} = \frac{-\hbar^2 k\sigma_{NN}}{2m_p}(i+\varepsilon_0)\ell^{\frac{-\beta_0^2 q^2}{2}} W(R)\sum_{n=0}^{3} a_n \frac{\partial^n I(q)}{\partial q^n} \quad (16)$$

where
$$I(q) = \int \ell^{i(\mathbf{q}-\mathbf{k}_p)\mathbf{R}} \psi_{nl}(R) Y_{lm}(\theta_p, \varphi_p) d\Omega \quad (17)$$

$$W(R) = \langle f | \int \psi^*_{A-1}(\mathbf{r})\ U(|\mathbf{R}-\mathbf{r}|) \psi_{A-1}(\mathbf{r}) d\mathbf{r} | i \rangle \quad (18)$$

An explicit expression of the potential at the center of the nucleus is $U(0)$, functions $a_n(q)$ and distorting parameters a, b that depend on the distribution of nucleon density in nuclei are given in [10].

When calculating the integral (17), we use the expansion of a plane wave in spherical functions [11]:

$$\ell^{i(\mathbf{q}-\mathbf{k_p})\mathbf{R}} = \sum_{L=0}^{\infty} i^L \sqrt{4\pi(2L+1)}\ j_L(|\mathbf{q}-\mathbf{k}_p|R) Y^*_{L0}(\theta), \quad (19)$$

after taking into account the orthogonality condition in (17), we obtain

$$I(q) = \sum_{L=0}^{\infty} i^L \sqrt{4\pi(2L+1)}\ j_L(|\mathbf{q}-\mathbf{k}_p|R) \psi_{nL}(R), \quad (20)$$

here $j_L(|\mathbf{q}-\mathbf{k}_p|R)$ is the Bessel function and

$$|\mathbf{q}-\mathbf{k}_p| = \sqrt{q^2 - 2|\mathbf{q}||\mathbf{k}_p|\cos(\tfrac{\pi}{2} + \vartheta_3 - \vartheta_1) + k_p^2}. \quad (21)$$

It is known that experiments on the angular distribution of the products of the reaction of direct interaction make it possible to judge the properties of the energy levels of nuclei. Therefore, expression (20) allows determining from the angular distribution – *L*, of the emitted protons.

Finally, we proceed to the calculation of the integral (7). Moreover, in the interaction potential of a knocked-out proton with nucleons of the residual nucleus, the two-particle interaction potential is taken into account in an asymptotic form due to single-peon exchange, which allows us to write:

$$W(R) = -\frac{4\pi\ \hbar^2}{m}\gamma \int \frac{\ell^{-k_0|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|} \rho(\mathbf{r}) d\mathbf{r}, \quad (22)$$

here $\gamma = g^2/\hbar c = 0.081 \pm 0.002$ the coupling constant, which determines the potential obtained by analyzing scattering π- mesons at nucleons $k_0 = \frac{m_\pi c}{\hbar}$, whose inverse value corresponds to the radius of action of nuclear forces.

To calculate the integral (22), the density distribution of nucleons in the ground state of the residual nucleus is selected in the form of a Fermi - function:

$$\rho_N(r) = \rho_0(1 + \ell^{\frac{r-R}{d}})^{-1} \quad (23)$$

Moreover, using the pole method [7] of calculating the integral (22), we have

$$W = W_0(d)\ell^{-k_0 R}[\cos(dk_0) - i\sin(dk_0)] \quad (24)$$

where
$$W_0(d) = i(2\pi)^3 \frac{8\hbar^2\gamma}{m} dR\rho_0\{1 + \frac{3i}{2}\frac{\pi d}{R} - \frac{1}{2}(\frac{\pi d}{R})^2\} (25)$$

We examined the knocking out of protons from nuclei by scattering of nucleons, i.e. reaction (n,np) type, however, the results can be directly applied to reactions (p,np), $(p,2p)$ and $(n,2n)$ as well because the Coulomb interaction is not taken into account. Since direct interaction is significant at medium and higher nucleon energies, the effect of the Coulomb interaction on the angular distribution is not significant.

