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MULTIPLE PARTON SCATTERING AT HIGH ENERGY PROTON-PROTON COLLISIONS

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Abstract

Multiple parton scattering occurs when two or more independent identified hard particle from each proton takes place in the same proton-proton collision. The multiparton scattering mechanism got a new impulse at the LHC, where the high energy provides access to a very small value at the fractional momentum.Double parton scattering (DPS) and triple parton scattering (TPS), are expected to occur frequently in the proton-proton collision at the LHC and future high-energy colliders. This can give rise to a sizeable background to certain rare single parton scattering (SPS) and an interesting signal process in their own right.In this article, we discuss the total cross sections in both double and triple parton scattering mechanism for different process.

1. Introduction

The multiple parton scattering got a new impulse with the operation of the LHC, extended nature of hadrons and their growing parton densities probed at increasingly higher energies, make it possible to simultaneously produce multiple particles in independent multiparton interactions in $(pp, p\bar{p})$ collisions [1-3].

The interest of MPI has increases in the last years not only as a source of particle production at hadron colliders, but due to information on partonic behavior observed in pp collisions.

Recently, double parton scattering mechanism is consider to be as an important sourse of background in searches for new physics. In the process such as $pp \rightarrow HZ + X \rightarrow b\bar{b}Z + X$, one hard scatter can produce the Z and the other a continuum $b\bar{b}$ pair [1]. The same holds for $pp \rightarrow HW + X \rightarrow b\bar{b}W + X$, where the DPS contribution to the background has been studied in detail for Tevatron kinematics in [2]. A process with DPS contributions of prominent size is the like charge weak gauge boson W^+W^+or W^-W^- production at hadron colliders [4-9].

The associated production of a Higgs boson with a $t\bar{t}$ pair at the LHC, $pp \rightarrow t\bar{t}h$, will play a very important role in the Higgs boson mass range, both for discovery and for precision measurements of the Higgs boson couplings. This process will provide a direct measurement of the top quark Yukawa coupling which could help to distinguish a SM Higgs boson from more complex Higgs sector and shed light on the details of the generation of fermions masses [7-9].

The $b\bar{b}$ channel is the most favorite Higgs boson decay mode when the Higgs boson mass is below the W^+W^- threshold . The confidence in the capability of identifying efficiently the b quark jets has therefore addressed towards the detection of $b\bar{b}$ pairs to observe the Higgs boson production at the LHC.

To reduce the huge QCD background to the $b\bar{b}$ pair production, the $b\bar{b}$ pair is detected in association with an isolated lepton from the decay of a W boson. The process of interest to detect the Higgs boson production through the $b\bar{b}$ decay channel is therefore $pp \rightarrow WH + X, W \rightarrow l\nu$and $H \rightarrow b\bar{b}$, where l is e, μ.

The purpose of the present note is to point out that to a sizeable background to certain rare single parton scattering (SPS) and an interesting signal process in their own right, namely by a double parton mechanism (DPS) process and triple parton scattering mechanism (TPS). In fact as a result of the present analysis we find that double and triple parton scatterings may represent a rather sizeable source for some processes and background to other different processes.

2. Single Parton Scattering Mechanism

The Drell–Yan process occurs in high-energy pp scattering as shown in Fig. (1). It takes place when a quark of one proton and an antiquark of another proton annihilate, creating a virtual photon or Z boson which then decays into a pair of oppositely-charged leptons.

Importantly, the energy of the colliding quark-antiquark pair, can be almost entirely transformed into the mass of new particles [10].

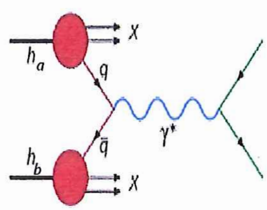

Figure 1 Drell–Yan process: a quark from one quark and an antiquark from another hadron annihilate to create a pair of leptons through the exchange of a virtual photon.

The distribution of the fractional momenta of quarks in hadrons is described by the structure functions, which were obtained from lepton-nucleon scattering experiment.

The total cross section for particle production in the Drell-Yan scattering, can be written in the form:

$$\sigma(p\bar{p} \rightarrow V + X) = \sum \int_0^1 d\, x_a dx_b \;\; G_{a/p}(x_a, Q^2) G_{a/\bar{p}}(x_b, Q^2)\, \hat{\sigma}_{q\bar{q}} \rightarrow V \;, \qquad (1)$$

Where $\hat{\sigma}_{q\bar{q}} \rightarrow V$ is the sub process cross-section and, V is the production of virtual particle.