Thus, the final expression of the differential cross section for quasi-elastic scattering of nucleons on nuclei can be writ

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_f} = \aleph_0(q)\sum_{L=0}^{\infty} i^{2L}(2L+1)\left|\psi_{n'L}(R)\sum_{n=0}^{3} a_n(q)\frac{\partial^n j_L(|\mathbf{q}-\mathbf{k}_p|)}{\partial q^n}\right|^2 \quad (26)$$

where

$$\mathcal{N}_0(q) = (2\pi)^5\hbar^4 m_p^2 \sigma_{NN}^2 (1+\varepsilon_0^2) e^{-\beta_0^2 q^2}\overline{W} \quad (27)$$

here

$$\overline{W} = |W(R)|^2 E_f^{1/2} (E_i - E_f - E_N - E_R)^{1/2}$$

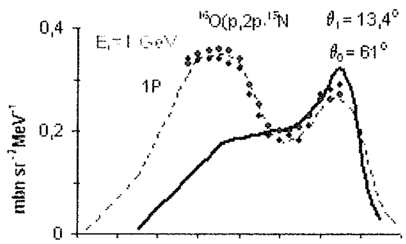
Since in the future these reactions will be studied on light nuclei, it is desirable to give explicit expressions of the radial wave functions of nucleons for the $\psi_{n'L}(R)$ для $1s$ and $1p$ states in the nuclei obtained from the solution of the non-relativistic Schrödinger equation for a spherically symmetric potential [12]:

$$\psi_{1s}(R) = 2\frac{R}{a_0}\pi^{-\frac{1}{4}} exp(-\frac{R^2}{2a_0^2}) \quad (28)$$

$$\psi_{1p}(R) = \left(\frac{8}{3}\right)^{\frac{1}{2}} \frac{R^2}{a_0^2}\pi^{-\frac{1}{4}} exp(-\frac{R^2}{2a_0^2}) \quad (29)$$

3 The results of the first part of theoretical calculations

The proposed approach allows one to calculate the differential cross section by knocking out nucleons with protons with energy $E_i = 1GeV$, using a variation of the parameter characterizing the thickness of the surface layer (*d*) of the nucleus.



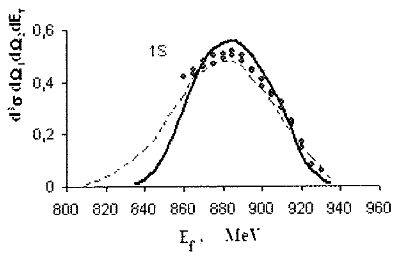


Fig.1: Experimental (dots) and theoretical (solid line) differential cross sections for quasi-elastic proton knockout reactions from sub shells $1p$ and $1s$ nucleus ^{16}O at angles $\vartheta_1 = 13,4^\circ$, $\vartheta_0 = 61^\circ$. The dashed curves are the results of [12] calculated with Hartree – Fock wave functions taking into account the excitation of the nucleus.

The results of specific calculations of the reactions A(p, 2p)B on nucleus ^{16}O and ^{12}C in comparison with experimental data are shown in Fig. 1 and 2.

The calculations were mainly performed for different angles of release of slow protons at a fixed scattering angle ($\vartheta_3 = 61^\circ, 64^\circ, 67^\circ, 73^\circ$) of fast protons $\vartheta = 13,4^\circ$. The figures show the results only for the emission angle $\vartheta_3 = 61^\circ$ of slow protons [13]. An analysis of the results shows that the scattering cross section weakly depends on the angles of the knocked out protons.

Nuclei ^{16}O and ^{12}C can emit protons from levels $1p, 1s$ therefore the differential cross section is calculated for each of these cases. In the experiment, slow protons were recorded at energies $E_p = (60 \div 105) MeV$ (the theoretical cross section coincides with those measured by the registration of fast protons at precisely these energies of slow protons).