3. Double Parton scattering Mechanism

The multiple parton scattering occurs when two or more different pairs of parton scattering independently in the same hadronic collision, as shown in Fig. (2).

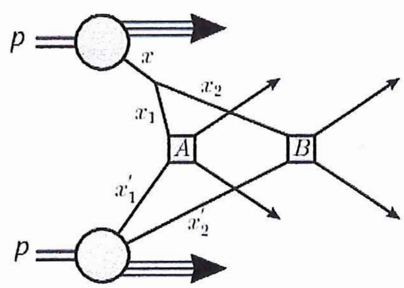

Figure 2 Mechanism od Double Parton Scattering in which two quarks from one hadron and two antiquarks from another hadron annihilate [11].

With the only assumption of factorization of the two hard parton processes A and B, the inclusive cross section of a double parton-scattering in a hadronic collision is expressed by [11-13]:

$$\sigma^{DPS}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{i,j} \;\; (x_1, x_2 : b) \hat{\sigma}^A_{ik} \; (x_1, x'_1) \; \hat{\sigma}^B_{jl}(x_2, x'_2) \; \Gamma_{k,l}(x'_1, x'_2 \,; b) \; dx_1 dx'_1 \; dx_2 dx'_2 \; d^2 b. \qquad (2)$$

Where $\Gamma_{i,j}(x_1, x_2 : b)$ are the double parton distribution functions which depends on x_1, x_2 the fractional momenta, and the relative transverse separation b of the two parton undergoing the hard processes A and B, the indices i and j refer to the different parton species and $\hat{\sigma}^A_{ik}$ and $\hat{\sigma}^B_{jl}$ are the partonic cross sections. The factor m is for symmetry, specifically $m = 1$ for indistinguishable parton processes and $m = 2$ for distinguishable processes.

The double distributions $\Gamma_{i,j}(x_1, x_2 : b)$ are the main reason of interest in multiparton collisions. This distribution contains in fact all the information of probing the hadron in two different points contemporarily through the hard processes A and B.

The cross section for multiparton process is sizable when the flux of parton is large, namely at small x. Given the large flux one may hence expect that correlations in momentum fraction will not be a major effect and partons to be rather correlated in transverse space. Neglecting the effect of parton correlations in x one writes the double distributions $\Gamma_{i,j}(x_1, x_2 : b)$:

$$\Gamma_{ij}(x_1, x_2; b) = \Gamma_i(x_1)\Gamma_j(x_2)F^i_j(b) \tag{3}$$

Where $\Gamma_{ij}(x_1)$ the usual one body parton distribution is function and $F^i_j(b)$ is a function normalized to one and representing the pair density in transverse space. The inclusive cross section hence simplifies

$$\sigma^{DPS}_{(A,B)} = \frac{m}{2}\sum_{ijkl}\Theta^{ij}_{kl}\hat{\sigma}_{ij}(A)\hat{\sigma}_{kl}(B), \tag{4}$$

Where $\hat{\sigma}_{ij}(A)$ and $\hat{\sigma}_{kl}(B)$ are the hadronic inclusive cross section for the two partons labeled i and j undergoes the hard interaction labeled A and for two partons k and l to undergo the hard interaction labeled B;

$$\Theta^{ij}_{kl} = \int d^2b F^i_k(b) F^j_l(b) \quad , \tag{5}$$

are the geometrical coefficients with dimension an inverse cross section and depending on various parton processes. These coefficients are the experimentally accessible quantities carrying the information of the parton correlation in transverse momentum. The cross section for multiple parton collisions has been further simplified as:

$$\sigma^D_{(A,B)} = \frac{m}{2}\frac{\hat{\sigma}(A)\hat{\sigma}(B)}{\sigma_{eff}} \tag{6}$$

Where all the information on the structure of the hadron in transverse space is summarized in the value of the scale factor. The experimental value measured by CDF yields [15] $\sigma_{eff} = 14.5 \pm 1.7^{+1.7}_{-2.3}\,mb$.

Fig. 4 shows effective cross sections measured by different experiments and energies.