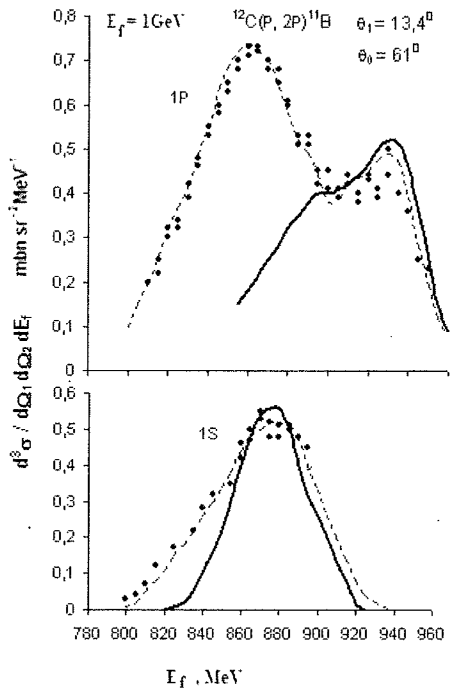


Fig.2: The same as in fig.1, but for the nucleus $^{12}_{6}C$.

As can be seen from Fig. 1 and 2, the theoretical curves obtained in this work for knocking out protons from the level $1p$, unlike the experiment, have only one maximum. Apparently, this is a consequence of the fact that the process considers absolute quasi-elastic proton knockout, that is, the residual nucleus is not excited. However, the shift of the maximum toward higher energies is due to the fact that distortion is not taken into account in the wave function of the knocked-out proton.

For comparison, the figures also show theoretical curves calculated in the distorted-wave pulse approximation, where the Hartree - Fock wave functions were used for nuclear nucleons. Moreover, the authors of [12] also took into account the excitation of the residual nucleus.

4 Analysis of the results

Based on the above-developed theory, the calculated differential cross sections for quasielastic proton knockout reactions from subshells $1p$ and $1s$ in nuclei ^{16}O and ^{12}C make it possible to determine the orbital moments of these nucleons in nuclei before scattering from the angular distribution of emitted protons.

Thus, we can come to the conclusion that such calculations are convenient for practical use with analytical wave functions of the nucleus.

From an analysis of the results obtained, it follows that the knockout reaction requires a more rigorous consideration of intranuclear wave functions, in particular, a more accurate consideration of the excitation of the residual nucleus, i.e. single-particle [14] and collective ([15],[16]) also distortions in the wave functions of knocked out nucleons from nuclei.

5 The second part of the proposed theory

Knocking out nucleons from nuclei taking into account excitation of the residual nucleus

Nucleons of medium and higher initial energies participating in nuclear reactions have a wavelength shorter than the size of the nucleus, which allows us to consider their motion quasiclassically and talk about the "location" of the nucleon entering the nucleus, as well as the "trajectory" of its motion inside the nucleus, which makes it convenient accounting for changes in the phase of the incident and outgoing waves.
In addition, for these incident nucleons the "transparency" of the nucleus increases and it becomes possible to interact with the transfer of energy to one nucleon sufficient to remove it outside the nucleus, not only with "surface" loosely coupled nucleons, but also with "deep" nucleons. This leads to a rearrangement of nucleons at energy levels on the surface of the target nucleus, i.e. excitation of the nucleus.

Considering the above, we consider the reaction of quasielastic knocking of protons by nucleons from the nucleus, and with the transition of nucleons in the nucleus of the residue to the upper energy level.
We write down the wave functions of the system for the reaction. In this case, the wave function of the initial state has the form [11]:

$$\psi_{A-1}(\mathbf{r},r_2)\varphi_{jm_2}(\theta_2,\varphi_2)\psi_{nl}(r_3)\varphi_{lm_3}(\theta_3,\varphi_3)\chi_{\frac{1}{2}\mu}\psi_{\mathbf{k}_i}(\mathbf{r}') \quad (30)$$

where is the r_3 - coordinate, $\varphi_{lm_3}(\theta_3,\varphi_3)$ - is the spin is the angular function of the knocked out proton in the nucleus-A, r_2 - is the coordinate and $\varphi_{jm_2}(\theta_2,\varphi_2)$ - is the spin is the angular function of the underlying proton in this nucleus; $\mathbf{r}'$, $\mathbf{k}_i$ and $\chi_{\frac{1}{2}\mu}$ - , are the coordinate, wave vector, and spin function of the incident nucleon, respectively. The final state corresponds to the nucleus, which is obtained after filling in the nucleus A, with the underlying proton ($\mathbf{r}_2$) of the proton emitted by the nucleus. The wave function of this state takes the following form

$$\psi_{A-1}(\mathbf{r},r_2)\varphi_{j'm_2'}(\theta_2,\varphi_2)\psi_{\mathbf{k}_3}(\mathbf{r}_3)\chi_{\frac{1}{2}m_1'}\psi_{\mathbf{k}_f}(\mathbf{r}')\chi_{\frac{1}{2}\mu'} \quad (31)$$

where $\varphi_{j'm_2'}$ is the spin - angular function occupying the underlying proton in the nucleus - B, $\mathbf{k}_3,\mathbf{r}_3$ and $\chi_{\frac{1}{2}m_1'}$ - are the wave vectors, coordinates, and spin function of the knocked-out proton, respectively; $\mathbf{k}_f$, $\mathbf{r}'$ and $\chi_{\frac{1}{2}\mu''}$ - correspond to the scattered nucleon.

If there is only one nucleon outside the closed shells A and B, then the spin - angular functions that determine the angular dependence of the wave functions of the initial and final states of the nucleus will take the form :

$$\varphi_{im_3} = \sum_{m_3''} (l\,\tfrac{1}{2} m_3'', m_3 - m_3'' | i m_3) \mathrm{Y}_{l m_3''} \chi_{\frac{1}{2}, m_3 - m_3''} \quad (32)$$

The transition matrix element is now written

$$T_{if} = \sum_{\sigma_N \sigma_p} \langle f | \int \psi_{k_f}^{(-)*}(\mathbf{r}') \psi_{\mathbf{k}_3}^{*}(\mathbf{r}_3) \varphi_{j'm'}^{*}(\theta_2, \varphi_2) \chi_{\frac{1}{2} m_3'}^{*} \chi_{\frac{1}{2} \mu'}^{*} \upsilon(|\mathbf{r}' - \mathbf{r}_3|)$$

$$\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}') \varphi_{jm}(\theta_2, \varphi_2) \psi_{nl}(\mathbf{r}_3) \varphi_{im_3}(\theta_3, \varphi_3) \chi_{\frac{1}{2} \mu'} d\mathbf{r}_3 W(\mathbf{r}_3) \delta(|\mathbf{r}_3 - R|) | i \rangle,$$

here the summation is carried out over the spin variables of the scattered nucleon and the knocked-out proton, and

$$W(\mathbf{r}_3) = \int \psi_B^{*}(r_2, \mathbf{r}) U(|\mathbf{r}_3 - \mathbf{r}|) \psi_{A-1}(r_2, \mathbf{r}) d\mathbf{r} \quad (33)$$

After a similar conduct of all mathematical operations applied above, the matrix element, simplifying, takes the form

$$T_{if} = \frac{-\hbar^2 k \sigma_{NN}}{m} (i + \varepsilon_0) \ell^{-\beta_0^2 q^2} W(R) \sum_{\sigma_N \sigma_p} \sum_{n=0}^{3} a_n(q, \beta) \frac{\partial^n J(q)}{\partial q^n} \quad (34)$$

$$J(q) = S(J) \sum_{L=0}^{\infty} i^L \sqrt{4\pi(2L+1)} \; j_L(|\mathbf{q} - \mathbf{k}_3| R) \psi_{nL}(R) \quad (35)$$ here

$$S(J) = \sum_{M=-J}^{J} \int \varphi_{j'm'}^{*}(\theta_2, \varphi_2) \mathrm{Y}_{JM}(\theta) \varphi_{i\mu}(\theta_1, \varphi_1) \varphi_{jm}(\theta_2, \varphi_2) d\Omega$$

Now, a few words about the calculation of the integral expression- $S(J)$. Moreover, the product of two spherical functions of the same angular variables (θ_2, φ_2) is expressed through a linear combination of spherical functions of the same variables. The calculation of the sum of the magnetic quantum numbers of the products of the vector addition coefficients is carried out according to the Raka method [11].