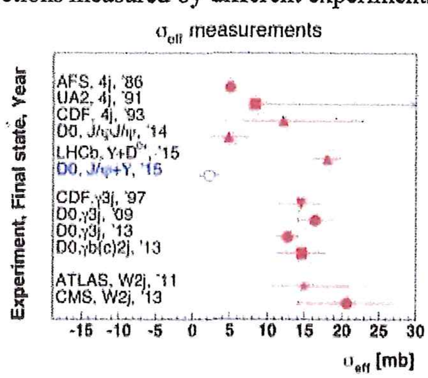

Figure 4 σ_{eff} measured by different experiments using different process. This figure was obtained from [15].

4. Triple Parton Scattering Mechanism

The triple parton scattering mechanism for $pp \to abc + X$ reaction is schematically illustrated in Fig. 3. The corresponding inclusive TPS cross section in a general form [16] can be written as follows:

$$\sigma^{TPS}_{pp\to abc} = \left(\frac{1}{3!}\right) \sum \int \Gamma^{qqq}_p(x_1, x_2, x_3;\ b_1, b_2, b_3) \times \hat{\sigma}^{qq}_a(x_1, x'_1)\, \hat{\sigma}^{qq}_b(x_2, x'_2)\, \hat{\sigma}^{qq}_c(x_3, x'_3) \times \Gamma^{qqq}_p(x'_1, x'_2, x'_3;\ b_1, b_2, b_3) \times dx_1\, dx_2\, dx_3\, dx'_1\, dx'_2\, dx'_3\, d^2b_1\, d^2b_2\, d^2b_3 \,. \quad (7)$$

where x_i, x'_i are the longitudinal momentum fractions, scales, $\hat{\sigma}^{qq}_a(x_i, x'_i)$ are the partonic cross sections for $qq \to a$ mechanism and $\frac{1}{3!}$ is the combinatorial factor relevant for the case of the three identical final states. The above TPS hadronic cross section is expressed in terms of the so-called triple-gluon distribution functions $\Gamma^{qqq}_p(x_1, x_2, x_3;\ b_1, b_2, b_3)$ that contain additional information about positions of the three corresponding partons in the transverse plane of the colliding protons.

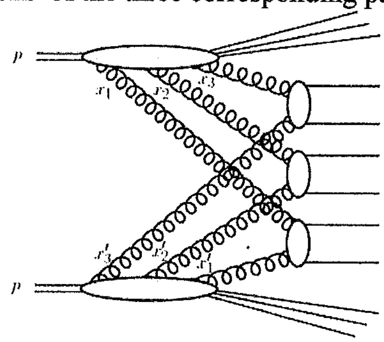

Figure.3. A diagrammatic illustration of the triple-parton scattering mechanism for triple production in proton–proton scattering. Only the dominant at high-energies [16].

Therefore, in practice one usually follows the so-called factorized Ansatz, where the correlations between partons are neglected and longitudinal and transverse degrees of freedom are separated.

Taking all together, the formula for inclusive TPS cross section can be simplified to the pocket form [16]:

$$\sigma^{TPS}_{pp\to abc} = \frac{1}{3!} \frac{\sigma^{SPS}_{pp\to a} \cdot \sigma^{SPS}_{pp\to b} \cdot \sigma^{SPS}_{pp\to c}}{\sigma^2_{eff,TPS}} \quad (8)$$

where the triple-parton scattering normalization factor $\sigma^2_{eff,TPS}$ contains only information about proton transverse profile and can be related to the overlap function from via the following expression:

$$\sigma^2_{eff,TPS} = \left[\int d^2b\ T^3(b)\right]^{-1} \quad (9)$$

The normalization factor $\sigma^2_{eff,TPS}$ contains all unknowns about the TPS dynamics. Its pure geometrical interpretation comes from the practical approximations of the factorized Ansatz mentioned above.