So, the differential cross section for quasi-elastic knocking out of nucleons by single-particle excitation of the residual nucleus can be written as follows:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_f} = \aleph(q) S^2(J_1, J, J_2) \sum_{\sigma_i \sigma_f} \sum_{L=0}^{\infty} i^{2L} (2L+1) \times$$

$$\times \left| \psi_{n'L}(R) \sum_{n=0}^{3} a_n(q) \frac{\partial^n j_L(|\mathbf{q} - \mathbf{k}_3| R)}{\partial q^n} \right|^2, \quad (36)$$

where

$$S^2 = (2L_2 + 1)(2L_1 + 1)(2J_1 + 1) W_R^2(L_2 J_2 L_1; \tfrac{1}{2} J) \qquad (37)$$

The coefficient Raka - $W_R(L_2 J_2 L_1; \frac{1}{2} J)$, is given in [11].

The energy of a knocked out nucleon is determined using the following expression

$$E_p = E_i - E_f - E_N - E_R - E_{A-1}^{\bullet}, \quad (38)$$

which is determined through the recoil energy of the residual core

$$E_R = \frac{P_R^2}{2M_{A-1}} \qquad (39)$$

and the law of conservation of momentum

$$\mathbf{P}_R = \hbar\mathbf{k}_i - \hbar\mathbf{k}_f - \hbar\mathbf{k}_3 \qquad (40)$$

Here $E^{\bullet}_{A-1}$ is the energy of the excited states of the daughter nucleus and is determined by the formula:

$$E^{\bullet}_{A-1} = E_N - B_N, \qquad (41)$$

where B_N - is the separation energy of the least bound nucleon in the daughter nucleus.

As noted above, the wave function of the slow nucleon (in this case, the knocked-out proton) is determined from the solution of the non-relativistic Schrödinger equation for a spherically symmetric potential, taking into account the spin-orbit interaction. The radial part of these functions for the studied nuclei is given in (28) and (29). The Eigen values $E_N = E_{nlj}$ of this equation, for specific nuclei, is defined as

$$E_{nlj} = E_{nl} + \Delta_{(n)l}. \qquad (42)$$

Here $\Delta_{(n)l} = |U_{ls}|(2l+1)/2$ is the distance between the split levels, which is determined from the following well-known expression [17]:

$$\Delta_{(n)l} = -20\mathbf{ls}A^{-2/3}\,MeV, \qquad (43)$$

where

$\mathbf{ls} = \frac{1}{2}\{j(j+1) - l(l+1) - s(s+1)$ – takes values

$\frac{l}{2}$ for $j = l + \frac{1}{2}$ and $\frac{l(l+1)}{2}$ for $j = l - \frac{1}{2}$.

$$E_{nl} = \hbar\varpi(2n + l + 3/2) \qquad (44)$$

where

$$\hbar\varpi = \frac{\hbar^2}{ma_0^2}. \qquad (45)$$

During quasi-elastic knocking out of nucleons, the recoil nucleus has a hole in the shell from which the proton is emitted, and the separation energy is equal to the energy of this single-particle state. Since at the beginning the core A was at rest, the momentum of the nuclear nucleon $(\mathbf{q}_0)$ before the interaction is assumed to be equal in magnitude, but opposite in direction to the momentum of the recoil nucleus in the final state, so its value can be found from the momentum conservation law (40),

$$\mathbf{q}_0 = -\mathbf{P}_R = -\hbar\mathbf{k}_{A-1}, \qquad (46)$$

$$\mathbf{q}_0 = \mathbf{k}_i - \mathbf{k}_f - \mathbf{k}_3 = \mathbf{q} - \mathbf{k}_3, \qquad (47)$$

equal in magnitude to the momentum of the nuclear nucleon before the act of interaction.

Here, $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$ the momentum of the transfer of the scattered proton $\mathbf{k}_3$ – is the momentum of the knocked out nucleon.

It should be noted that when calculating the cross section for the sum over the spinors χ_i and χ_f - corresponding to the nucleon moment $\mathbf{k}_i$ and $\mathbf{k}_f$ before and after scattering, we obtain

$$\sum_{\sigma_i \sigma_f} \left| \chi_f^* \chi_i \right|^2 = \cos^2(\vartheta/2) \quad . \qquad (48)$$

However, taking into account the above, for the sum over the spins of the knocked-out protons we get unity. Therefore, for the factor in (40) we obtain

$$\aleph(q) = \aleph_0(q) \cos^2 \vartheta/2 \qquad (49)$$

In the residual nucleus, the proton transition occurs between states characterized by certain values of the angular momentum and parity. In this case, the selection rules arising from the law of conservation of moments and parity allow us to determine possible values L from the so-called triangle relation.

$$\left| |J_1 - J_2| - \frac{1}{2} \right| \le L \le J_1 + J_2 + \frac{1}{2}$$

$L_1 + L_2 + L$ - even number

6 Applying Theory to Individual Nucleus

The nucleus ^{16}O can emit protons from the levels $1P_{3/2}, 1P_{1/2}, 1S_{1/2}$ and the nucleus ^{12}C – from the $1P_{3/2}$ and $1S_{1/2}$ levels. Therefore, the differential cross sections calculated for each of these cases are shown in Fig.3 and 4, respectively. In addition, in these figures, for comparison, the theoretical curves calculated in [12] with the Hartree – Fock wave functions are given.

Thus, knowing the energies of the emitted nucleons and the angles of their escape, we can directly determine the energies and momentum distributions of single-particle states in specific shells of the nucleus.

The differential cross section for quasi-elastic scattering of protons with an energy of 1GeV was calculated for ^{16}O and ^{12}C. The analysis was mainly reduced to studying the shape of the correlation distributions and relative yields of protons from various nuclear shells.

The calculations were mainly performed for the emission angle $\theta_p = 61^0$ of slow protons at a fixed scattering angle $\theta_1 = 13.4^0$ of the fast proton. In this case, the oscillator parameter was "free" with the help of which a noticeable similarity of the theoretical differential cross section with the experimental ones was obtained.

As can be seen from Fig.3, the proton energy distribution for the levels $P_{1/2}$ and $P_{3/2}$, corresponds to two maxima in the energy range of *865MeV* and *925MeV*, which can be explained on the basis of the momentum distribution of protons at the corresponding single-particle level.

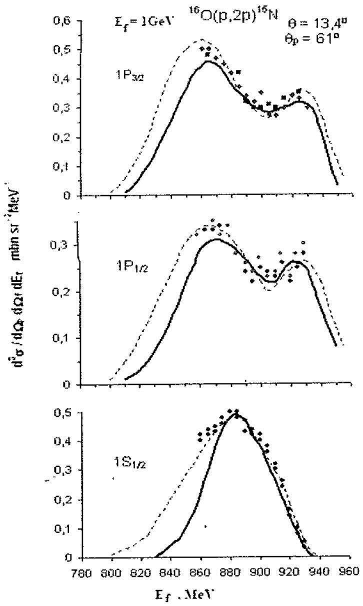


Fig.3: Differential cross sections for quasi-elastic proton knockout reactions from shells $1P_{3/2}$, $1P_{1/2}$ and $1S_{1/2}$ nuclei $^{16}_{8}O$. The points are experimental data [12], the solid line is the theoretical calculations, the dashed line is the theoretical results with the Hartree-Fock wave functions.