5. n-Parton Scattering Mechanism

In a hadronic collision, the inclusive cross section to produce n hard particles in n independent parton scattering, $hh' \rightarrow a_1 \ldots . a_n,$ can be written as of a generalized n-parton distribution functions and elementary partonic cross sections over all involved partons [17]:

$$\sigma^{NPS}_{hh' \rightarrow a_1 \ldots\ldots a_n} = \frac{m}{n!} \sum_{i_1,\ldots i_n, i'_{1\ldots} i'_n} \int \Gamma_h^{i_1 \ldots i_n}(x_1, \ldots\ldots x_n; b_1 \ldots\ldots b_n) \times \hat{\sigma}_{a_1}^{i_1 i'_1}(x_1, x'_1) \ldots\ldots \hat{\sigma}_{a_n}^{i_n i'_n}(x_n, x'_n) \times \Gamma_{h'}^{i'_1 \ldots\ldots i'_n}(x'_1, \ldots\ldots\ldots x'_n; b_1, \ldots\ldots b_n) \times dx_1 \ldots dx_n dx'_1, \ldots\ldots dx'_n \, d^2b_1, \ldots\ldots d^2b_n d^2b, \quad (10)$$

Here, $\Gamma_h^{i_1 \ldots i_n}(x_1, \ldots\ldots x_n; b_1 \ldots\ldots b_n)$ are n parton generalized distribution functions on the momentum $x$and energy Q scales at transverse b of i parton, producing final states a with subprocess cross sections $\hat{\sigma}$. $\frac{m}{n!}$ Factor takes into account the different cases final states.

follows the so-called factorized Ansatz, where the correlations between partons are neglected and longitudinal and transverse degrees of freedom are separated. The n-parton cross section cross section can be expressed as the nth product of the corresponding SPS cross sections for the production of each single final-state particles, normalized by the $(n-1)$ power of an effective cross section

$$\sigma^{NPS}_{hh' \rightarrow a_1 \ldots\ldots a_n} = \frac{m}{n!} \frac{\sigma^{SPS}_{hh' \rightarrow a_1} \ldots\ldots \sigma^{SPS}_{hh' \rightarrow a_n}}{\sigma^{n-1}_{eff,NPS}} \quad (11)$$

Where all the information on the structure of the hadron in transverse space is summarized in the value of the scale factor.

6. Cross section calculation

Table 1. The SPS and DPS cross sections for different processes at the LHC

Final State	$\sigma^{SPS}_{pp \rightarrow a+X}$	$\sigma^{DPS}_{(qq)(qq) \rightarrow ab+X}$
$\sigma\,(W^+W^-$ +X)	6 (pb)	1.06 (pb)
$\sigma\,(ZZ$ +X)	1 (pb)	0.28 (pb)
$\sigma\,(W^+W^+$ +X)	5 (pb)	3.2 (pb)
$\sigma\,(W^-W^-$ +X)	2 (pb)	1.2 (pb)
$\sigma\,(W^{\pm}H$ +X)	1 pb	1.4 (nb)
$\sigma\,(ZH$ +X)	0.5 pb	0.17 (nb)
$\sigma\,(b\bar{b}H$ +X)	2 pb	0.12 (fb)
$\sigma\,(t\bar{t}H$ +X)	380 fb	120 (fb)

Table2. The SPS and TPS cross sections for different processes at the LHC

Final State	$\sigma^{SPS}_{pp\to a+X}$	$\sigma^{DPS}_{(qqq)(qqq)\to abc+X}$
$\sigma\,(b\bar{b}$ +X)	0.50 (mb)	0.12 (μb)
$\sigma\,(WWW$ +X)	0.1 (pb)	0.28 (pb)
$\sigma\,(WWZ$ +X)	0.08 (pb)	3.2 (pb)
$\sigma\,(WZZ$ +X)	0.03 (pb)	1.2 (pb)
$\sigma\,(ZZZ$ +X)	0.008 (pb)	0.5 (pb)
$\sigma\,(ZH$ +X)	0.28 fb	0.03 (fb)
$\sigma\,(b\bar{b}H$ +X)	2 pb	0.12 (fb)
$\sigma\,(t\bar{t}H$ +X)	600 fb	120 (fb)

7. Discussion

When looking at extrapolation of the cross section at high energies, one finds that the results is effected by several uncertainties, as the knowledge of the parton structure functions at very small X and the values of the heavy quark mass and of the running coupling constant.
The study presented that Double and Triple parton scattering are a non-negligible source of perturbative particle production in pp collision at increasingly higher energyies at the LHC. The formula derived for Double, triple and n-Parton scattering yields for any final state of interest. More integrated luminosity needed for the new chanal, also higher center of mass energy would increase multiple parton scattering mechanism.

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