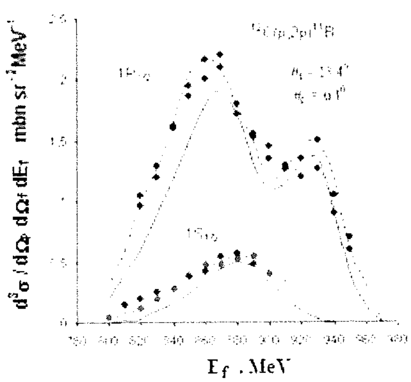


Fig.4: The same as in fig.3, but for the nucleus $^{12}_{6}C$.

The first maximum corresponds to the population of the ground state $(1/2-)$ and $(3/2-)$.The second maximum corresponds to the excited state in the core of the residue $^{15}_{7}N$.

This figure also shows a curve describing the knockout of a proton from a state with a maximum in the energy region of *875MeV.*

In the case of a nucleus (Fig. 3and 4), the theory reproduces well the relative course of differential cross sections upon transition from one nuclear shell to another ($1P_{3/2}$, $1S_{1/2}$).The experiment for this nucleus was performed in a wide range of kinematic allowable energies - E_p, therefore, it becomes possible to analyze in more detail the behavior of the correlation distributions of the emitted fast and slow protons.

As can be seen from the figures, the theoretical values of the cross sections at the maxima and minima are in good agreement with the experimental cross sections. This agreement is also achieved by correctly taking into account

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As can be seen from the figures, the theoretical values of the cross sections at the maxima and minima are in good agreement with the experimental cross sections. This agreement is also achieved by correctly taking into account the distortion in the wave functions of protons, which are manifested in a shift in the position of the minima to the region of high energies E_p.

Calculated and experimental energies
single-particle states in nuclei.

Nucleus	Conditin	a (Fm)	E_{nlj} (MeV)	E^*_{nlj} (MeV)	E_{nlj} (exp.) (MeV)
$^{16}_{8}O$	1P $_{3/2}$	2.27	18.6	15.98	19.6
	1P $_{1/2}$	2.55	12.8	10.74	13.0
	$1S_{1/2}$	1.28	38.1	33.19	39.2
$^{12}_{6}C$	1P $_{3/2}$	2.43	15.6	12.53	16.0
	$1S_{1/2}$	1.33	35.2	29.09	33.9

Table1

Table-1 shows the energies calculated for single-particle levels in the nuclei $^{16}_{8}O$ and $^{12}_{6}C$ for the corresponding values of the oscillator parameter, and the energy of the excited hole state. In addition, for comparison, the experimental values of the energy of single-particle levels are also given [18].

7 Conclusion

It should be noted that the obtained values of the energy of single-particle excitations of residual nuclei are somewhat overestimated. This is possibly due to the fact that the contributions from collective excitations were not taken into account in the residual nuclei.

Thus, we can conclude that a satisfactory description of the shape of the correlation distributions and relative proton yields, on the one hand, indicates that the method of accounting for wave distortions and the mathematical methods used in calculating the process amplitude are quite accurate. On the other hand, the agreement of single-particle energy spectra with both experimental data and cross-section curves calculated with the Hartree-Fock wave functions allows us to conclude that this theoretical mechanism describes well the *A(p,Np)B* reactions, as well as and core structures. Therefore, it can be used for the quantitative analysis of differential cross sections of quasi-elastic interaction of nucleons in a number of nuclei.

The effective *NN*- interaction, reconstructed from data on elastic scattering, allows one to obtain a qualitative description of quasi-free scattering and other inelastic processes. The results of a theoretical analysis of the data on quasi-elastic scattering of protons in the region of intermediate energies shows that the peripheral nature of the interaction of protons with nuclei at these energies makes this process a powerful tool for studying the surface modes of nuclear excitation.

Having made certain conclusions, it can be said that the experimental data obtained to date do not allow an exhaustive analysis and preference for one or another theoretical model of the interaction of intermediate-energy nucleons with nuclei.

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